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Bands of localized electromagnetic waves in random collections of dielectric particles

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Abstract

Anderson localization of electromagnetic waves in random arrays of dielectric cylinders is studied. An effective theoretical approach based on the finite size scaling analysis of transmission is developed. The disordered dielectric medium is modeled by a system of randomly distributed 2D electric dipoles. The appearance of the band of localized waves emerging in the limit of an infinite medium is discovered. It suggests deeper insight into existing experimental and theoretical results.

1. Introduction

The concept of Anderson localization in solid-state physics originates from investigations of transport properties of electrons in noncrystalline systems such as amorphous semiconductors or disordered insulators. In a sufficiently disordered *infinite* material an entire band of electronic states can be spatially localized [1]. Thus for any energy from this band the stationary solution of the Schrödinger equation is localized for almost any realization of the random potential. Prior to the work of Anderson, it was believed that electronic states in infinite media are either extended, by analogy with the Bloch picture for crystalline solids, or are localized around *isolated* spatial regions such as surfaces and impurities [2].

Usually experiments related to electron localization deal with such measurable quantities as transmission, diffusion constant, or a transport coefficient (e.g., electrical conductivity). For electronic problems the natural quantity to look for is the static (dc) conductivity. Intuitively, localized states are basically bound to stay in a finite region of space for all times, whereas extended ones are free to flow out of any finite region.

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Therefore, it is natural to expect that the material in which an entire band of electronic states is localized will be an insulator, whereas the case of extended states will correspond to a conductor. In this way the phenomenon of Anderson localization may be referred to a dramatic inhibition of the propagation of an electron when it is subject to a spatially random potential. This connection is not proven on a general basis but it is certainly valid within reasonable physical models [3].

Recently disordered dielectric structures with typical length scale matching the wavelength of electromagnetic radiation in the microwave and optical part of the spectrum have attracted much attention. Propagation of electromagnetic waves in these structures resembles the properties of electrons in disordered semiconductors and many generalizations of electron localization to electromagnetic waves have been proposed [4,5,6,7,8]. A convincing experimental demonstration that strong localization could be possible in two-dimensional disordered dielectric structures has been given [9]. The strongly scattering medium has been provided by a set of dielectric cylinders randomly placed between two parallel aluminum plates on half the sites of a square lattice.

Better understanding of the Anderson localization of electromagnetic waves requires sound theoretical models. Such models should be based directly on the Maxwell equations and they should be simple enough to provide calculations without too many approximations. In this paper we investigate a simple yet reasonably realistic model describing the scattering of electromagnetic waves from a collection of randomly distributed 2D dielectric particles. The main advantage of the presented approach is that we do not need to perform any average over the disorder. Averaging of the scattered intensity over some random variable leads to a transport theory of localization [10,11,12]. However, to perform any meaningful averaging procedure the assumption of infinite medium is needed. On the other hand within our approach we can easily see how localization "sets in" for increasing number of scatterers by studying the finite size scaling of transmission.

2. Basic Assumptions

In the following we study the properties of the stationary solutions of the Maxwell equations in two-dimensional media consisting of randomly placed parallel dielectric cylinders of infinite height. This means that one (y) out of three dimensions is translationally invariant and only the remaining two (x, z) are random. The main advantage of two-dimensional localization is that we can use the scalar theory of electromagnetic waves:

$$\vec{E}(\vec{r}, t) = \mathrm{Re}\left\{\vec{e}_y\, \mathcal{E}(x, z)\, e^{-i\omega t}\right\}, \tag{1}$$

and can still try to compare, at least qualitatively, the model predictions with experimental results [9] and rigorous numerical simulations [13,14,15]. Consequently, the polarization of the medium takes the form:

$$\vec{P}(\vec{r}, t) = \mathrm{Re}\left\{\vec{e}_y\, \mathcal{P}(x, z)\, e^{-i\omega t}\right\}. \tag{2}$$

Localization of electromagnetic waves in 2D media is studied experimentally in microstructures consisting of dielectric cylinders with diameters and mutual distances being comparable to the wavelength [9]. It is a reasonable assumption that what really counts for the basic features of localization is the scattering cross-section and not the real geometrical size of the scatterer. Therefore we will represent the dielectric cylinders located at the points (x_a, z_a) by *single* 2D electric dipoles:

$$\mathcal{P}(x,z) = \sum_{a=1}^{N} p_a \, \delta^{(2)}(x - x_a, z - z_a). \tag{3}$$

Although this approximation is strictly justified only when the diameter of the cylinders is much smaller than the wavelength, in practical calculations many multiple-scattering effects have been obtained qualitatively for coupled electrical dipoles [16,17,18].

3. Metallic Waveguide

In the present model we position the 2D dipoles (3) between two infinite, perfectly conducting mirrors described by the equations $x = 0$ and $x = d$. For simplicity we consider only the case where the dipoles are oriented parallel to the mirrors. Moreover, our discussion will be restricted to the frequencies from the following range:

$$\pi < k\,d < 2\pi, \tag{4}$$

where $k = \omega/c$ is the wave number in vacuum. Thus in the planar waveguide formed by the two parallel mirrors separated by a distance d only *one* guided TE mode exists [19]:

$$\mathcal{E}^{(0)}(x,z) = \frac{2}{\sqrt{\beta\, d}} \sin(\alpha\, x)\, e^{i\beta\, z}, \tag{5}$$

where the propagation constants are given by:

$$\alpha = \frac{\pi}{d}, \quad \beta = \sqrt{k^2 - \alpha^2}. \tag{6}$$

The total field far from the dipoles is fully described by the reflection and transmission coefficients. Using the Lorentz theorem and repeating the straightforward but lenghtly calculations (see, e.g., [19]) we finaly arrive at the following expressions determining the transmission

$$\tau = 1 + i\pi\, k^2 \sum_{a=1}^{N} p_a\, \mathcal{E}^{(0)*}(x_a, z_a), \tag{7}$$

and reflection coefficients

$$\rho = i\pi\, k^2 \sum_{a=1}^{N} p_a\, \mathcal{E}^{(0)}(x_a, z_a), \tag{8}$$

for a given dipole moments p_a. In the following section we will relate p_a to the incident field $\mathcal{E}^{(0)}(x,z)$.

4. Method of Images

A simple way to reproduce the boundary conditions of parallel mirrors on the electromagnetic field is to use the method of images. This technique has been used, i.e., in QED calculations of spontaneous emission in cavities [20,21]. To reproduce the correct boundary conditions on the radiation field of each dipole (3) the mirrors are replaced by an array of image dipoles whose phases alternate in sign:

$$\mathcal{P}(x,z) = \sum_{a=1}^{N} \sum_{j=-\infty}^{\infty} (-1)^j p_a \, \delta^{(2)}(x - (-1)^j x_a - jd, z - z_a). \tag{9}$$

To use safely the point dipole approximation it is essential to use a representation for the scatterers that fulfills the optical theorem rigorously and conserves energy in the scattering processes. This requirements give the following form of the coupling between the dipole moment p_a and the electric field incident on the dipole $\mathcal{E}'(x_a, z_a)$ [22]:

$$i\pi \, k^2 \, p_a = \frac{1}{2}(e^{i\phi} - 1)\mathcal{E}'(x_a, z_a). \tag{10}$$

The field acting on the ath dipole:

$$\mathcal{E}'(x_a, z_a) = \mathcal{E}^{(0)}(x_a, z_a) + \frac{1}{2}(e^{i\phi} - 1) \sum_{b=1}^{N} G_{ab} \, \mathcal{E}'(x_b, z_b), \tag{11}$$

is the sum of the incident guided mode $\vec{\mathcal{E}}^{(0)}$ and waves scattered by all *other* dipoles and their images. In the present model the G matrix from Eq. (11) is defined by:

$$i\pi \, G_{ab} = 2 \sum_{\rho_{ab}^{(j)} \neq 0} (-1)^j \, K_0(-ik\rho_{ab}^{(j)}), \tag{12}$$

where

$$\rho_{ab}^{(j)} = \sqrt{(x_a - (-1)^j x_b - jd)^2 + (z_a - z_b)^2} \tag{13}$$

denotes the distance between the ath dipole and the jth image of the bth dipole. Eqs. (11) determine the field acting on each dipole $\mathcal{E}'(x_a, z_a)$ for a given field of the guided mode $\mathcal{E}^{(0)}(x_a, z_a)$ incident on the system. If we solve this system of linear equations and use Eqs. (10) then we are able to find the transmission and reflection coefficients given by Eqs. (7) and (8).

Note that analogous relationships between the stationary outgoing wave and the stationary incoming wave are known in general scattering theory as the Lippmann-Schwinger equation [23]. In this framework localized waves correspond to nonzero solutions of the Lippmann-Schwinger equation (or in our case Eqs. (11)) for the incoming wave equal to zero [24]. Also different arguments supporting this statement, based on the analysis of the behavior of the energy density of the field, can be elaborated [22].

5. Transmission Experiment

The actual properties of physical systems are observed experimentally not from the properties of the stationary solutions of the Maxwell equations. These are only theoretical

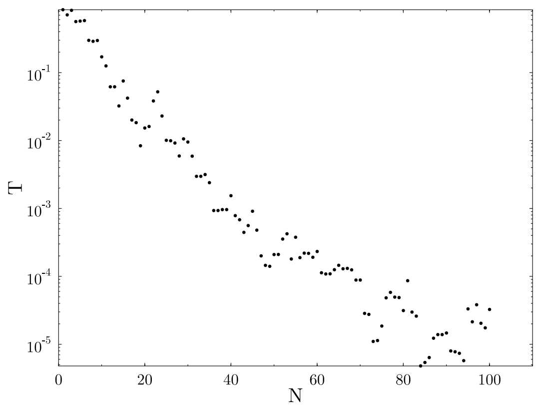

Figure 1. Transmission T of the system of dielectric cylinders described by the phase shifts $\phi = -1$ placed randomly in a planar metallic waveguide ploted as a function of the number of cylinders N.

tools. Experiments deal rather with *measurable* quantities. For many practical problems, a natural quantity to look for is the transmission $T = |\tau|^2$ of a *finite* system of characteristic size L and its dependence on L. Usually propagation of electromagnetic waves in weakly scattering random media can be described by a diffusion process. Thus the equivalent of Ohm's law holds and the transmission decreases linearly with the size of the system $T \propto L^{-1}$. When the fluctuations of the dielectric constant become large enough, due to interference the electromagnetic field ceases to diffuse and becomes localized. The Anderson transition can be best observed in the transmission properties of the system. In the localized state the transmission decreases exponentially with the thickness of the sample $T \propto e^{-L/\xi}$ [5].

As a simple example let us consider a system of N cylinders placed between the mirrors separated by a distance $k\,d = 3\pi/2$. The cylinders were distributed randomly with constant uniform density $n = 1$ cylinder per wavelength squared. Therefore for each N the size of the system is proportional to the number of cylinders $L \propto N$.

In Fig. 1 we present on the log-log plot the transmission T as a function of the number of cylinders N. It follows from inspection of this figure that $T \propto e^{-L/\xi}$. This proves that in the limit $L \to \infty$ our system indeed goes into a localized state where it behaves as an optical insulator.

Let us imagine a typical configuration of an *infinite* medium exhibiting Anderson localization. When we consider some finite part of length L of this system, as we are investigating in the case of a transmission experiment, the localized modes become *reso-*

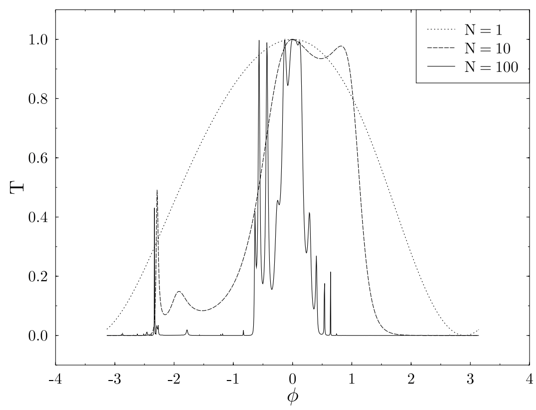

Figure 2. Transmission T of the system of N dielectric cylinders placed randomly in a planar metallic waveguide plotted as a function of the phase shift of a single cylinder ϕ.

nances. These resonances can help the wave to tunnel through the system for frequencies near to those of the localized modes of the infinite medium. This will lead to specific, sample dependent values of the phase shifts ϕ_j, for which we have maxima of transmission.

To illustrate this statement, in Figure 2 we have plotted the transmission T as a function of the phase shift of a single cylinder ϕ. We see that for sufficiently large N incident waves are totally reflected for almost any ϕ except the discrete set $\phi = \phi_j$ for which the transmission is close to unity.

It is reasonable to expect that in the case of a *random* and *infinite* system, a countable set of phase shifts ϕ_j corresponding to localized waves becomes dense in some finite interval. Therefore, an entire *band* of spatially localized electromagnetic waves appears. Anderson localization occurs when this happens. Physically speaking this means that different realizations of sufficiently large system of randomly placed cylinders are practically (i.e., by a transmission experiment) indistinguishable from each other. E.g., for a certain realization of a random and infinite one-dimensional system one can prove mathematically [25] that incident waves are totally reflected for "almost any" energy, i.e., except the discrete set (of zero measure) for which the transmission is equal to unity. This dense set of energies exceptional in the Furstenberg theorem [26] corresponds to the band of localized waves.

According to the scaling theory of localization [27], the dimension of the disordered medium is a crucial parameter. In one and two dimensions any degree of disorder will lead to localization, while in three dimensions a certain critical degree of disorder is needed before localization will set in. Our calculations do not exclude the possibility that in an *infinite* 2D medium the band of localized waves may appear even for small ϕ. However, in all experiments we can investigate only systems confined to certain *finite* regions of space. As follows from Fig. 2 (dealing with finite media), with increasing size of the system the band of localized waves appears *faster* for $|\phi| \simeq \pi$, than for other values of ϕ. This means that the scattering cross-section of individual scatterers $k\,\sigma = 2(1-\cos\phi)$ should be made maximal (for example by tuning the frequency to match the internal resonances of the cylinders). Let us stress that the situation can be different for 3D random media.

6. Summary

In summary, we have further developed and refined a quite realistic coupled-dipole model describing scattering of electromagnetic waves by a disordered dielectric medium. Its relative simplicity allowed us to discover some new features of the Anderson localization of electromagnetic waves in 2D dielectric media without using any averaging procedures. Within our theoretical approach one can easily see how localization "sets in" for increasing size of the system. For the first time (to our knowledge) the appearance of the band of localized electromagnetic waves in 2D was demonstrated. Connection between this phenomenon and a dramatic inhibition of the propagation of electromagnetic waves in a spatially random dielectric medium has been sketched. It can be understood as a counterpart of Anderson transition in solid state physics.

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