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The concept of free electromagnetic field in quantum domain

Alexander SHUMOVSKY* **and Özgür MÜSTECAPLIOĞLU**
Physics Department, Bilkent University
06533 Ankara-TURKEY

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Abstract

By virtue of the consideration of polarization and phase properties of dipole radiation in the quantum domain, it is shown that the concept of free electromagnetic field should be considered as a quite risky approximation in the description of quantum fluctuations of some physical observables.

1. Introduction

For a long time, the electromagnetic theory was considered as an exemplary physical theory both in classical and quantum domains because of its perfect logical structure and excellent agreement with the experiments. Nevertheless, as we know from history it is the fate of any physical theory that, after some time of success, difficulties or limitations of its applicability become apparent and lead to the necessity of revision of some aspects which had been initially considered as the "dogmas of faith". In our opinion, electromagnetic theory does not represent an exception to the rules.

In spite of the fact that no one observed an electromagnetic field which had not been generated by a source, the concept of *free electromagnetic field* or *pure radiation field* is widely used both in classical and quantum theories. This concept is based on the existence of nontrivial solution of the homogeneous wave equation formed by superposing transverse waves and representing the transport of energy from one point to another. In some textbooks one can meet the statements such as the following:
"*These equations* [the Maxwell equations in the absence of sources] *possess nonzero solutions. This means that an electromagnetic field can exist even in the absence of any charge.*" [1].

*e-mail: shumo@fen.bilkent.edu.tr

The concept of free field plays the key part in the definition of the field energy, field momentum, field angular momentum, polarization etc. (e.g., see [1,2]). Just the expansion of the vector potential of free electromagnetic field in terms of transverse waves leads to the definition of the dynamical variables and canonical conjugate momenta within the conventional scheme of field quantization (see [3,4]). Nevertheless, this concept seems to be inadequate to some properties of the electromagnetic field which can be observed. In the present paper we argue this statement by the consideration of polarization and quantum phase of light.

2. Polarization of electromagnetic radiation

The polarization properties of electromagnetic field are determined by slowly varying bilinear forms with respect to the complex field amplitudes, forming a Hermitian tensor of polarization [1,5]. In the case of a free electromagnetic field, this is a tensor of rank 2 with three independent components which are specified by the conventional set of Stokes parameters determined either in the linear polarization basis or in the circular polarization basis [5]. The quantum counterpart is provided by the Stokes operators which can be obtained from the classical Stokes parameters with the aid of standard quantization of the field amplitudes [6].

At the same time, within quantum optics, treating the electromagnetic radiation as a beam of photons, the polarization is determined as a given spin state of the beam [7]. The spin of a photon is defined as the minimum value of the angular momentum and is equal to 1. Thus, it should have three projections, two of those correspond to the circular polarizations with opposite helicities while the third one describes the linear longitudinal polarization.

In many texts on quantum electrodynamics and field theory one can meet the statement that spin of a photon has only two projections ± 1 and that this fact reflects the transversality of classical free electromagnetic field (e.g., see [7]).

However, it is well known that even the simplest classical dipole radiation always has the longitudinal component in addition to the transversal components [2]. In the case of electric dipole radiation, for example, the magnetic filed is always orthogonal to the direction of propagation while the electric field has both the transversal and longitudinal components [2]. Similarly, the magnetic dipole radiation has the longitudinal component of magnetic field. Since the longitudinal component has an intensity noticeable in some vicinity of the source and decaying much faster than the transversal components, the dipole radiation can be **approximated** by the transversal field but at far distance only.

In turn, the quantum description of dipole radiation is based on the selection rules according to which the angular momentum of an excited state must exceed the angular momentum of a ground state by a unit $j \to j' = j - 1$ [7]. Due to the conservation of angular momentum in the process of radiation this unit is taken away by a photon. Moreover, the photon takes away the projections of the angular momentum as well.

Therefore, it seems to be unreasonable to neglect the longitudinal component a priori. It is natural to first determine the polarization properties of the dipole radiation, taking into consideration all possible components, and then to find the limits of applicability of the free field approximation.

In the case of dipole radiation with three components, the Hermitian tensor of polarization has the rank 3 and contains five independent components [8,9]. The set of five generalized Stokes parameters can be obtained in a standard way as follows [9]:

$$\begin{array}{lcl} s_0 & = & \sum_{m=-1}^{1} |\vec{E}_m^* \cdot \vec{E}|^2 \\ s_1 & = & \Re \sum_m (\vec{E}_m^* \cdot \vec{E})(\vec{E}_{m+1}^* \cdot \vec{E}) \\ s_2 & = & \Im \sum_m (\vec{E}_m^* \cdot \vec{E})(\vec{E}_{m+1}^* \cdot \vec{E}) \\ s_3 & = & |\vec{E}_-^* \cdot \vec{E}|^2 - |\vec{E}_+^* \cdot \vec{E}|^2 \\ s_4 & = & |\vec{E}_+^* \cdot \vec{E}|^2 + |\vec{E}_-^* \cdot \vec{E}|^2 - 2|\vec{E}_0^* \cdot \vec{E}|^2 \end{array} \tag{1}$$

where

$$\vec{E} = \sum_{\lambda jm} a_{\lambda jm} \vec{E}_{\lambda jm} + c.c., \qquad a_{\lambda jm} = \int \vec{E}_{\lambda jm}^* \cdot \vec{E} r^2 dr d\Omega, \tag{2}$$

and $\vec{E}_m \equiv \vec{E}_{\lambda 1m}$ is the λ-type (electric or magnetic) dipole term in the multipole expansion of the complex electric field amplitude. In the limit of $\vec{E}_0 \to 0$ the equations (1) clearly coincide with the conventional Stokes parameters determined in the circular polarization basis. In turn, the generalized Stokes operators can be obtained from (1) by quantization in the representation of spherical photons [3] as follows:

$$\begin{array}{lcl} S_0 & = & \sum_m \hat{a}_m^+ \hat{a}_m \\ S_1 & = & \frac{1}{2}[(\hat{a}_+^+ + \hat{a}_-^+)\hat{a}_0 + \hat{a}_-^+ \hat{a}_+ + h.c.] \\ S_2 & = & \frac{-i}{2}[(\hat{a}_+^+ - \hat{a}_-^+)\hat{a}_0 + \hat{a}_-^+ \hat{a}_+ - h.c.] \\ S_3 & = & \hat{a}_-^+ \hat{a}_- - \hat{a}_+^+ \hat{a}_+ \\ S_4 & = & \hat{a}_+^+ \hat{a}_+ + \hat{a}_-^+ \hat{a}_- - 2\hat{a}_0^+ \hat{a}_0 \end{array} \tag{3}$$

One can see that $[S_1, S_2] = 0$ and therefore these two parameters can be measured at once. It is possible to show [10,11] that these two parameters describe respectively the cosine and sine of the azimuthal phase of the angular momentum of radiation. The set (2) can also be obtained directly from the conservation of total angular momentum in the atom+radiation system [11].

Let us try to neglect the longitudinal component contribution into (2). Then the operators S_1, S_2 are transformed into

$$\begin{array}{lcl} S_1^T & = & \frac{1}{2}(\hat{a}_-^+ \hat{a}_+ + \hat{a}_+^+ \hat{a}_-) \\ S_2^T & = & \frac{-i}{2}(\hat{a}_-^+ \hat{a}_+ - \hat{a}_+^+ \hat{a}_-), \end{array} \tag{4}$$

which clearly coincides with the conventional Stokes operators obtained by quantization of the Stokes parameters of free electromagnetic field (e.g., see [12]). Now we get

$$[S_1^T, S_2^T] = \frac{i}{2}(\hat{a}_-^+ \hat{a}_- - \hat{a}_+^+ \hat{a}_+) \neq 0 \tag{5}$$

so that S_1^T and S_2^T cannot be measured at once. Thus, the neglect of the contribution of longitudinal component in the quantum domain leads to the change of the algebraic properties of Stokes operators. Let us note that the classical counterpart of (3) determine the cosine and sine of the phase difference between two circularly polarized components of free field [2,5]. Therefore, one might expect the commutativity of S_1^T and S_2^T. In fact, the definition of the quantum phase difference in terms of the conventional Stokes operators is based on a quite refined but unrigorous approach [13].

Let us stress that the difference in algebraic properties of the Stokes operators (2) and (3) causes an important effect independent of the distance from the source. Suppose that the radiation is considered in the far zone where the state of longitudinal component can be approximated by the vacuum state. Suppose for simplicity that both circularly polarized components are in the coherent states. It is a straightforward matter to arrive at the following relations [8]:

$$\langle S_l \rangle = \langle S_l^T \rangle = 2|\alpha_+ \alpha_-| \begin{cases} \cos \Delta_{+-} & l = 1 \\ \sin \Delta_{+-} & l = 2. \end{cases} \tag{6}$$

Here α_m is the parameter of corresponding coherent state and $\Delta_{mm'} = \arg \alpha_m - \arg \alpha_{m'}$. Thus, in the far zone, both definitions describe one and the same expectation value of the Stokes operators. At the same time, for the variances we get

$$\begin{array}{rcl} V(S_1^T) & = & |\alpha_+|^2 + |\alpha_-|^2 \\ V(S_1) & = & 2(|\alpha_+|^2 + |\alpha_-|^2 + |\alpha_+ \alpha_-| \cos \Delta_{+-}) \end{array} \tag{7}$$

Hence, the quantum fluctuations of S_1 are much stronger than that for S_1^T. Moreover, they are qualitatively different because of dependence on the phase difference Δ_{+-}. Similar result can be easily obtained for S_2 and S_2^T.

Thus, the longitudinal component of a dipole radiation strongly influences the quantum fluctuations, even in the case when this component does not contribute into the total intensity. In other words, the use of the free field approximation leads to incorrect results for the polarization properties of the dipole radiation in the quantum domain.

3. Quantum phase of radiation

The existence of a well-behaved Hermitian quantum phase operator is a controversy surviving from the early days of quantum mechanics (e.g., see [14]). The recent analysis of various schemes of phase measurements has shown that there is no unique phase operator at all [15]. Actually, the possible phases can be divided into two classes - the "geometrical" phases, depending on the concrete scheme of measurement, and the "intrinsic" phase which is the quantum property of a photon per se. Since the vacuum state of electromagnetic field should have a uniform phase distribution, we might imagine that the phase properties of radiation are obtained in the process of generation. Actually, there is one-to-one correspondence with the situation, well known in the quantum statistical mechanics and quantum field theory as quasi-averaging [16]. The degeneration of a quantum state is registered by interaction with some physical "source". The interaction of the vacuum field with an excited atomic transition registers the phase degeneration. Then, the quantum phase of radiation can be determined via the conservation laws, corresponding to the process of generation [10]. Energy conservation cannot be responsible for the transmission of the "phase information" from the source to the radiation because of its scalar nature. Linear momentum conservation might be responsible for the geometrical phase only. Therefore, angular momentum conservation should be examined as the most plausible candidate.

The phase properties of the atomic transition can easily be determined by the polar decomposition of $SU(2)$ algebra, describing the angular momentum of the excited state [10,11]. At the same time, the $SU(2)$ sub-algebra in the Weyl-Heisenberg algebra, describing the angular momentum of the radiation field, has no isotype representation. In other words, the envelope algebra has not a uniquely determined unit operator in the whole Hilbert space. Therefore, the direct polar decomposition of the angular momentum of a photon is impossible. To avoid this difficulty, we introduce the radiation cosine and sine operators as complements of the corresponding atomic operators with respect to the integrals of motion for the standard Hamiltonian of atom-filed interaction [10]. Then, the "radiation" cosine and sine operators of the azimuthal phase are defined as follows [10]:

$$C_R = KS_1, \qquad S_R = KS_2, \tag{8}$$

where $S_{1,2}$ are the operators (2). The normalization constant K is determined by the condition [8,9]

$$\langle C_R^2 + S_R^2 \rangle = 1. \tag{9}$$

In the case of a single-atom radiation $K = 1$ [11]. It is not hard to see that the definition (7) leads to the natural behavior of expectation values and variances both in the vacuum state and in the classical limit [9]. At the same time, it leads to a new effect which cannot be described within the framework of approaches based on consideration of free electromagnetic fields.

Suppose for simplicity that the longitudinal mode $m = 0$ is in the vacuum state while two transversal modes are in the coherent states with low intensities. Suppose that one of the intensities, say $I_- = |\alpha_-|^2$, is fixed while the second one can be varied. We have

$$V(C_R) = \frac{I_+ + I_- + \sqrt{I_+ I_-}\cos\delta_{+-}}{2(I_+ + I_- + \sqrt{I_+ I_-})}. \tag{10}$$

Clearly, $V(C_R) \to 1/2$ at $I_+ \to 0$ and fixed I_-. Under the condition $1 \geq \cos\Delta_{+-} > \sqrt{I_+ I_-}$ which can be realized in the strong quantum case of low intensities, the value of the variance (8) can exceed 1/2 at some I_+. The maximum of (8) can be achieved at

$$I_+ = I_- \left[\frac{\sqrt{I_-^2 + (1 + I_-)\cos\Delta_{+-}} - I_-}{(1 + I_-)\cos\Delta_{+-}} \right]. \tag{11}$$

This effect can be explained qualitatively in the following way [17]. At $I_+ = 0$ there is a uniform probability distribution in the system. In other words, because $\arg\alpha_+$ can have any value with one and the same probability, the difference Δ_{+-} is indefinite. Creation of the first photons of the mode $m = +1$ leads to the formation of some domains with almost equal probabilities to have the phase difference Δ_{+-} and $\Delta_{+-} + \pi$. Further increase of I_+ leads to formation of more or less sharp probability distribution centered at Δ_{+-}. In analogy to well-known photon bunching effect one can say that there is the "phase bunching" in the case under consideration. Let us stress that this effect cannot be described within the approaches based on the consideration of either purely transversal free field or approximations connected with the use of the finite sub-spaces of the Hilbert space [17].

4. Summary

Thus, the above considered examples show that it is necessary to be very careful with the use of free electromagnetic field, at least, in the quantum domain. In particular, the quantum properties of polarization, as well as the behavior of the azimuthal phase of the angular momentum (the radiation phase), are very sensitive to the presence of the longitudinal component of radiation field connected with the $m = 0$ projection of the photon spin. In other words, the free electromagnetic field should be considered as a quite risky approximation in the consideration of quantum fluctuations of some properties of a real electromagnetic radiation.

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