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Gauge theory of phase and scale

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Abstract

Old Weyl's the idea of scale recalibration freedom and Infeld's and van der Waerden's (IW) ideas concerning geometrical interpretation of natural spinor phase gauge symmetry are discussed in the context of modern models of fundamental particle interactions. It is argued that (IW) gauge symmetry can be naturaly identified with the $U(1)$ symmetry of the Weinberg-Salam model. It is also argued that there are no serious reasons to reject Weyl's gauge theory from consideration. Its inclusion enriches the original Weinberg-Salam theory and leads to prediction of new phenomena that do not contradict experiments.

1. Introduction

Gauge theories are fundamental tools in the contemporary physics of particles and their interactions. The standard Model of fundamental interactions (SM) reasonably describes particle physics at present accelerator energies via quantum gauge theory of $U(1) \times SU(2) \times SU(3)$ symmetry group of electroweak and strong forces. The model leads to cosmological scenarios that seems to be consistent with observational astrophysics. There are many extensions and modifications of SM. A gauge theory schema is at the base of all of them. These gauge theories are in fact gauge theories of generalized phase of spinorial field multiplets. All of them are formulated in flat space time but are argued (and sometimes proved) to be generalizable (at least locally) to an arbitrary Riemann space.

The first consistent formulation of $U(1)$ gauge theory of spinor phase in curved space was given soon after Dirac's theory was proposed. This model is reviewed shortly in section 2. The notion of gauge symmetry is even older. It was introduced by Weyl before the notion of spinors had been defined. Today we can call this theory a gauge theory

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of scale. A short review is presented in section 3. Both early gauge theories of phase and scale were based on abelian gauge groups and was to incorporate electromagnetism into the geometrical scheme of general relativity. Those attempts were admitted to be unsuccessful. The reasons and arguments for that are shortly reviewed in section 4 where a critical discussion of those arguments is also given. In section 5 a general model with scale and phase gauge symmetry is described. Its features and physical consequences are discussed in section 6.

2. Gauge theory of phase in Infeld and van der Waerden formulation.

Soon after the appearance of Dirac's theory of quantum relativistic electron in the flat space [1] the general relativistic extension of Dirac's theory was also proposed [2, 3, 4]. A canvas for such description is a four dimensional manifold M. A copy of two dimensional complex vector field F_pM is attached to every point p of M. In principle, independent pairs of affine and metric structures can be implemented on M. The tangent boundle TM can be equipped with an affine connection Γ and the field of metric g. Independently, a connection γ can be defined in the bundle FM. For generic two dimensional complex vector space there is a natural class of antisymmetric Levi-Civita metrics that differ by a complex factor. Thus arbitrary field ε of Levi-Civita mertic can be chosen at FM. The important observation is that the Levi-Civita metric ε induces Lorentz metric $\varepsilon \otimes \overline{\varepsilon}$ at every fiber of $FM \otimes \overline{FM}$ (see e.g. [5] for definition of complex conjugation structure and for further details). Thus the real part of $FM \otimes \overline{FM}$ (which is a four dimensional real vector bundle which we denote as $F\bar{F}M$) can be naturally related with TM, the tangent vector bundle of M.

In Einstein's general relativity theory the affine and metric structures of TM are related by the metricity condition

$$\nabla g = 0, \tag{1}$$

and the torsion free condition

$$\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu} = T^{\lambda}_{\mu\nu} = 0. \tag{2}$$

Keeping those restrictions and relating TM with $F\bar{F}M$, Infeld and van der Waerden [4] found that the metric structure ε of FM is given by the metric structure g of TM up to an arbitrary phase factor, while the affine structure γ of FM is given by the affine structure Γ of TM up to an arbitrary vector field. It is clear that this new field is a compensating potential for the $U(1)$ local symmetry group of phase transformations of all Dirac fields in the theory. The authors have identified this new field with electromagnetic potential.

3. Gauge theory of scale: conformal Weyl's model.

The Infeld and van der Waerden model was not the first example of gauge theory. The idea and notion of gauge invariance was introduced by Weyl [6] as a consequence of natural generalization of Riemann geometry. Weyl assumed that the metricity condition

(1) can be replaced by a less restrictive condition

$$\nabla g_{\mu\nu} \sim g_{\mu\nu}. \tag{3}$$

Thus he supposed that, for a vector transported around a closed loop by parallel displacement although the direction and the length change, the angle between two parallelly transported vectors must be conserved.

If Einstein's torsion free condition (2) is kept, then again there is a renewed relation between the metric and the affine structures of TM, however the connection is not uniguely given by the Christophel symbol. It depends also on an arbitrary vector field. This field is the compensating potential for the local gauge group of length changes for all dimensional fields in the theory. Originally Weyl interpreted this new field as an electromagnetic vector potential. He soon abandoned both the electromagnetic interpretation and the whole idea that his new symmetry (called the conformal symmetry, as it conserves angles) plays any role in physics.

4. Abandoned models.

Both models - the Weyl's gauge theory of scale and the Infeld and van der Waerden gauge theory of phase - despite their geometrical beauty, were abandoned for physical reasons.

Weyl's theory that in which there is freedom for the space-time dependent choice of length standards was rejected on the argument that it clashes with quantum phenomena that provide an absolute standard of length. Thus, at least, there is no need for the arbitrary metric standards of Weyl's theory. On the other hand, the electromagnetic interpretation of this theory seemed not to be satisfactory by itself (see, however, [7]).

The physical reasons for rejection of Infeld and van der Waerden interpretation of their vector potential as a medium for electromagnetic interaction was the fact that the obtained potential couples universally to all fermions. Thus it should also couple to neutrinos that are electrically neutral Dirac particles. Consequently the $U(1)$ gauge symmetry of fermion phase should be considered as a possibly new independent gauge symmetry. As there is no other long range interactions observed in nature except electromagnetism and gravity, it is assumed that such a gauge interaction is not realized or is extremely small.

Let us critically revise all the arguments mentioned above.

There were two kinds of arguments against Weyl's theory. The first laid a contradiction between the theory and quantum phenomena. Those convictions are mostly based on a misunderstanding or misinterpretation of Weyl's gauge symmetry. In fact, the freedom to set arbitrary length standards along an atomic path does not mean that atomic frequencies will depend on the atomic histories, was the most popular argument in early literature. In Weyl's theory atomic frequency depends on the length standard at a given point but simultaneously all other dimensional quantities measured at this point depend on this standard in the same way. There is no contradiction with experiment as dimensionless ratios are standard and of course do not depend on the history of a particular atom.

More serious arguments against Weyl's theory were based on the reasonable claim that, any acceptable theory should not introduce needles objects and notions. If atomic clocks measure time in an absolute way and velocity of light is an absolute (or at least definite) physical quantity then the relativism of length is unnatural and redundant. Argument is very reasonable except one subtle question: Which atomic clock provides the absolute time and length standard? The fast answer is: ALL! But here, further problems begin. We know form our "almost flat" experience that "free atomic clock" frequency ratios are external independent conditions; should we really extrapolate those experience to all conditions and times? A naive extrapolation could be evidently wrong as we know from solid state physics. We can imagine very strong sources of gravity producing such extremal conditions that neither known atomic nor quantum clocks will exist there. And what about the radiation age of Universe when there was no matter at all? Observe that neither so called "distant" nor "isolated" standards are helpful in the case of gravity as there is no screening of interactions. Of course we are free to assume that - roughly speaking - the ratios of electron mass to proton mass and to other quantum standards are always and everywhere the same, but we should remember (especially when we interpret such effects like red shift or other distant signals) that this is only our assumption and it could be and it should be a subject to experimental verification. Weyl's theory does have room to relax from such, if not definitely confirmed, suppositions. Can we judge *a priori* that it is really needless?

The arguments formulated against Infeld's and van der Waerden's interpretation of a vector gauge potential (the potential that arises when the affine structure of a tangent bundle is extended to spinor bundle) are based on the fact that neutrinos are chargeless. Those arguments were important before Weinberg-Salam theory (WS) had been proposed. WS predicts that all fermions couple to $U(1)$ gauge field. There is a second nonabelian gauge group $SU(2)$ in the theory acting only on left components of Dirac bispinors. Due to the structure of couplings and the effective mass matrix for gauge bosons the massless field - naturally identified with photon - is a combination of original $U(1)$ and $SU(2)$ bosons. It does not couple to neutrinos despite the fact that the original abelian vector potential does. Thus we are free to identify the Infeld - van der Waerden potential with $U(1)$ gauge group potential of the WS model without any conflict with theory and experiment.

We see that the arguments raised against Weyl's and Infeld - van der Waerden models are not ultimately and definitively convicting. On the other hand both theories realize in a sense an old and beautiful idea that physical interactions should be ascribed to geometrical properties of space itself, instead of being merely something embedded in space. Observe that both these theories are complementary and correlated. The Weyl potential can be raised to spinorial level according to Infeld and van der Waerden prescription. Then it can be collected (together with the derivative of $\log|\det\varepsilon|$) to be the real part of a complex vector potential that has an imaginary part found by Infeld and van der Waerden [8]. The Infeld - van der Waerden correlation between geometrical structures of TM and FM

leads immediately to Weyl's conformal metricity condition (3). This result is independent of any assumption about relation between metrical and affine structures of TM (e.g. is independent of (2)) and follows only from the fact that metric g is related to spinorial metric ε by an arbitrary Infeld - van der Waerden relation which, for selfconsistency of the model, must be covariantly constant. Thus the correlations between gauge theories of phase and scale are rich and universal.

5. Classical gauge theory model of phase and scale.

Despite the controversies around the geometrical origin of $U(1)$ gauge theory of fermion phase, its role in physics is not disputed. The conformal gauge theory of scale is less lucky but many authors none the less return to Weyl's original ideas in various contexts (see.g. [9] and also [7, 10, 11, 12]). Let us write down a general model respecting both those symmetries. But first let us fix the notation.

Weyl's potential will be denoted by S_μ. Then, if torsion free condition (2) is assumed, the connection is given by

$$\Gamma^\rho_{\mu\nu} = \{^{\rho}_{\mu\nu}\} + f(S_\mu g^\rho_\nu + S_\nu g^\rho_\mu - S^\rho g_{\mu\nu}), \tag{4}$$

where f is an arbitrary coupling constant. (In principle it could be absorbed at this level by redefinition of S_μ, but it is convenient to keep it here and set its value later.) Consequently,

$$\nabla_\mu \hat{g} = -2fS_\mu \hat{g}. \tag{5}$$

Equations (4) and (5) are invariant with respect to Weyl transformations

$$g_{\mu\nu} \to \Omega^2 g_{\mu\nu} = e^{2\lambda} g_{\mu\nu} \tag{6}$$

$$S_\mu \to S_\mu - \frac{1}{f}\partial_\mu \lambda. \tag{7}$$

Thus the metric tensor is covariant with respect to Weyl transformations with degree 2. The Riemann and Ricci tensors constructed from (4) are conformally invariant objects but the scalar curvature R is not. R can enter linearly to a conformally invariant expression of dimension of action if it is combined with a scalar field ϕ which transforms according to

$$\phi \to e^{-\lambda}\phi. \tag{8}$$

Then the combination $\phi^2 R$ is conformally invariant. The conformal covariant derivative of ϕ is given by

$$\nabla_\mu \phi = (\partial_\mu - fS_\mu)\phi \tag{9}$$

and it transforms according to (8).

The most general conformally invariant Lagrangian that leads to second order equations of motion for the metric-Weyl-scalar system reads [10]:

$$L_g = -\frac{\alpha_1}{12}\phi^2 R + \frac{\alpha_2}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{\alpha_3}{4}H_{\mu\nu}H^{\mu\nu} - \frac{\lambda}{4!}\phi^4, \tag{10}$$

where

$$H_{\mu\nu} = \partial_\mu S_\nu - \partial_\nu S_\mu. \tag{11}$$

The coupling constants α_1, α_2 and α_3 are arbitrary but the last two can be absorbed in ϕ and S_μ by a suitable redefinition of the fields. Observe however, that we are not able to absorb simultaneously α_3 and f. Thus the last coupling remains arbitrary and has to be fixed by experiment.

Now we can include fermions. First we should recall [8] that Weyl's vector potential S_μ do not couple directly to Dirac fermions if they transform according to the rule

$$\Psi \to e^{-\frac{3}{2}}\Psi. \tag{12}$$

If we want to fit to the SM prescription we must admit that except for the $U(1)$ gauge symmetry group of fermion phase (for which we shall denote the gauge potentiaby B_μ) other internal nonabelian gauge symmetry groups are also present in the model. The scalar field ϕ that had been introduced in (10) can be extended to a complex scalar multiplet. The ordinary derivative in (9) must be replaced by $D_\mu = \nabla_\mu - ieB_\mu + ...$ being the convariant derivative with respect to $U(1)$ (we assume that it couples universally to the phase of ϕ) and with respect to some other internal symmetry groups:

$$\nabla_\mu \phi = (D_\mu - fS_\mu)\phi. \tag{13}$$

Thus the curvilinear versions of Dirac Lagrangian L_Ψ and Yukawa Lagrangian L_Y can be easily written. We can also select Maxwell Lagrangian $L_B = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ for $U(1)$ vector potential and a general Yang-Mills lagrangian L_V for other gauge potentials. As there are two abelian gauge groups in the model the mixed term

$$L_{SB} = \alpha_4 H_{\mu\nu}F^{\mu\nu} \tag{14}$$

obeys all symmetries and must be admitted. The total Lagrangiam can be written as a sum of these terms:

$$L_T = L_g + L_\Psi + L_Y + L_B + L_V + L_{SB}. \tag{15}$$

6. Discussion.

The theory given by (15) has interesting properties that depend on the value of coupling constants α_i. But its special property is its conformal gauge invariance. If a gauge theory is to be solved, some additional gauge fixing conditions must also be supposed in order to make the evolution definite. This choice is arbitrary within the whole class of gauge equivalent conditions. The physical results are gauge independent. There are customary procedures to handle this freedom in the case of gauge symmetry of generalized phase. Theary case of scale gauge symmetry is specially interesting. The dimensional scale reference can be chosen arbitrary but it is reasonable to choose it in a way that is most practical and convenient. If we are focused on laboratory phenomena where gravitational effects are negligible then there is no reason to doubt the universality of length standards provided by the whole class of quantum phenomena (please recall the discussion of section 4). We are free to chose the length standards that lead to constant, space independent particle masses. If theory (15) is a conformal modification of SM then the conformal gauge fixing condition that provides correspondence with the ordinary description is the condition [11, 12]

$$|\phi|^2 = v^2 = (246GeV)^2. \tag{16}$$

It leads to a mass spectrum that is the same as obtained from the mechanism of spontaneous symmetry breaking in WS, but those mechanism is absent in the minimal version of conformal theory. As a result Weyl's vector field S_μ acquires mass

$$m_S^2 = \frac{1}{2} f(\alpha_2 - \alpha_1) v^2 \tag{17}$$

which is equal zero only in the special case when $\alpha_2 = \alpha_1$ and an additional symmetry is realized in the model.

The striking feature of the described theory is the lack of ordinary Einstein term in (15). Observe however, that with condition (16) this term can be easily reproduced [11]. It is sufficient to demand that

$$-\frac{\alpha_1}{12} v^2 = \frac{1}{8\pi G}. \tag{18}$$

It leads to the Weyl vector mass

$$m_S = 0.5 \cdot 10^{19} f \cdot GeV. \tag{19}$$

It was already mentioned that in the case $\alpha_2 = \alpha_1$ the model has an additional symmetry. The Weyl potential decouples from scalar field and, if $\alpha_4 = 0$, it is coupled only to gravity. Transformations (6), (8) and (12) are the symmetries of the theory independent of (7). We get Penrose-Chernikov-Tagirov teory of scalar field conformally coupled to gravity [13]. We are free to further include other terms in (15) that respects the new symmetry. Thus, despite the fact that the coefficient in front of R in the original Lagrangian is negative we are able to reproduce the appropriate Newtonian limit for the whole theory [12, 14].

The very new feature of Lagrangian (15) is the mixed term (14) that leads to interaction of Weyl and $U(1)$ vector potentials. At quantum level it would result in a mixing of Weyl boson with photon and weak bosons - the effect in a sense similar to the known $\gamma - Z$ mixing. As the mass of S_μ and the coupling α_4 is not predicted by the theory the strength of the mixing effect could be small as well as very large. Also, the mass m_S cannot easily be estimated from known data as there is no interaction of fermions with the Weyl potential. Thus definite answers concerning the presence and interactions of Weyl potential should be examined in experiments.

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