Tr. J. of Physics
22 (1998) , 1139 – 1147.

Coulomb Breakup of Nucleus ^{6}Li on Ion ^{208}Pb

B. F. IRGAZIEV, H. T. ERGASHBAEV
Department of Theoretical Physics,
Tashkent State University,
Tashkent 700095, UZBEKISTAN

Received 25.09.1997

Abstract

In the framework of the three-body approach the $A(a, bc)A$ Coulomb breakup has been investigated. The three-body Coulomb dynamic is taken into account to derive the expression for the reaction matrix element. The mechanism of the breakup includes the direct process and the excitation of resonance state of the particle a. The calculation of the triple differential cross section of the $^{208}Pb(^{6}Li, \alpha d)^{208}Pb$ Coulomb dissociation have been performed in the energy region $E_{\alpha d} < 1MeV$. Calculations for the Coulomb dissociation $^{208}Pb(^{6}Li, \alpha d)^{208}Pb$, including consideration of the triple cross section going through the first resonance of ^{6}Li have been performed. The results of the calculations are compared with experimental data.

1. Introduction

From theoretical and experimental investigations there is now the extensive information for the Coulomb breakup of light nuclei on heavy ions [1-5]. Similar studies have been stimulated by the opportunity to extract the astrophysical S-factor. However, there is the question of the influence of the three-body Coulomb dynamic on the energy dependence of the differential cross section in the final and intermediate states [6-9]. In addition, the reaction going through resonance close to threshold has attracted special attention. Influence of the three-body Coulomb effects on the energy dependence of the differential cross section near resonance was investigated in Ref. [6]. However, in the final expression the Coulomb interaction in an asymptotic approach was carried out in Ref.[8] for reactions $A(x, cy)B$. In that investigation the analytical formula describing the influence of the Coulomb interaction between the spectator-particle and the resonance in the intermediate state and the Coulomb rescattering of three particles in the final state to the absolute value of the differential cross section is obtained. In Ref.[9] for a short-lifetime resonance the contribution of the polarization potential in the eikonal approach is taken into account. The authors of Ref.[4] note that there is some quantitative suppression of the acceleration of particles in the final state when a projectile transits to a resonance state of narrow width. Such conclusion demands careful analysis. The investigation of the $E1$ and $E2$ electric transitions for the Coulomb dissociation of nuclei is of special interest for the case when the $E1$ transition is suppressed accordingly to the isotopic spin [10].

In the present work the Coulomb dissociation of a light particle in the field of a heavy multicharged ion is investigated. Concrete numerical calculations are reported for the reaction $^{208}Pb(^{6}Li, \alpha d)^{208}Pb$. The results of the calculations are compared with the experimental data from Ref.[2].

2. The general formula

Let us consider the reaction in the framework of a model of charged particles 1, 2 and 3,

$$(12) + 3 \to (12)^* + 3 \to 1 + 2 + 3, \tag{1}$$

where (12) is a bound state of particles 1 and 2. This reaction can go directly and through resonance state $(12)^*$. As the Coulomb effects do not depended on spin, the particles are considered spinless. We use the following notation for indices;$\alpha = 1, 2$ and 3, where i denotes pair (jk). We make the further assumption that the particles are structureless. Such application can be used in the framework of the cluster model. The amplitude of the Coulomb transition for reaction (1) can be written down in the form

$$M =< \Psi_f^{(-)} |\Delta V| \Phi_i^{(+)} >, \tag{2}$$

Here, $\Phi_i^{(+)}$ is the wave function of the initial state; $\Psi_f^{(-)}$ is the wave function of the final state; $\Delta V = V_1^C + V_2^C - U_i^C$ is the transition potential; $U_i^C = \frac{(Z_1+Z_2)Z_3e^2}{R}$ is the Coulomb "optical" potential of the interaction between particle 3 and the center of mass of pair (12);Z_1, Z_2 and Z_3 are charges of particles 1, 2 and 3, respectively, V_1^C, V_2^C are the long-range Coulomb potentials

$$V_1^C = \frac{Z_2 Z_3 e^2}{|\vec{R} + \frac{m_1}{m_{12}}\vec{r}|}, V_2^C = \frac{Z_1 Z_3 e^2}{|\vec{R} - \frac{m_2}{m_{12}}\vec{r}|}, \tag{3}$$

where $\vec{R}$ is the radius-vector of the relative motion of particle 3 and the center of mass of particles 1 and 2, $\vec{r}$ is a radius-vector describing the relative motion of particles 1 and 2, $m_1(m_2)$ is the mass of particle 1 (2) and $m_{12} = m_1 + m_2$.

The wave function $\Psi_f^{(-)}$ of the final state can be taken in the form

$$\Psi_f^{(-)} = \Psi_C^{(-)} + G(E - i0)V_3^N \Psi_C^{(-)}, \tag{4}$$

where $G(E - i0)$ is the complete Green function, $\Psi_C^{(-)}$ is the three-body Coulomb wave function of the continuum spectrum describing motion of particles 1, 2 and 3 in the final state, V_3^N is the potential of the nuclear interaction between particles 1 and 2.

The wave function $\Phi_i^{(+)}$ is a solution of the equation:

$$[-\frac{1}{2\mu_{12}}\Delta_r - \frac{1}{2\mu_3}\Delta_R + V_3(\vec{r}) + U_i(\vec{R})]\Phi^{(+)}(\vec{r},\vec{R}) = (E_i - \varepsilon_{12})\Phi^{(+)}(\vec{r},\vec{R}), \tag{5}$$

where $\mu_{12} = m_1 m_2/m_{12}, \mu_3 = m_3 m_{12}/(m_3+m_{12}), \varepsilon_{12}$ is the binding energy of the system (12) and E_i is the energy of the projectile.

Substituting expression (3) into formula (2) we receive:

$$M =< \Psi_C^{(-)}|\Delta V|\Phi_i^{(+)} > + < \Psi_C^{(-)}|V_3^N G(E+i0)\Delta V|\Phi_i^{(+)} >, \tag{6}$$

The first term of (4) corresponds to the direct process, while the second term describes the reaction going through resonance.

Using the procedure described in Ref.[8, formula (14)] we obtain the expression for the resonance term of process (1):

$$M_r = \int \frac{d\vec{k}}{(2\pi)^3} \frac{< \Psi_C^{(-)}|V_3^N|\varphi_{3r}\Psi_{3\vec{k}}^{(-)} >< \Psi_{3\vec{k}}^{(-)}\tilde{\varphi}_{3r}|\Delta V|\Phi_i^{(+)} >}{E - k^2/2\mu_3 - E_r}. \tag{7}$$

Here, $< \Psi_{3k}^{(-)}\tilde{\varphi}_{3r}|\Delta V|\Phi_i^{(+)} >$ is the transition amplitude to the resonance state $(12)^*$ with taking into account the Coulomb scattering of particle 3 and resonance $(12)^*$ in the intermediate state; φ_{3r} is the wave function of the resonance in system (12) (the Gamov function); $\tilde{\varphi}_{3r}$ is the conjugate wave function of the resonance; $\Psi_{3\vec{k}}^{(-)}$ is the Coulomb wave function in the continuum describing the relative motion of particle 3 and the center of mass of pair (12) with relative energy $k^2/2\mu_3$; wave function $\Phi_i^{(+)}$ is a product of wave function φ_3 of the bound state of pair (12) and the Coulomb wave function $\Psi_{3\vec{k}_i}^{(-)}$, which describes the continuum of the relative motion between particle 3 and pair (12) in the initial state; $E_r = E_0 - i\Gamma/2$ is the resonance energy in system (12); and E_0 and Γ are parameters of the resonance. In amplitude (5), only one resonance is chosen and summation by projection of the orbital moment l_r is used.

The matrix element $< \Psi_C^{(-)}|V_3^N|\varphi_{3r}\Psi_{3\vec{k}}^{(-)} >$ responds to decay of resonance state $(12)^* \to 1+2$ when the Coulomb interaction in the final state is into account, and it can be connected with the vertex function of decay of resonance $(12)^* \to 1+2$ in the Coulomb field of particle 3 [8]. As wave function $\varphi_3(\vec{r})$ of the bound pair is an exponentially decreasing function, in integration with respect to r in the first matrix element of Eq.(4) and the second matrix element of Eq. (5), we limit r in the range $r \sim 1/\kappa_{12}$, where $\kappa_{12} = (2\varepsilon_{12}\mu_{12})^{1/2}$. Therefore the potential of the transition can be expanded in a power series in r/R:

$$\Delta V = Z_3 e^2 \sum_{l,m}^{\infty} \frac{4\pi}{(2l+1)} \frac{r^l}{R^{l+1}} \left[\left(\frac{m_2}{m_{12}}\right)^l Z_1 + (-1)^l \left(\frac{m_2}{m_{12}}\right)^l Z_2 \right] Y_{lm}^*(\hat{R}) Y_{lm}(\hat{r}), \tag{8}$$

where $\hat{b} = \vec{b}/b$.

In integration with respect to $\vec{R}$ the lower limit will be taken to be more than the sum of the radiuses of system (12) and particle 3, the Coulomb functions $\Psi^{(+)}_{3\vec{k}_i}(\vec{R}), \Psi^{(-)}_{3\vec{k}}(\vec{R})$ can be taken in the asymptotic expressions:

$$\Psi^{(+)}_{3\vec{k}_i}(\vec{R}) \approx \exp(i\vec{k}_i\vec{R} + i\eta_i \ln(k_i R - \vec{k}_i\vec{R})), \tag{9}$$

$$\Psi^{(-)}_{3\vec{k}}(\vec{R}) \approx \exp(i\vec{k}\vec{R} - i\eta \ln(kR + \vec{k}\vec{R})), \tag{10}$$

where $\eta_i = Z_3 Z_{12} e^2 \mu_3 / k_i$, $\eta = Z_3 Z_{12} e^2 \mu_3 / k$. $Z_{12} = Z_1 + Z_2, \vec{k}_i(\vec{k})$ is the momentum of the relative motion of system (12) and particle 3 in the initial (intermediate) state.

Further we use the approximate calculation procedure of the integral with respect to k in matrix element (5) describing in Ref. [8]. Omitting rather cumbrous calculation we write down the final result, which is analogous to formula (31) from Ref.[8] for the resonance term of amplitude (5).

$$M_r = -\frac{G_r M_3(\vec{k}_f, \vec{k}_i)}{\varepsilon - i\Gamma/2}\tilde{N}(\varepsilon), \tag{11}$$

where $\vec{k}_f$ is the momentum of the relative motion of system (12) and particle 3 in the final state,

$$\tilde{N}(\varepsilon) = \frac{\exp[-\frac{1}{2}\pi(\eta_{13} + \eta_{23} + \eta_0)]\Gamma(1 + i\xi)k(\varepsilon)}{[2\mu_3(\varepsilon - i\Gamma/2)]^{i\xi}} \tag{12}$$

Here

$$\varepsilon = k_f^2/2\mu_3 - E + E_0, \tag{13}$$

$$k(\varepsilon) = \frac{\sqrt{2}[2\tilde{\gamma}]^{i\eta_{13}}[2(\tilde{\alpha} + \tilde{\beta}]^{i\eta_{23}}}{[(1+\gamma^2)^{1/4} + (1+\gamma^2)^{-1/4}]^{1/2}} \times \tag{14}$$

$$\times \frac{\Gamma(1 + i\eta_{13})\Gamma(1 + i\eta_{23})}{\Gamma[1 + i(\eta_{13} + \eta_{23})](2k_0)^{2i\eta_0}} F(-i\eta_{13}, -i\eta_{23}; 1; \omega), \tag{15}$$

$k_0^2 = 2\mu_3(E - E_r)$, $E_r = E_0 - i\Gamma/2 = q_r^2/2\mu_{12}$ is the resonance energy of system (12), $\gamma = \Gamma/2E_0$,

$$G_r = \sqrt{\frac{2\pi\Gamma}{\mu_{12} q_0}} \frac{\exp[-\frac{1}{2}\pi\eta_r - \frac{1}{4} i arctan\gamma]}{[(1+\gamma^2)^{1/4} + (1+\gamma^2)^{-1/4}]^{1/2}} \tag{16}$$

is the vertex constant for decay $(12)^* \to 1 + 2$,

$$q_0 = \sqrt{2\mu_3 E_0}, \quad \eta_{ij} = Z_i Z_j e^2 \mu_{ij}/k_{ij},$$

$$\eta_r = Z_1 Z_2 e^2 \mu_{12}/q_r, \quad \eta_0 = Z_3 Z_{12} e^2 \mu_3/q_0,$$

$$\omega = (\tilde{\beta}\tilde{\gamma} - \tilde{\alpha}\tilde{\delta})/\tilde{\gamma}(\tilde{\alpha} + \tilde{\beta}),\ \tilde{\alpha} = (k_f^2 - k_0^2)/2,\ \tilde{\beta} = -(\vec{k}_{23}\vec{k}_f + k_{23}k_f),$$

$$\tilde{\gamma} = -(\vec{k}_{13}\vec{k}_f - k_{13}k_f) + \tilde{\alpha}, \quad \tilde{\delta} = \vec{k}_{13}\vec{k}_{23} - k_{23}k_{13} + \tilde{\beta},$$

$$\xi = \eta_{13} + \eta_{23} - \eta_0,$$

$F(a, b, c; x)$ is the hypergeometric function

The expression $M_3(\vec{k}_f, \vec{k}_i)$ for the electrical quadrupole transition $E2$ has the form

$$M_3(\vec{k}_f, \vec{k}_i) = \sqrt{2}(2\pi)^{3/2}\mu_{12}^2 \left[\frac{Z_1}{m_1^2} + \frac{Z_2}{m_2^2}\right] Z_3 e^2 \int_{r0}^{\infty} \chi_{3r}(r) r^4 \chi_3 dr$$
$$\int_{R>R_{\min}}^{\infty} \int_{\theta_{\min}}^{\theta_{\max}} \frac{1}{R} \exp[i(k_i \cos\theta - k_f) R \cos\theta_R + 2i\eta_f \ln(k_f R \sin\Theta_R] \times,$$
$$\times J_0(k_i R \sin\theta \sin\theta_R) P_2(\cos\theta_R) \sin\theta_R d\theta_R dR, \tag{17}$$

where $\chi_{3r}(r), \chi_3(r)$ are the radial parts of the resonance wave functions of (12)* and bound state (12), respectively; cos $= \frac{\vec{k}_i\vec{k}_f}{|\vec{k}_i||\vec{k}_f|}$; θ_R is the angel between radius-vector $\vec{R}$ and $\vec{k}$; $J_0(x)$ is the Bessel function; and $P_2(x)$ is the Legander polynomial.

The integration with respect to angle θ_R is carried out from $\theta_{\min}$ up to $\theta_{\max} < 180°$, to exclude the singular direction. For the derivation of formula (14), we took in to account that $k_i \approx k_f$ and the angle between them is very small.

For the amplitude of the direct breakup (the first term of Eq. (14)) we obtain the following expression using the result of Ref. [11].

$$M_{dir} = \frac{\sqrt{4\pi}}{k_{12}} Z_3 e^2 \sum_l C_l \mu_{12}^{\ell} \left[\frac{Z_1}{m_1^l} + (-1)^l \frac{Z_2}{m_2^l}\right] i^l e^{i\sigma_l} \int_{r_0}^{\infty} \chi_3(r) r^{l+1} F_l(k_{12} r) dr$$
$$\int_{R>R_{\min}}^{\infty} \int_{\theta_{\min}}^{\theta_{\max}} \frac{1}{R^{l-1}} \exp[i(k_i \cos\theta - k_f) R \cos\theta_R + 2i\eta_f \ln(k_f R \sin\theta_R)] \times$$
$$\times J_0(k_i R \sin\theta \sin\theta_R) P_1(\cos\theta_R) \sin\theta_R d\theta_R dR, \tag{18}$$

where $F_l(x)$ is the spherical Coulomb function, σ_l is the Coulomb phase shift, C_l is a factor depending on the direction of $\vec{k}_{12}$ and $\vec{k}_f$. If $\vec{k}_{12}$ and $\vec{k}_f$ have the same direction then $C_l = (-1)^l$, and for the opposite direction $C_l = 1$. The main contributions to amplitude (15) give the $E1$ and $E2$ transitions.

For the concrete calculation of the matrix elements $M_3(\vec{k}_f, \vec{k}_i)$ and M_{dir} the wave functions of the bound state and the resonance were chosen in the following asymptotic form:

$$\chi_3(r) \approx \frac{C}{r} W_{-\eta_b, l_b+1/2}(2\kappa_{12} r), \tag{19}$$

$$\chi_{3r}(r) \approx \frac{C_r}{r} W_{-i\eta_r, l_r+1/2}(2iq_r r), \kappa_{12} r), \tag{20}$$

where $W_{\lambda,\nu}(x)$ is the Whittaker function;$\eta_b = Z_1 Z_2 e^2 \mu_{12}/\kappa_{12}$; and l_b and l_r are the orbital moment of bound state (12) and resonance $(12)^*$, respectively. On integration with respect to r, the lower limit r_0 was taken equal to 3.7 Fm for the reaction $^{208}Pb(^6Li, \alpha d)^{208}Pb$ [12]. Indeed, the contribution to the integral from $0 \leq r \leq 3.7$ Fm is small in this region because the wave function $\chi_3(r)$ of the ^6Li ground state oscillates [13]. The asymptotic coefficients C and C_r are related with the vertex constants of the virtual decay of the resonance through the following expressions [14]:

$$G = -\exp[\frac{i\pi}{2}(l_b + \eta_b)] \frac{\sqrt{\pi}}{\mu_{12}} C, \tag{21}$$

$$G_r = -\exp[\frac{i\pi}{2}(l_r + i\eta_r)] \frac{\sqrt{\pi}}{\mu_{12}} C_r, \tag{22}$$

For the triple differential cross section we have:

$$\frac{d^3\sigma}{d\Omega_{12} d\omega_f dE_{12}} = \frac{\mu_3^2 \mu_{12}}{(2\pi)^5} \frac{k_f k_{12}}{k_i} |M_{dir} + M_r|^2 \tag{23}$$

3. Numerical Calculation of the reaction $^{208}Pb(^6Li, \alpha d)^{208}Pb$ and the Discussion of the Results

The nucleus ^6Li has as strongly pronounced (αd) cluster structure in the ground state as in the lower excited states [13]. The first excited 3^+ state is a narrow resonance with $E_0 = 2.185$ MeV and $\Gamma = 0.02$ MeV lying near to the $\alpha + d$ threshold. The disintegration of ^6Li through the 3^+ resonance state is determined by the $E2$ transition because the ground state is the 1^+ state. The kinematics of the reaction was taken in accordance with Ref.[2]. The Rutherford scattering angle θ was taken equal to $3°$ to allow the assumption that the disintegration has pure Coulomb character. The ^6Li projectile energy was equal to $E_i = E_{^6Li} = 156$ MeV. The α particle and deuteron were fixed under the same angle after breakup, While the velocity of the relative motion $(v_{\alpha d} < 0)$ as positive $(v_{\alpha d} > 0)$ to the direction of motion of the (αd) system center mass.

The value of the vertex constant $|G|^2$ for the $^6Li \to \alpha + d$ vertual decay xas taken equal to 0.41 Fm as in Ref.[12]. The limits of the integration respect with θ_R were taken to be $\theta_{\min} = 20°$, $\theta_{\max} = 160°$.

In Ref. [2] the differential cross section was measured up to the energy $E_{\alpha d} = 1$ MeV. The selection of these energies is related to the difficulties of performing the measurements.

Figure 1 shows the results of the calculation using only resonance amplitude (9). Notice that the height of the resonance peak coincides with the observed significance. However, the width is narrower than the experimental curve. The resonance curve exhibits symmetry about point $v_{\alpha d} = 0$. This can be easily explained in that $k_f \gg k_{12}$ over the whole investigated region of the $E_{\alpha d}$ relative energy. It is difficult to be concrete about the difference between the theoretical and the experimental results on the right side of Figure 1, because in this region the experiment was only carried out to 610 keV, but the resonance is at $E_{\alpha d} = 710$ keV as on the left side of the figure.

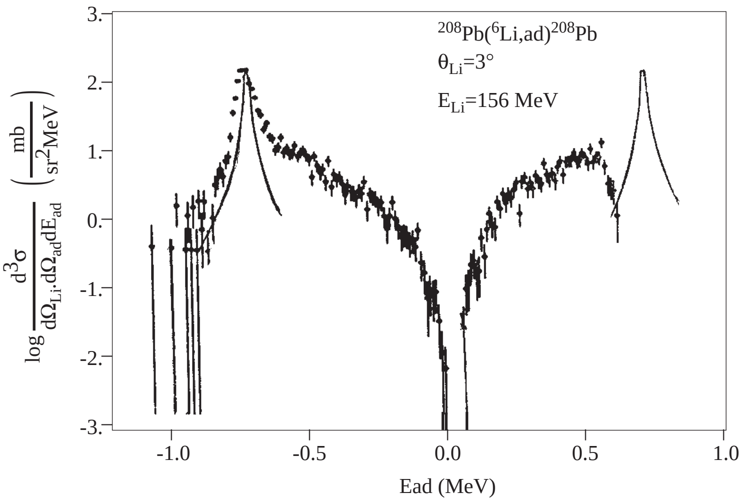

Figure 1. The triple differential cross section of the Coulomb breakup $^{208}Pb(^6Li, \alpha d)^{208}Pb$ in the center of mass of fragments as a function of the relative energy when the resonance mechanism is only included to the amplitude of the breakup. Negative and positive relative energies denote backward ($v_{\alpha d} < 0$) and forward ($v_{\alpha d} > 0$) emission, respectively, of the α particle in the ^6Li center mass system. The experimental data is taken from Ref.[2].

The result, when the amplitude is the sum of the direct M_{dir} and the resonance M_r terms, is represented in Figure 2. The result shows good coincidence with experimental data in the energy region up to $E_{\alpha d} = 710$ keV. The difference between our calculation and the experimental data in region $E_{\alpha d} > 710$ keV can be explained in that we did not take into account the nuclear interaction between α and d in the final state which gives the contribution in this energy region. It should be noted that the influence of Coulomb effects occurs due to the different particle accelerations in the field of particle 3 to be small, since the mass to charge ratio for α particle and a deuteron is almost the same.

Summarizing our discussion, we can conclude that it is necessary to take into account the Coulomb interaction between particle 3 and system (12), as in the intermediate so in the final state to describe the reaction energy spectrum when we deal with three charged particles.

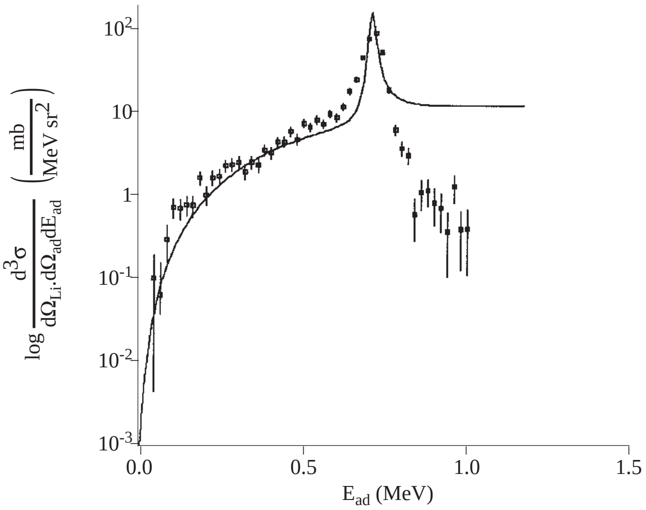

Figure 2. The triple differential cross section of the Coulomb breakup $^{208}Pb(^6Li, \alpha d)^{208}Pb$ in the center of mass of fragments as a function of the relative energy. The amplitude of the reaction consists of the sum of the direct and the resonance terms for the breakup.

References

[1] G. Baur, C. A. Bertulani, H. Rebel, *Nucl. Phys. A* **458** (1986) 188; C. A. Bertulani, G. Baur, *Phys. Rep.,* **163** 299.

[2] J. Kiener, H. J. Gills, H. Rebel et al., *Phys. Rev., C* **44** (1991) 2195.

[3] S. Typel, G. Baur, *Phys. Rev., C***49** (1994) 379; *Phys. Rev., C***50** (1994) 2104.

[4] C. A. Bertulani, G. F. Betsch, *Phys. Rev., C***49** (1994) 2839.

[5] H. Utsunomiya, Y. W. Lui, R. Haenni et al., Preprint. Cyclotron Institute College Station. Texas. 77843-3966. 1990.

[6] Sh. D. Kunikeev, V. S. Senashenko, Pisma v ZHTF, **14** (1988) 1811.

[7] M. Yu. Kuchiev, S. A. Sheynerman, ZHETF, **90** (1986) 1680.

[8] A. R. Ashurov et. al., *Yad. Fiz.,***53** (1991) 151.

[9] V. V. Komarov et. al., ECHAYA, **23** (1992) 1035.

[10] S. Typel, G. Bluge, and K. Langanke, *Z. Phys. A. Hadrons and Nuclei.* **339** (1991) 335.

[11] E. O. Alt, B. F. Irgaziev, A. T. Muminov, A. M. Mukhamedzhanov, *Yad. Fiz.,* **58** (1995) 1967.

[12] S. Igamov, R. Yarmukhamedov, *Yad. Fiz.,* **58** (1995) 1403.

[13] V. I. Kukulin et al., *Nucl. Phys., A* **586** (1995) 151.

[14] L. D. Blokhintsev, A. M. Mukhamedzhanov, A. N. Safronov, ECHAYA, **15** (1984) 1296.