

A Mean Field Model For the Coexistence of Nematic, Smectic A and Smectic C phases in Liquid Crystals

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Abstract

We present here a mean field model for the nematic-smectic A-smectic C coexistence of a liquid crystal near the NAC point which is a multicritical point. The critical exponents calculated from our model are tricritical exponents for the order parameters, susceptibility and the specific heat. Our results show that if the NAC point is the tricritical point, then our mean field model describes adequately the liquid crystalline system in the coexistence region of the nematic-smectic A-smectic C phases.

1. Introduction

In our earlier work [1] we have developed a mean field model with two order parameters for three-phase coexistence near the multicritical point. This model generalizes the mean field model with a single order parameter for fluid mixtures [2], liquid mixtures [3] and also for a liquid crystal system exhibiting the nematic-smectic A tricritical phase transition [4,5]. On the other hand, there have been given in the literature some studies dealing with the mean field models with two order parameters for liquid crystals near the NAC point [6-9].

In this study we have developed a mean field model with two order parameters to be applicable to a liquid crystalline system in the coexistence region of the nematic-smectic A-smectic C phases near the NAC point. In our model the NAC point has been taken as a multicritical point. The order parameter Ψ describes the ordering of the smectic A and smectic C phases, whereas η is the order parameter of the smectic C phase only. For the nematic phases these order parameters Ψ and η are zero. Using the free energy given in our model, we calculate the critical exponents for the order parameters, susceptibility and the specific heat. We find that our calculated critical exponents are the tricritical exponents.

In Section 2 we give the form of the free energy of our mean field model and we obtain the critical exponents for the order parameters, susceptibility and the specific heat. In Section 3 we discuss our results and finally in Section 4 our conclusions are given.

2. Theory

Here we introduce a mean field model for liquid crystals near the NAC point which is taken as a multicritical point. This model is valid in the region where the nematic, smectic A and smectic C phases coexist. We denote the three different phases α, β and γ as nematic, smectic A and smectic C phases, respectively. In the nematic phase we have the order parameters $\Psi_\alpha = \eta_\alpha = 0$. In the smectic A phase we have the order parameters $\Psi_\beta \neq 0, \eta_\beta = 0$. In the smectic C phase we have the order parameters $\Psi_\gamma \neq 0, \eta_\gamma \neq 0$. $\Psi_i (i = \beta, \gamma)$ is the order parameter which characterizes the appearance of layers in the smectic phases. η_γ is the order parameter which can be identified with the tilt angle.

The free energy of our model has three components. The first component which is due to the order parameter Ψ , is given by

$$\begin{aligned}
 F_1 = & \Psi(\Psi - \Psi_\beta)(\Psi - \Psi_\gamma)^4 + \Psi(\Psi - \Psi_\beta)^2(\Psi - \Psi_\gamma)^3 \\
 & + \Psi(\Psi - \Psi_\beta)^3(\Psi - \Psi_\gamma)^2 + \Psi(\Psi - \Psi_\beta)^4(\Psi - \Psi_\gamma) \\
 & + \Psi^2(\Psi - \Psi_\beta)(\Psi - \Psi_\gamma)^3 + \Psi^2(\Psi - \Psi_\beta)^2(\Psi - \Psi_\gamma)^2 \\
 & + \Psi^2(\Psi - \Psi_\beta)^3(\Psi - \Psi_\gamma) + \Psi^3(\Psi - \Psi_\beta)(\Psi - \Psi_\gamma)^2 \\
 & + \Psi^3(\Psi - \Psi_\beta)^2(\Psi - \Psi_\gamma) + \Psi^4(\Psi - \Psi_\beta)(\Psi - \Psi_\gamma).
 \end{aligned} \tag{1a}$$

The second component of the free energy which is due to the order parameter η , is given by

$$\begin{aligned}
 F_2 = & \eta(\eta - \eta_\gamma)^5 + \eta^2(\eta - \eta_\gamma)^4 + \eta^3(\eta - \eta_\gamma)^3 \\
 & + \eta^4(\eta - \eta_\gamma)^2 + \eta^5(\eta - \eta_\gamma).
 \end{aligned} \tag{1b}$$

The third component of the free energy which is due to the coupling between the order parameters Ψ and η is given by

$$\begin{aligned}
 F_3 = & \Psi(\Psi - \Psi_\beta)(\Psi - \Psi_\gamma)\eta^2(\eta - \eta_\gamma) \\
 & + \Psi(\Psi - \Psi_\beta)(\Psi - \Psi_\gamma)\eta(\eta - \eta_\gamma)^2.
 \end{aligned} \tag{1c}$$

Hence, the total free energy for the system is

$$F = F_1 + F_2 + F_3. \tag{1d}$$

This free energy given by Eq. (1d) can be rewritten as

$$F = a_0 + a_1\Psi + a_2\Psi^2 + a_3\Psi^3 + a_4\Psi^4 + a_5\Psi^5 + a_6\Psi^6$$

$$\begin{aligned}
 &+ b_1\eta + b_2\eta^2 + b_3\eta^3 + b_4\eta^4 + b_5\eta^5 + b_6\eta^6 \\
 &+ c_1\Psi\eta + c_2\Psi^2\eta + c_3\Psi\eta^2 + c_4\Psi^2\eta^2 + c_5\Psi^3\eta \\
 &+ c_6\Psi\eta^3 + c_7\Psi^2\eta^3 + c_8\Psi^3\eta^2 + c_9\Psi^3\eta^3.
 \end{aligned} \tag{2}$$

In this equation we give the coefficients, $a_i(i = 1, \dots, 6)$, $b_i(i = 1, \dots, 6)$ and $c_i(i = 1, \dots, 9)$, in terms of $\Psi_i(i = \beta, \gamma)$ and $\eta_i(i = \gamma)$ in Appendix. We assume that the temperature dependence of a_5 is

$$a_5 = a_{50}(T - T_c)^x.$$

Here, T_c denotes the critical temperature. According to the Landau theory, by taking the temperature dependence of a_2 as

$$a_2 = a_{20}(T - T_c)$$

We find that using the equations for a_2 and a_5 given in Appendix, the power x equals to $1/4$. Hence, using the equations for a_i given in Appendix, according to

$$a_i \sim (T - T_c)^y$$

we get the values of y as: $5/4$ for a_1 ; $4/4$ for a_2 ; $3/4$ for a_3 ; $2/4$ for a_4 and $1/4$ for a_5 . Similarly, by taking

$$b_i \sim (T - T_c)^z$$

we get the z values as: $5/4$ for b_1 ; $4/4$ for b_2 ; $3/4$ for b_3 ; $2/4$ for b_4 and $1/4$ for b_5 . Using the equations for c_i given in Appendix, by taking

$$c_i \sim (T - T_c)^r$$

we get the values of r as: $4/4$ for c_1 ; $3/4$ for c_2 and c_3 ; $2/4$ for c_4 , c_5 and c_6 ; and $1/4$ for c_7 and c_8 .

Using the temperature dependences of a_5 and b_5 , we get the critical exponent $\beta = 1/4$ for the order parameters Ψ_i and η_i .

The susceptibilities can be defined as

$$\chi_i^{-1} = \left(\frac{\partial^2 F}{\partial \Psi^2} \right)_{\psi=\psi_i, \eta=\eta_i}$$

and

$$\chi_i'^{-1} = \left(\frac{\partial^2 F}{\partial \eta^2} \right)_{\psi=\psi_i, \eta=\eta_i}$$

From the temperature dependences of a_i , b_i and c_i , we obtain the critical exponent for the susceptibilities as $\gamma = 1$.

The specific heat can be defined as

$$C = T \left(\frac{\partial^2 F}{\partial T^2} \right) \Big|_{T=T_c}.$$

Using the temperature dependences a_i , b_i and c_i we get the critical exponent for the specific heat as $\alpha = 1/2$.

3. Results and Discussion

The free energy given by Eq.(1d) is valid near the NAC point which is taken as a multicritical point. Using this free energy we have calculated the critical exponents. We find these exponents as $\beta = 1/4$ for the order parameters, $\gamma = 1$ for the susceptibilities and $\alpha = 1/2$ for the specific heat. Hence, these exponents are the tricritical exponents. Therefore, the free energy given by Eq.(1d) should be valid near the tricritical point. In fact, the NAC point can be taken as a tricritical point. This is true because the nematic-smectic A transition is usually considered as a second order transition, the smectic A-smectic C transition may be first order or second order and the nematic-smectic C transition is always first order. Therefore, taking the NC and AC transitions as first order and the NA transition as a second order, we can consider the NAC point as a tricritical point.

4. Conclusions

In this study we have developed a mean field model with two order parameters for a liquid crystalline system in the coexistence region of the nematic, smectic A and smectic C phases. Our results show that this mean field model describes correctly the tricritical behaviour of a liquid crystalline system near the NAC point.

Appendix

If we expand Eq.(1d) we obtain Eq.(2) with the coefficients a_i, b_i and c_i as follows:

$$\begin{aligned} a_0 &= 0 \\ a_1 &= -\Psi_\beta^4 \Psi_\gamma - \Psi_\beta^3 \Psi_\gamma^2 - \Psi_\beta^2 \Psi_\gamma^3 - \Psi_\beta \Psi_\gamma^4 \\ a_2 &= \Psi_\beta^4 + \Psi_\gamma^4 + 7\Psi_\beta^3 \Psi_\gamma + 7\Psi_\beta \Psi_\gamma^3 + 7\Psi_\beta^2 \Psi_\gamma^2 \\ a_3 &= -6\Psi_\beta^3 - 6\Psi_\gamma^3 - 21\Psi_\beta^2 \Psi_\gamma - 21\Psi_\beta \Psi_\gamma^2 \\ a_4 &= 15\Psi_\beta^2 + 15\Psi_\gamma^2 + 35\Psi_\beta \Psi_\gamma \\ a_5 &= -20\Psi_\beta - 20\Psi_\gamma \\ a_6 &= 10 \\ b_1 &= -\eta_\gamma^5 \\ b_2 &= 6\eta_\gamma^4 \\ b_3 &= -15\eta_\gamma^3 \end{aligned}$$

$$\begin{aligned}
b_4 &= 20\eta_\gamma^2 \\
b_5 &= -15\eta_\gamma \\
b_6 &= 5 \\
c_1 &= \Psi_\beta \Psi_\gamma \eta_\gamma^2 \\
c_2 &= -\Psi_\beta \eta_\gamma^2 - \Psi_\gamma \eta_\gamma^2 \\
c_3 &= -3\Psi_\beta \Psi_\gamma \eta_\gamma \\
c_4 &= 3\Psi_\beta \eta_\gamma + 3\Psi_\gamma \eta_\gamma \\
c_5 &= \eta_\gamma^2 \\
c_6 &= 2\Psi_\beta \Psi_\gamma \\
c_7 &= -2\Psi_\beta - 2\Psi_\gamma \\
c_8 &= -3\eta_\gamma \\
c_9 &= 2
\end{aligned}$$

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