Turkish Journal of Physics

http://journals.tubitak.gov.tr/physics/

Research Article

Turk J Phys
(2020) 44: 465 – 471
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doi:10.3906/fiz-2008-11

Weak deflection angle of black-bounce traversable wormholes using Gauss–Bonnet theorem in the dark matter medium

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Received: 23.08.2020 • **Accepted/Published Online:** 27.10.2020 • **Final Version:** 30.10.2020

Abstract: In this paper, we first use the optical metrics of black-bounce traversable wormholes to calculate the Gaussian curvature. Then we use the Gauss–Bonnet theorem to obtain the weak deflection angle of light from the black-bounce traversable wormholes. We investigate the effect of dark matter medium on weak deflection angle using the Gauss–Bonnet theorem. We show how weak deflection angle of wormhole is affected by the bounce parameter a. Using the Gauss–Bonnet theorem for calculating weak deflection angle shows us that light bending can be thought as a global and topological effect.

Key words: Relativity and gravitation, gravitational lensing, deflection angle, wormholes, Gauss–Bonnet theorem

1. Introduction

Root of gravitational lensing is the deflection of light by gravitational fields such as a planet, a black hole, or dark matter predicted by Einstein's general relativity, in the weak field limit [1, 2]. Weak deflection is used to detect dark matter filaments, and it is important topic because it helps to understand the large scale structure of the universe [1, 3].

One of the important method to calculate the weak deflection angle using optical geometry is proposed by Gibbons and Werner (GW), which is known as Gauss–Bonnet theorem (GBT) [4, 5]. In the method of GW, deflection angle is considered as a partially topological effect and can be found by integrating the Gaussian optical curvature of the black hole space using [4]:

$$\hat{\alpha} = -\int\int_{D_\infty} K \mathrm{d}S, \tag{1}$$

where $\hat{\alpha}$ is a deflection angle, K is a Gaussian optical curvature, dS is an optical surface and the D_∞ stands for the infinite domain bounded by the light ray, excluding the lens. Since the GW method provides a unique perspective, it has been applied to various types of black hole spacetime or wormhole spacetime [6–55].

In this paper, our main motivation is to explore weak deflection angle of black-bounce traversable wormholes [56, 57] using the GBT and then extend our motivation of this research is to shed light on the effect of dark matter medium on the weak deflection angle of black-bounce traversable wormhole using the GBT. Note that the refractive index of the medium is supposed that it is spatially nonuniform but one can consider that it is uniform at large distances [58–64]. To do so, the photons are thought that may be deflected

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through dark matter due to the dispersive effects, where the index of refractive $n(\omega)$ which is for the scattering amplitude of the light and dark matter in the forward.

2. Calculation of weak deflection angle from black-bounce traversable wormholes using the Gauss–Bonnet theorem

The "black-bounce form" for the spacetime metric of traversable wormhole is [56]:

$$ds^2 = -\left(1-\frac{2M}{\sqrt{r^2+a^2}}\right)dt^2 + \left(1-\frac{2M}{\sqrt{r^2+a^2}}\right)^{-1}dr^2 + \left(r^2+a^2\right)\left(d\theta^2+\sin^2\theta d\phi^2\right). \tag{2}$$

It is noted that the parameter a stands for the bounce length scale and when $a=0$, it reduces to the Schwarzschild solution. We restrict ourselves to the equatorial coordinate plane ($\theta=\frac{\pi}{2}$), so that the black-bounce traversable wormhole spacetime becomes

$$ds^2 = -\left(1-\frac{2M}{\sqrt{r^2+a^2}}\right)dt^2 + \left(1-\frac{2M}{\sqrt{r^2+a^2}}\right)^{-1}dr^2 + \left(r^2+a^2\right)d\phi^2. \tag{3}$$

Then the optical geometry of the black-bounce traversable wormhole spacetime is found by using

$$g^{opt}{}_{\alpha\beta} = \frac{g_{\alpha\beta}}{-g_{00}}, \tag{4}$$

$$\mathrm{d}t^2 = \frac{\mathrm{d}r^2}{\left(1-\frac{2M}{\sqrt{r^2+a^2}}\right)^2} + \frac{(r^2+a^2)\mathrm{d}\varphi^2}{\left(1-\frac{2M}{\sqrt{r^2+a^2}}\right)}. \tag{5}$$

The Gaussian optical curvature K for the black-bounce traversable wormhole optical space is calculated as follows:

$$K \simeq 3\,\frac{m^2}{r^4} - 2\,\frac{m}{r^3} - 15\,\frac{a^2m^2}{r^6} + 10\,\frac{a^2m}{r^5} - \frac{a^2}{r^4}. \tag{6}$$

Then we should use the Gaussian optical curvature in GBT to find deflection angle because the GBT is a theory which links the intrinsic geometry of the 2-dimensional space with its topology (D_R in M, with boundary $\partial D_R = \gamma_{\tilde{g}} \cup C_R$) [4]:

$$\int_{D_R} K\,\mathrm{d}S + \oint_{\partial D_R} \kappa\,\mathrm{d}t + \sum_i \epsilon_i = 2\pi\chi(D_R), \tag{7}$$

in which κ is defined as the geodesic curvature ($\kappa = \tilde{g}\left(\nabla_{\dot{\gamma}}\dot{\gamma},\ddot{\gamma}\right)$), so that $\tilde{g}(\dot{\gamma},\dot{\gamma})=1$. It is noted that $\ddot{\gamma}$ is an unit acceleration vector and ϵ_i is for the exterior angles at the i^{th} vertex. Jump angles are obtained as $\pi/2$ for $r\to\infty$. Then we find that $\theta_O+\theta_S\to\pi$. If D_R is a nonsingular, the Euler characteristic becomes $\chi(D_R)=1$, hence GBT becomes

$$\iint_{D_R} K\,\mathrm{d}S + \oint_{\partial D_R} \kappa\,\mathrm{d}t + \theta_i = 2\pi\chi(D_R). \tag{8}$$

The Euler characteristic number χ is 1, then the remaining part yields $\kappa(C_R) = |\nabla_{\dot{C}_R}\dot{C}_R|$ as $r \to \infty$. The radial component of the geodesic curvature is given by

$$\left(\nabla_{\dot{C}_R}\dot{C}_R\right)^r = \dot{C}_R^{\varphi}\,\partial_{\varphi}\dot{C}_R^r + \Gamma^r_{\varphi\varphi}\left(\dot{C}_R^{\varphi}\right)^2. \tag{9}$$

For large limits of R, $C_R := r(\varphi) = r = const.$ we obtain

$$\left(\nabla_{\dot{C}_R^r}\dot{C}_R^r\right)^r \to -\frac{1}{r}, \tag{10}$$

so that $\kappa(C_R) \to r^{-1}$. After that it is not hard to see that $\mathrm{d}t = r\,\mathrm{d}\,\varphi$, where $\kappa(C_R)\mathrm{d}t = \mathrm{d}\,\varphi$. The GBT reduces to this form

$$\iint\limits_{D_R} K\,\mathrm{d}S + \oint\limits_{C_R} \kappa\,\mathrm{d}t \overset{r\to\infty}{=} \iint\limits_{S_\infty} K\,\mathrm{d}S + \int\limits_{0}^{\pi+\hat{\alpha}} \mathrm{d}\varphi. \tag{11}$$

The light ray follows the straight line so that, we can assume that $r_\gamma = b/\sin\varphi$ at zeroth order. The weak deflection angle can be calculated using the formula:

$$\hat{\alpha} = -\int\limits_{0}^{\pi}\int\limits_{r_\gamma}^{\infty} K\,r\,\mathrm{d}r\,\mathrm{d}\varphi. \tag{12}$$

Using the Gaussian optical curvature (6), we calculate the weak deflection angle of black-bounce traversable wormholes up to second order terms:

$$\hat{\alpha} \approx 4\,\frac{m}{b} + \frac{a^2\pi}{4b^2}. \tag{13}$$

Note that it is in well agreement with the [57] in leading order terms.

3. Deflection angles of photon through dark matter medium from black-bounce traversable wormholes

In this section, we investigate the effect of dark matter medium on the weak deflection angle. To do so, we use the refractive index for the dark matter medium [58]:

$$n(\omega) = 1 + \beta A_0 + A_2\omega^2. \tag{14}$$

Note that $\beta = \frac{\rho_0}{4m^2\omega^2}$, ρ_0 is the mass density of the scattered dark matter particles, $A_0 = -2\varepsilon^2 e^2$ and $A_{2j} \geq 0$. The terms in $\mathcal{O}\left(\omega^2\right)$ and higher terms are related to the polarizability of the dark matter candidate.

The order of ω^{-2} is for the charged dark matter candidate and ω^2 is for a neutral dark matter candidate. In addition, the linear term in ω occurs when parity and charge-parity asymmetries are present. The 2-dimensional optical geometry of the wormhole is:

$$d\sigma^2 = n^2\left(\frac{dr^2}{\left(1-\frac{2M}{\sqrt{r^2+a^2}}\right)^2} + \frac{(r^2+a^2)}{\left(1-\frac{2M}{\sqrt{r^2+a^2}}\right)}d\varphi^2\right), \tag{15}$$

and

$$\left.\frac{d\sigma}{d\varphi}\right|_{C_R} = n\left(\frac{r^2+a^2}{\left(1-\frac{2M}{\sqrt{r^2+a^2}}\right)}\right)^{1/2}. \tag{16}$$

Using the GBT within optical geometry of black-bounce traversable wormhole, we obtain the weak deflection angle in a dark matter medium:

$$\lim_{R\to\infty}\int_0^{\pi+\alpha}\left[\kappa_g\frac{d\sigma}{d\varphi}\right]\Bigg|_{C_R}d\varphi = \pi - \lim_{R\to\infty}\int\int_{D_R}\mathcal{K}dS. \tag{17}$$

First we calculate the Gaussian optical curvature at linear order of M:

$$\mathcal{K} \approx 10\,\frac{csgn\,(r)\,a^2M}{\left(A_2\,\omega^2+\beta\,A_0+1\right)^2r^5} - 2\,\frac{csgn\,(r)\,M}{\left(A_2\,\omega^2+\beta\,A_0+1\right)^2r^3} - \frac{a^2}{\left(A_2\,\omega^2+\beta\,A_0+1\right)^2r^4}. \tag{18}$$

After that we find

$$\lim_{R\to\infty}\kappa_g\left.\frac{d\sigma}{d\varphi}\right|_{C_R} = 1. \tag{19}$$

Then for the limit of $R\to\infty$, the deflection angle in dark matter medium can be calculated using the GBT as follows:

$$\alpha = -\lim_{R\to\infty}\int_0^{\pi}\int_{\frac{b}{\sin\varphi}}^{R}\mathcal{K}dS. \tag{20}$$

Hence, we obtain the weak deflection angle in dark matter medium as follows:

$$\alpha = 4\,\frac{M}{b\Psi}+\frac{a^2\pi}{4b^2\Psi}, \tag{21}$$

where

$$\Psi = {A_2}^2\omega^4+2\,A_0\,A_2\,\beta\,\omega^2+{A_0}^2\beta^2+2\,A_2\,\omega^2+2\,\beta\,A_0+1, \tag{22}$$

which agrees with the known expression found using another method. Of course, in the absence of the dark matter medium ($\Psi=0$), this expression reduces to the known vacuum formula $\alpha\approx 4\,\frac{m}{b}+\frac{a^2\pi}{4b^2}$. Hence, we find that the deflected photon through the dark matter around the black-bounce traversable wormhole has large deflection angle compared to black-bounce traversable wormhole without dark matter medium.

4. Conclusion

In this paper, we have studied the weak deflection angle of black-bounce traversable wormholes using the GBT. Then we have investigated the effect of dark matter medium on the weak deflection angle of black-bounce traversable wormholes. Note that refractive index is taken spatially nonuniform, and it is uniform at large distances. Hence it is concluded that the deflection angle by black-bounce traversable wormholes increases with increasing the bounce parameter a, on the other hand the deflection angle decreases in a increasing medium of dark matter. It is showed that how weak deflection angle of wormhole is affected by the bounce parameter a. Moreover we use the Gauss–Bonnet theorem for calculating weak deflection angle which proves us that light bending can be thought as a global and topological effect.

References

[1] Bartelmann M, Schneider P. Weak gravitational lensing. Physics Reports 2001; 340 (4-5): 291-472.

[2] Bozza V. Gravitational lensing by black holes. General Relativity Gravitation 2010; 42: 2269-2300.

[3] Keeton CR, Kochanek CS, Falco EE. The optical properties of gravitational lens galaxies as a probe of galaxy structure and evolution. Astrophysical Journal 1998; 509: 561-578.

[4] Gibbons GW, Werner MC. Applications of the Gauss-Bonnet theorem to gravitational lensing. Classical and Quantum Gravity 2008; 25 (23).

[5] Werner MC. Gravitational lensing in the Kerr-Randers optical geometry. General Relativity Gravitation 2012; 44: 3047-3057.

[6] Takizawa K, Ono T, Asada H. Gravitational lens without asymptotic flatness. Physical Review D 2020; 6: 102.

[7] Jusufi K, Werner MC , Banerjee A, Övgün A. Light deflection by a rotating global monopole spacetime. Physical Review D 2017; (10): 95.

[8] Crisnejo G, Gallo E. Weak lensing in a plasma medium and gravitational deflection of massive particles using the Gauss-Bonnet theorem. A unified treatment. Physical Review D 2018; 97 (12).

[9] Jusufi K, Sakalli İ, Övgün A. Effect of Lorentz symmetry breaking on the deflection of light in a cosmic string spacetime. Physical Revial D 2017; 96 (2): 024040.

[10] Övgün A, Jusufi K, Sakalli İ. Gravitational lensing under the effect of Weyl and bumblebee gravities: applications of Gauss–Bonnet theorem. Annals of Physics 2018; 399: 193-203.

[11] Jusufi K, Övgün A. Effect of the cosmological constant on the deflection angle by a rotating cosmic string. Physical Review D 2018; 6: 97.

[12] Övgün A. Weak field deflection angle by regular black holes with cosmic strings using the Gauss-Bonnet theorem. Physics Revial D 2019; 10: 99.

[13] Javed W, Babar R, Övgün A. Effect of the dilaton field and plasma medium on deflection angle by black holes in Einstein-Maxwell-dilaton-axion theory. Physical Review D 2019; 100 (2): 104032.

[14] Javed W, Abbas J, Övgün A. Deflection angle of photon from magnetized black hole and effect of nonlinear electrodynamics. The European Physical Journal C 2019; 8: 694.

[15] Sakalli I, Ovgun A. Hawking radiation and deflection of light from Rindler modified Schwarzschild black hole. EPL (Europhysics Letters) 2017; 118 (6): 60006.

[16] De Leon K, Vega I. Weak gravitational deflection by two-power-law densities using the Gauss-Bonnet theorem. Physical Review D 2019; 99 (12): 124007.

[17] Li Z, Zhou T. Equivalence of Gibbons-Werner method to geodesics method in the study of gravitational lensing. Physical Review D 2020; 101 (4): 044043.

[18] Övgün A, Sakallı İ, Saavedra J. Weak gravitational lensing by Kerr-MOG black hole and Gauss–Bonnet theorem. Annals of Physics 2019; 411: 167978.

[19] Övgün A, Sakalli I, Saavedra J. Shadow cast and deflection angle of Kerr-Newman-Kasuya spacetime. Journal of Cosmology and Astroparticle Physics 2018; 10: 041.

[20] Jusufi K, Övgün A. Light d eflection by a quantum improved Kerr black hole pierced by a cosmic string. International Journal Geometric Methods in Modern Physics 2019; 16 (8): 1950116.

[21] Kumar R, Islam SU, Ghosh SG. Gravitational lensing by charged black hole in regularized $4D$ Einstein-Gauss-Bonnet gravity. arXiv 2020; arXiv:2004.12970 [gr-qc].

[22] Kumaran Y, Övgün A. Weak deflection angle of extended uncertainty principle black holes. Chinese Journal of Physics 2020; 44 (2): 025101.

[23] Kumar R,Ghosh SG,Wang A. Gravitational deflection of light and shadow cast by rotating Kalb-Ramond black holes. Physical Review D 2020; 101 (10): 104001.

[24] Javed W, Abbas J, Övgün A. Effect of the hair on deflection angle by asymptotically flat black holes in Einstein-Maxwell-dilaton theory. Physical Review D 2019; 100 (4): 044052.

[25] Javed W, Hazma A , Övgün A. Effect of non-linear electrodynamics on weak field deflection angle by black hole. Physical Review D 2020; 101 (10):103521.

[26] Kumar R, Ghosh SG, Wang A. Shadow cast and deflection of light by charged rotating regular black holes. Physical Review D 2019; 100 (12): 124024.

[27] Ishihara A, Suzuki Y, Ono T, Asada H. Finite-distance corrections to the gravitational bending angle of light in the strong deflection limit. Physical Review D 2017; 95 (4): 044017.

[28] Ono T, Ishihara A, Asada H. Gravitomagnetic bending angle of light with finite-distance corrections in stationary axisymmetric spacetimes. Physical Review D 2017; 96 (10): 104037.

[29] Ono T, Ishihara A, Asada H. Deflection angle of light for an observer and source at finite distance from a rotating global monopole. Physical Review D 2019; 99 (12): 124030.

[30] Ono T, Asada H. The effects of finite distance on the gravitational deflection angle of light. Universe 2019; 5 (11): 218.

[31] Arakida H. Light deflection and Gauss–Bonnet theorem: definition of total deflection angle and its applications. General Relativity Gravitation 2018; 50 (5): 48.

[32] Li Z, Övgün A. Finite-distance gravitational deflection of massive particles by a Kerr-like black hole in the bumblebee gravity model. Physical Review D 2020; 101 (2): 024040.

[33] Li Z, Zhang G, Övgün A. Circular orbit of a particle and weak gravitational lensing. Physical Review D 2020; 101 (12): 124058.

[34] Javed W, Khadim MB, Övgün A, Abbas J. Weak gravitational lensing by stringy black holes. The European Physical Journal Plus 2020; 135 (3): 314.

[35] Javed W, Khadim MB, Övgün A. Weak gravitational lensing by Bocharova–Bronnikov–Melnikov–Bekenstein black holes using Gauss–Bonnet theorem. The European Physical Journal Plus 2020; 135: 595.

[36] Li Z, Jia J. The finite-distance gravitational deflection of massive particles in stationary spacetime: a Jacobi metric approach. The European Physical Journal C 2020; 80 (2): 157.

[37] Li Z, Zhou T. Kerr black hole surrounded by a cloud of strings and its weak gravitational lensing in Rastall gravity. arXiv 2020; arXiv:2001.01642 [gr-qc].

[38] Takizawa K, Ono T, Asada H. Gravitational deflection angle of light: definition by an observer and its application to an asymptotically nonflat spacetime. Physical Review D 2020; 101 (10): 104032.

[39] Gibbons GW. The Jacobi-metric for timelike geodesics in static spacetimes. Classical and Quantum Gravity 2016; 33 (2): 025004.

[40] Islam SU, Kumar R, Ghosh SG. Gravitational lensing by black holes in $4D$ Einstein-Gauss-Bonnet gravity. arXiv 2020; arXiv:2004.01038 [gr-qc].

[41] Pantig RC, Rodulfo ET. Weak lensing of a dirty black hole. arXiv 2020; arXiv:2003.00764 [gr-qc].

[42] Tsukamoto N. Non-logarithmic divergence of a deflection angle by a marginally unstable photon sphere of the Damour-Solodukhin wormhole in a strong deflection limit. Physical Review D 2020; 101 (10): 104021.

[43] Crisnejo G, Gallo E, Villanueva JR. Gravitational lensing in dispersive media and deflection angle of charged massive particles in terms of curvature scalars and energy-momentum tensor. Physical Review D 2019; 100 (4): 044006.

[44] Jusufi K, Ovgün A, Banerjee A. Light deflection by charged wormholes in Einstein-Maxwell-dilaton theory. Physical Review D 2017; 96 (8): 084036.

[45] Jusufi K, Övgün A, Saavedra J, Vasquez Y, Gonzalez PA. Deflection of light by rotating regular black holes using the Gauss-Bonnet theorem. Physical Review D 2018; 97 (12): 124024.

[46] Övgün A, Jusufi K, Sakallı İ. Exact traversable wormhole solution in bumblebee gravity. Physical Review D 2019; 99 (2):024042.

[47] Javed W, Babar R, Övgün A. The effect of the Brane-Dicke coupling parameter on weak gravitational lensing by wormholes and naked singularities. Physical Review D 2019; 99 (8): 084012.

[48] Jusufi K, Övgün A. Gravitational lensing by rotating wormholes. Physical Review D 2018; 97 (2): 024042.

[49] Jusufi K, Övgün A, Banerjee A, Sakallı İ. Gravitational lensing by wormholes supported by electromagnetic, scalar, and quantum effects. The European Physical Journal Plus 2019; 134 (9): 428.

[50] Övgün A, Gyulchev G, Jusufi K. Weak gravitational lensing by phantom black holes and phantom wormholes using the Gauss-Bonnet theorem. Annals of Physics 2019; 406: 152-172.

[51] Övgün A. Deflection Angle of photons through dark matter by black holes and wormholes using Gauss–Bonnet theorem. Universe 2019; 5: 115.

[52] Övgün A. Light deflection by Damour-Solodukhin wormholes and Gauss-Bonnet theorem. Physical Review D 2018; 98 (4): 044033.

[53] Li Z, He G, Zhou T. Gravitational deflection of relativistic massive particles by wormholes. Physical Review D 2020; 101 (4): 044001.

[54] Goulart P. Phantom wormholes in Einstein–Maxwell-dilaton theory. Classical and Quantum Gravity 2018; 35 (2): 025012.

[55] Ono T, Ishihara A, Asada H. Deflection angle of light for an observer and source at finite distance from a rotating wormhole. Physical Review D 2018; 98 (4):044047.

[56] Simpson A, Martin-Moruno P, Visser M. Vaidya spacetimes, black-bounces, and traversable wormholes. Classical and Quantum Gravity 2019; 36 (14): 145007.

[57] Nascimento JR, Petrov AY, Porfirio PJ, Soares AR. Gravitational lensing in a black-bounce traversable wormhole spacetime. Physical Review D 2020; 102 (4):044021.

[58] Latimer DC. Dispersive light propagation at cosmological distances: matter effects. Physical Review D 2013; 88: 063517.

[59] Latimer DC. Anapole dark matter annihilation into photons. Physical Review D 2017; 95 (9): 095023.

[60] Bisnovatyi-Kogan GS,Tsupko OY. Gravitational lensing in a non-uniform plasma. Montly Notices of the Royal Astronomical Society 2010; 404: 1790-1800.

[61] Bisnovatyi-Kogan GS, Tsupko OY. Gravitational lensing in plasmic medium. Plasma Physics Reports 2015; 41: 562-581.

[62] Bisnovatyi-Kogan GS, Tsupko OY. Gravitational lensing in presence of plasma: strong lens systems, black hole lensing and shadow. Universe 2017; 3 (3): 57.

[63] Tsupko OY, Bisnovatyi-Kogan GS. Gravitational lensing in plasma: relativistic images at homogeneous plasma. Physical Review D 2013: 87 (12): 124009.

[64] Er X, Mao S. Effects of plasma on gravitational lensing. Montly Notices of the Royal Astronomical Society 2014; 437 (3): 2180-2186.