

## The calculation of total fragment excitation energy for photofission of Uranium isotopes

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**Abstract:** The total excitation energy as a function of fragment mass (TXE/A) are calculated for neutron induced fission of  $^{235}\text{U}$  and photofission of  $^{238}\text{U}$  by using the statistical model. With comparing the calculated results and the experimental data, the statistical model is modified. In new modified model, TXE are calculated by adding the neutron binding energy to the deformation energy. The calculated results using modified statistical model are in good agreement with the experimental data. Then, the total excitation energy as a function of fragment mass (TXE/A) are evaluated for photofission of Uranium isotopes.

**Key words:** Photofission, deformability parameter, total excitation energy, neutron binding energy, uranium isotopes

### 1. Introduction

The nuclear fission process still represents a major challenge for fitting theoretical calculation results with experimental data. Meanwhile, the total excitation energy (TXE) has been little discussed as an important value in fission process. Most of the available energy is released as total kinetic energy (TKE), but the other part is turned up as total excitation energy of the fragments. Ruben [1] presented the energy balance equation that TXE values can be obtained with TKE values. Another way to calculate the TXE requires the dissipative energy ( $E_{dis}$ ), while the calculation of dissipative energy is not much discussed and not easily achieved. The total prompt neutron ( $\nu(A)$ ) is evaluated with TXE value at low excitation energy (E). Brosa [2, 3] divided TXE by 8 MeV (as the mean value of neutron binding energy) to obtain an approximate number of prompt neutrons. While some researchers [1, 4] have been used the gamma emission and average center of mass system energy to calculate the neutron multiplicity.

The Point by Point (PbP) model [5–8] calculated the TXE by using fragments level densities which were calculated by the generalized superfluid model of Ignatyuk [9]. The PbP model is used to calculate the excitation energy of complementary fully accelerated fission fragments and  $\nu(A)$ . To calculate TXE of fission fragments, Faust assumed that fission products are excited as an exponential distribution function, which depends on the Q-value and the level density parameter [10]. Ivanyuk [11] calculated TXE for neutron fission of  $^{235}\text{U}$  with the calculated values of TKE. Also, TXE values are evaluated with the experimental values of total kinetic energy by [13, 14]. Mirea [12] obtained TXE values by using the microscopic equations of motion with time dependent pairing equations. Here, we want to calculate TXE in a systematic method whose results are consistent with the experimental data.

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With increasing the excitation energy, TXE increases very slowly for photofission of  $^{238}\text{U}$  and  $^{232}\text{Th}$  [3, 15]. Of course, Piessens [3] divided the value of TXE by 8.6 MeV and presented the number of neutrons in Table 4. Therefore, the change of TXE with increasing the gamma energy is neglected in this work.

The theoretical framework and details of calculations are presented in section 2. In section 3, the calculated results of TXE for neutron fission of  $^{235}\text{U}$  are investigated to reach the proper results, then the calculated values of TXE for photofission of  $^{238}\text{U}$  are discussed by modifying the model. At last, the TXE of Uranium isotopes are calculated and plotted as a function of fission fragments by modified model. A summary is given in section 4.

## 2. Theoretical framework

The basic energy balance equation describes all partition available energy as

$$Q(A_L/A_H) + E = E_{pre} + E_{coul}(A_L/A_H) + E_{def}(A_L, A_H) + E_{dis} + E_h, \quad (1)$$

where  $A_L$  and  $A_H$  are the mass number of light and heavy fission fragments, respectively.  $E_{coul}$  is Coulomb potential energy at scission between two nascent fragments,  $E_{def}$  is deformation energy of the light and heavy nascent fragment at scission.  $E_{dis}$  and  $E_h$  are dissipative energy and intrinsic excitation energy, respectively.  $E_{pre}$  is prescission kinetic energy, and  $E$  is excitation energy. Ruben [1] and Gönmenwein [16] showed that  $E_{pre}$  is between 10 and 50 MeV and  $E_{dis}$  increases from 6.2 MeV for  $^{235}\text{U}(n,f)$  reaction to 9.6 MeV for  $^{252}\text{Cf}(sf)$ . The total excitation energy of the complementary fragments is given by

$$TXE(A_L, A_H) = E_{def}(A_L, A_H) + E_h + E_{dis}. \quad (2)$$

The intrinsic excitation energy is assumed as [1]

$$E_h = E - \Delta_{cn} - E_{f,b}, \quad (3)$$

where  $E_{f,b}$  is height of outer fission barrier.  $\Delta_{cn}$  is pairing energy of compound nucleus. By combining Eqs. 3 and 2, we have

$$TXE(A_L, A_H) = E_{def}(A_L, A_H) + E - \Delta_{cn} - E_{f,b} + E_{dis}. \quad (4)$$

The pairing energy of compound nucleus ( $\Delta_{cn}$ ) is calculated by a famous relation as

$$\Delta_{cn} = \begin{cases} 0 & \text{for } Z \text{ and } N \text{ even} \\ 12/\sqrt{Z} & \text{for } Z \text{ even and } N \text{ odd} \\ 12/\sqrt{N} & \text{for } N \text{ even and } Z \text{ odd} \\ 24/\sqrt{A} & \text{for } Z \text{ and } N \text{ odd.} \end{cases} \quad (5)$$

The deformation energy which could be defined by minimizing the nuclear potential in deformation space as ([1, 17])

$$E_{def} = \frac{E_{coul}^4}{4\alpha e^2 Z_1^2 (Z - Z_1)^2}, \quad (6)$$

where  $\alpha$  is the deformability parameter of each fragment and it is calculated in liquid drop model as [17]

$$\alpha = \alpha_0 - 0.063 \frac{Z^2}{A}. \quad (7)$$

By fitting the results of calculation to experimental data,  $\alpha_0$  equals to 4.86. The atomic number of fission fragments are obtained with the unchanged charge-density distribution as [6, 18]

$$Z_{UCD} = \frac{Z_{cn}(A + \nu)}{A_{cn}}, \quad (8)$$

where postscission neutrons  $\nu$  is defined by [19].  $\nu$  value equals to one by fitting the results of this equation for photofission are compared with the experimental data of [20].

In photofission phenomena, maximum energy of electron is called the end-point energy. To calculate the fission fragments mass distribution, this energy is used in formalisms. Otherwise, the values of peak-to-valley and average mass number are calculated by the average electron energy from zero to end-point energy ( $\langle E \rangle$ ) [21]. Thus, the excitation energy is the average excitation energy (i.e.  $E = \langle E \rangle$ ).

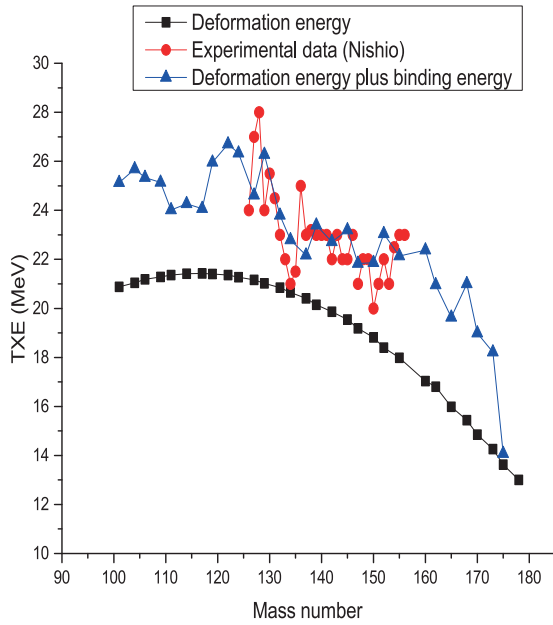
### 3. Results and discussion

The TXE for thermal neutron-fission of  $^{235}\text{U}$  as a function of fission fragments is presented in Figure 1. The squares represent the calculated results using Eq. 4 and the circles represent the experimental data associated with Figure 13 of [24]. The value of dissipative energy are taken from [1], which is equal to 6.2 MeV ( $E_{dis} = 6.2$  MeV). The height of outer fission barrier is taken from [22], which is equal to 5.9 MeV ( $E_{f,b} = 5.9$  MeV). The pairing energy of compound nucleus is calculated by Eq. 5, which is equal to 0.78 MeV ( $\Delta_{cn} = 0.78$  MeV). According to [1], the effect of pairing energy is not considered because the height of inner fission barrier for  $^{235}\text{U}$  nucleus is higher than the height of outer fission barrier. The average value of deformation energy ( $E_{def}$ ) is calculated for complementary fission fragments i.e. the sum of the deformation energy of both pair fragments is divided by 2. Also,  $E$  is definitely neglected for thermal neutron fission.

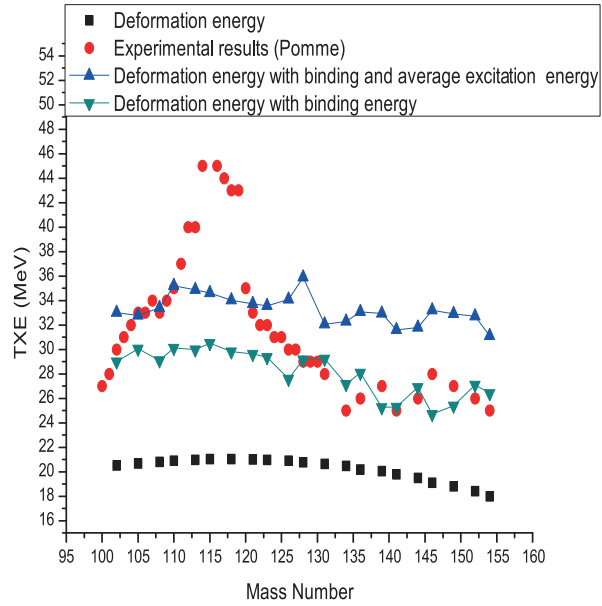
It is seen from Figure 1 that the experimental values are fluctuated, while the calculated results (square symbols) have a smooth changing. Beside, the experimental and calculated values are very different. So, we added neutron binding energy to fit the calculated and experimental values of TXE. Schmitt [25] applied neutron binding energy to calculate the total excitation energy of fission fragments. Manailescu [6] also presented the TXE relation with neutron binding energy. In the statistical model, the total excitation energy is discussed at the scission point, so the fission fragments represent preneutron-emission fragments (i.e. before neutron emission). Therefore, TXE values of these fission fragments include the binding energy of fragments. Then, the values of neutron binding energy are taken from [27] and the calculated results (triangular symbols) using the modified model are presented in Figure 1. This shows that by adding neutron binding energy, the calculated results are closer to the experimental data.

The calculated values of TXE for photofission of  $^{238}\text{U}$  as a function of mass number of fission fragments are calculated and plotted in Figure 2. For this nucleus, we have  $E_{dis} \simeq 6.6$  MeV (taken from [1]),  $E_{f,b} = 6.1$  MeV (taken from [22]). The pairing energy of compound nucleus using Eq. 5 equals to 1.54 MeV, but the height of inner fission barrier of  $^{238}\text{U}$  is higher than height of outer fission barrier, so  $\Delta_{cn}$  was chosen as zero in accordance with [1]. The squares represent the calculated results using Eq. 4 and the circles represent the

experimental data [15] in Figure 2. It is seen that the calculated (squares) and experimental (circles) values are very different, so the values of TXE are recalculated by adding the neutron binding energy. The upside-down triangular symbols in Figure 2 represent the calculated results using the neutron binding energy for photofission of  $^{238}\text{U}$ . Also, the average excitation energy ( $\langle E \rangle$ ) is added for comparing the calculated results (triangular) with the experimental data. It is seen from this Figure that the calculated results of total excitation energy for each fragment are closer to the experimental data only by using the neutron binding energy.



**Figure 1.** Calculated total excitation energy as a function of fragment mass for thermal neutron-fission of  $^{235}\text{U}$  accompany with experimental data [24].

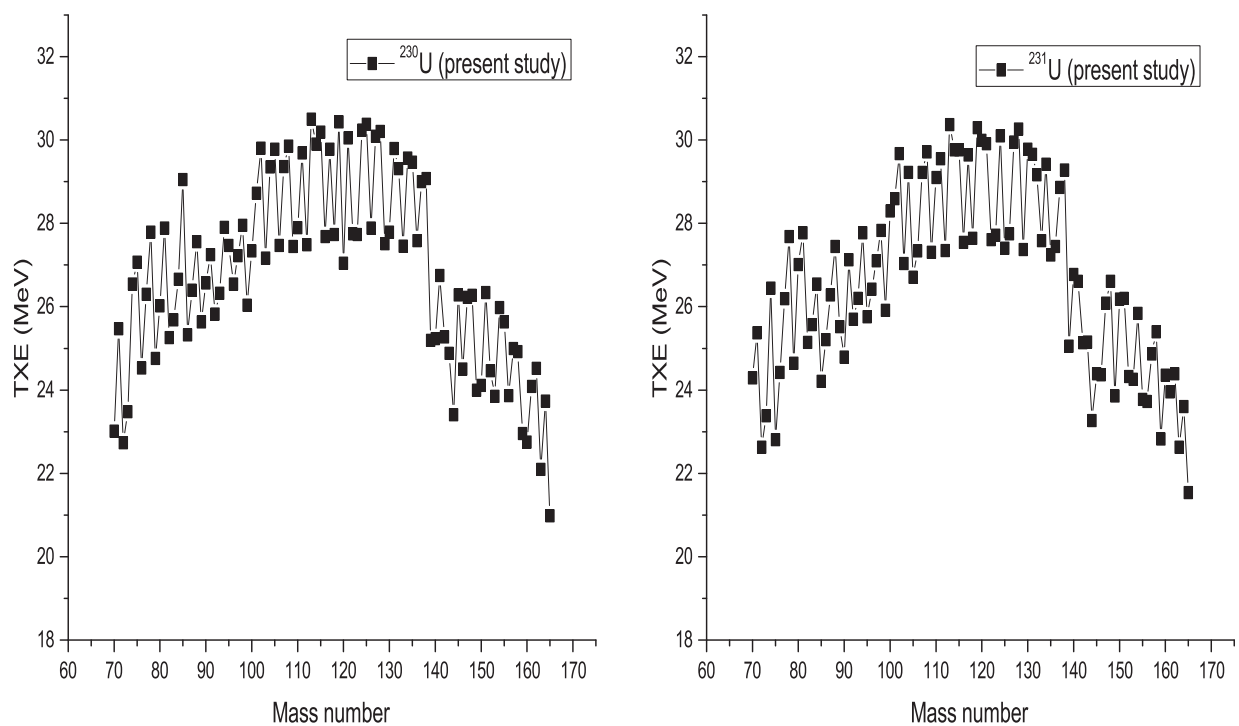


**Figure 2.** Calculated total excitation energy as a function of fragment mass for photofission of  $^{238}\text{U}$  at 13.15 MeV bremsstrahlung energy accompany with experimental data [15].

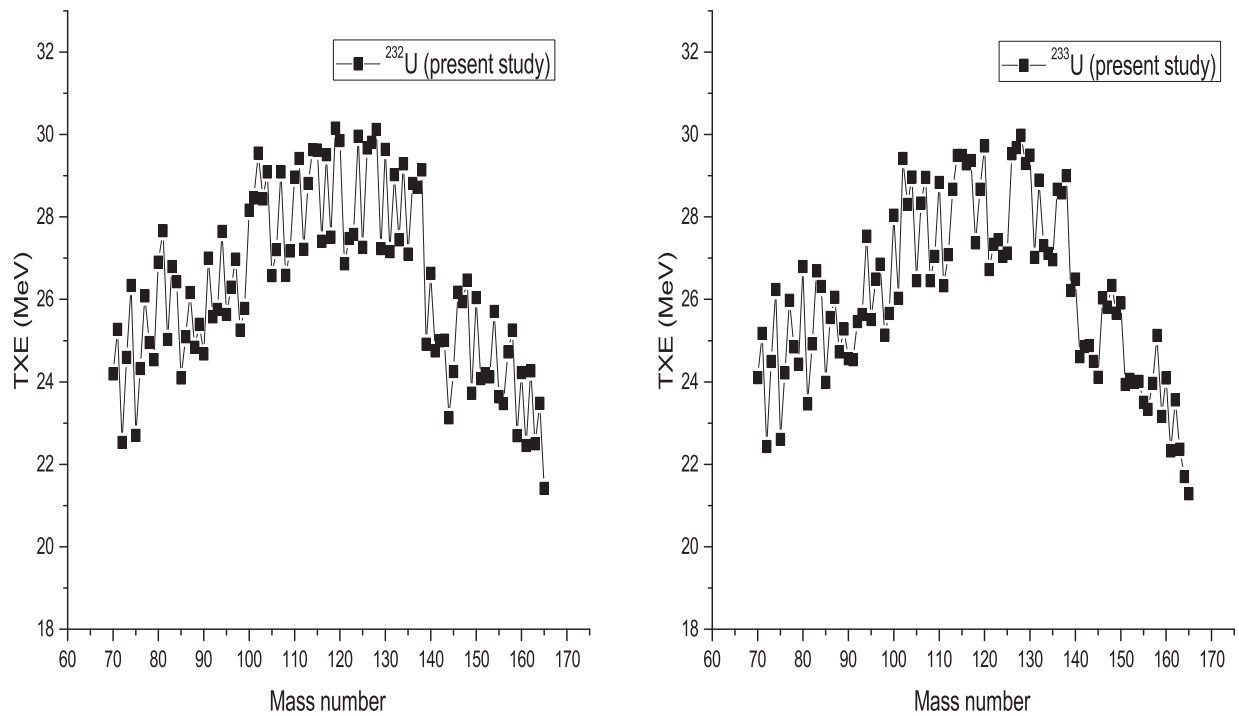
According to [15], a huge increase in values of TXE has been obtained for the symmetric mass region ( $A_{cn}/2$ ), but there is no huge increase in the energy values of this region ( $A=110-130$ ) in our calculations. This difference may be due to the large variation in the deformability parameter ( $\alpha$ ) in mass region  $A=110$  to  $130$ . In this mass range, the appropriate alpha values are not obtained with the liquid drop Eq. 7.

The values of neutron binding energy depend on the mass and atomic number of fission fragments. For more accuracy, three points are taken into account: 1. Initially, the atomic number of each fission fragment is calculated with Eq. 8. 2. The fission fragments that are closest to the experimental data are selected (Table 2 of [26]). 3. The fission fragments are selected with the maximum value of neutron binding energy. The calculated results for the fission fragment considering these three points are in good agreement with the experimental data.

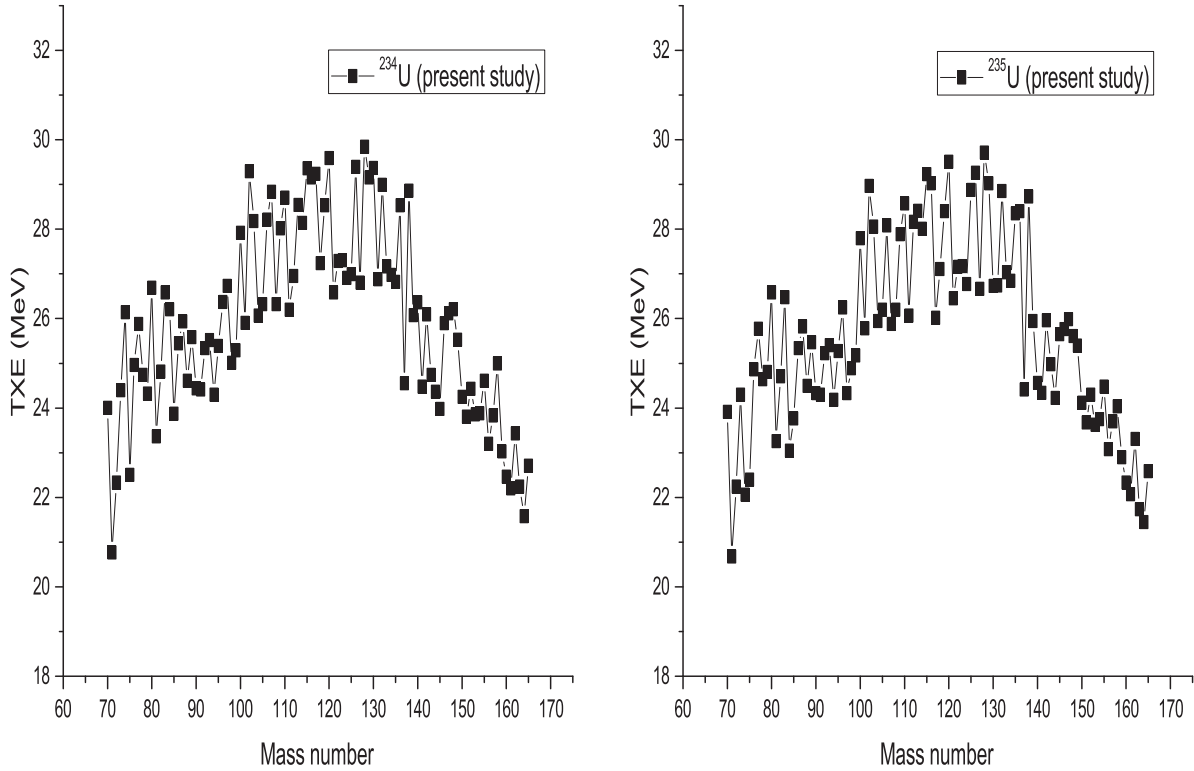
By this method, the TXE values for photofission of Uranium isotopes as a function of mass number of fission fragments are presented in Figures 3–6. The fluctuations in these figures are in range of 2 MeV and are related to the neutron binding energy values of fission fragments. Also, the calculated values for fission fragments with mass numbers between 110 and 130 are probably unreliable due to variation in deformability parameter values. However, the maximum calculated values occur in the symmetric region, but the experimental data are probably higher than these values. Also, the TXE values decrease with increasing and decreasing mass number of fission fragments from the symmetric region. The values of TXE range from 20 to 30 MeV.



**Figure 3.** Calculated total excitation energy as a function of fragment mass for photofission of  $^{230}\text{U}$  (left side) and  $^{231}\text{U}$  (right side).



**Figure 4.** Calculated total excitation energy as a function of fragment mass for photofission of  $^{232}\text{U}$  (left side) and  $^{233}\text{U}$  (right side).



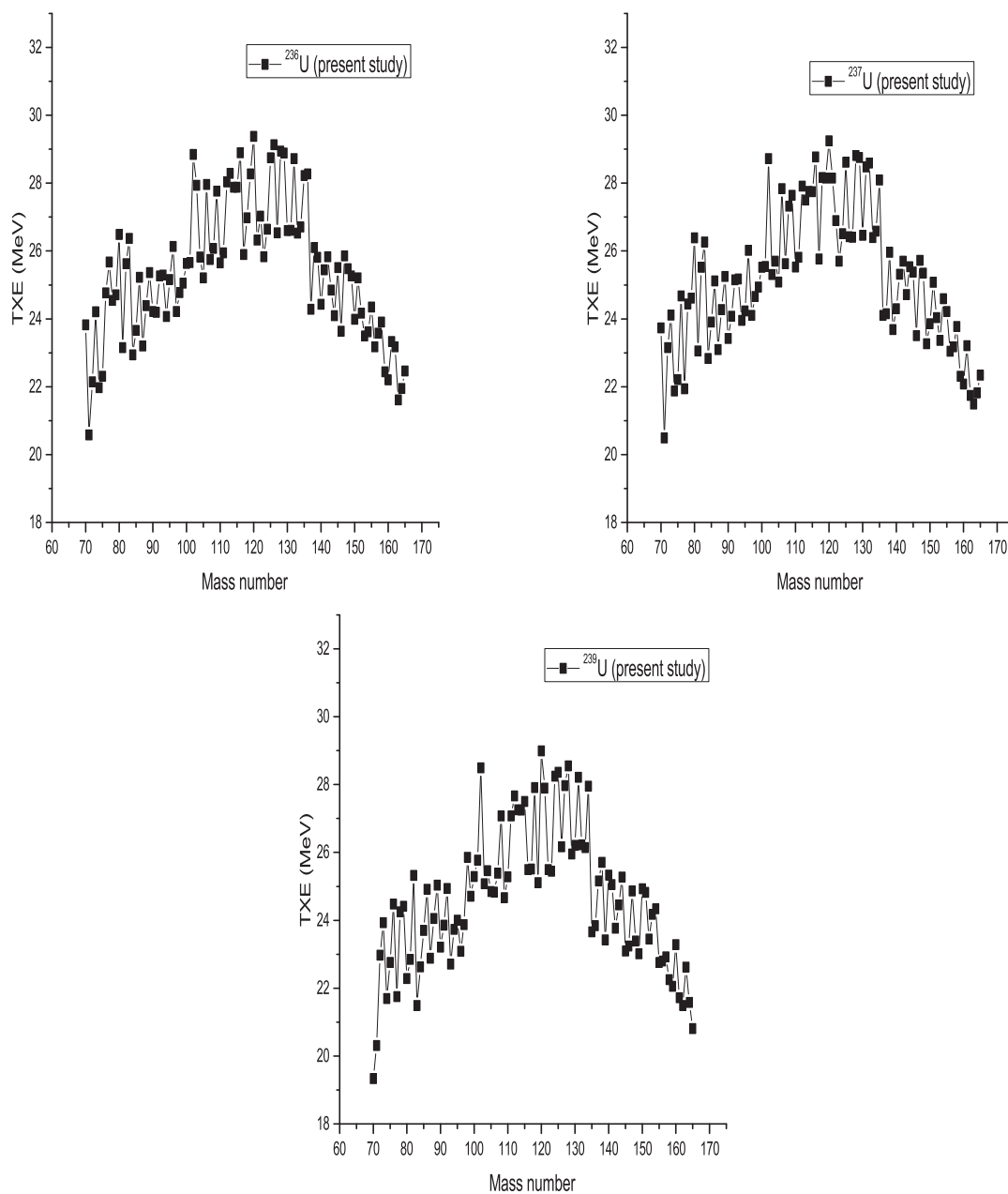
**Figure 5.** Calculated total excitation energy as a function of fragment mass for photofission of  $^{234}\text{U}$  (left side) and  $^{235}\text{U}$  (right side).

As it is seen in Figures 3–6 that the TXE values for photofission of  $^{232}\text{U}$  to photofission of  $^{239}\text{U}$  vary between 20 MeV and 30 MeV. According to [3, 15], it can be seen that with increasing the mass number of compound nucleus, the average of TXE values increases. This increase is seen for the photofission of  $^{238}\text{U}$  and  $^{232}\text{Th}$  when the average values of TXE for photofission of  $^{238}\text{U}$  is more than the average values of TXE for photofission of  $^{232}\text{Th}$  at the same excitation energy. (Of course, to obtain TXE values at different energies from Table 4 of [3], the number of emitted neutrons must be multiplied by 8.6 MeV.) On the other hand, the value of deformation energy decreases with increasing mass number, so the value of dissipative energy must increase as Ruben [1] prediction.

Figure 1 shows the TXE distribution of neutron fission of  $^{235}\text{U}$  and Figure 5 (right side) shows the TXE distribution of photofission of  $^{235}\text{U}$ , the results of these figures for same nucleus are so close together. This confirms that TXE values depend on the mass number of the compound nucleus, not the type of fission.

#### 4. Summary

The satisfactory agreement between the calculated and experimental values of TXE have been seen for photofission of  $^{238}\text{U}$  and neutron fission of  $^{234}\text{U}$  using the modified statistical model. This indicates that the calculated results are in good agreement with the experimental results with the addition of neutron binding energy of the fission fragments to the deformation energy. Then, total excitation energy for photofission of Uranium isotopes are calculated by adding the neutron binding energy. Of course, there may be a large increase in TXE values for the symmetrical region, which is not seen in our calculated results. This is because the liquid drop relation (Eq. 7) for deformability parameter is not proper for mass region  $A = 110$  to  $130$ .



**Figure 6.** Calculated total excitation energy as a function of fragment mass for photofission of  $^{236}\text{U}$  (left upper side) and  $^{237}\text{U}$  (right upper side) and  $^{239}\text{U}$  (lower side).

To fit the calculated and experimental values, the fission fragments are selected with the highest values of neutron binding energy. In new modified method, the gamma external energy and average center of mass system are not used in calculations. Also, because the dissipative energy values and the height of outer fission barrier are in the same range, TXE can only be calculated with deformation energy and neutron binding energy of fragments. Also, it is seen that TXE values depend on the mass number of the compound nucleus, not the type of fission.

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