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The field blocking on the interface with nonlinear response between nonlinear focusing media

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Abstract: We consider the nonlinear nonsymmetrical spatially distributed excitations near the thin layer with nonlinear properties separated with two media with Kerr-type focusing (positive) nonlinearity. We use the nonlinear Schrödinger equation with nonlinear potential containing two parameters to describe the new types of nonlinear stationary states. The problem is reduced to the solution of nonlinear Schrödinger equations on half spaces with the nonlinear boundary conditions at the interface plane. We obtain and analyze the two types of nonlinear excitations describing the effect of the field blocking passing through the interface into one half-space from another one. The field blocking effect is the localization of the nonlinear spatially periodic wave during the transition from one half-space to another one where the wave function is monotonically damping from the interface. We derive the exact energy levels of states of special kinds in explicit form. The conditions of the field blocking effect existence's dependence on the interface and media characteristics are found.

Key words: Planar defect, interface, nonlinear wave, interface state, soliton, localized state, nonlinear Schrödinger equation

1. Introduction

The theoretical and experimental study of nonlinear surface waves in solids has been carried out for a long time [1,2]. Such investigations remain relevant in connection with the wide application of the properties of nonlinear surface states in various optical devices and data storage systems [3–6].

Many authors have considered the localization near the contact of linear and nonlinear media [7–9]. Most often, the interface between media is taken into account in modeling by means of boundary conditions corresponding to the continuity of the field and its tangential component (normal derivative) [10,11]. In Refs. [12,13] we considered the nonlinear states near the boundary of linear and nonlinear media, taking into account interaction of excitations with the interface using the short-range potential.

The investigations of physical properties of nonlinear media boundaries (interfaces) have important applications in optic heterostructured systems [14] and magnetic multilayered structures [15–17]. The resonance scattering of nonlinear excitations on nonlinear media boundaries was considered in Refs. [18,19]. Localization and scattering of excitations near the interface of nonlinear media with spatial dispersion in continuous models were considered in Refs. [20,21]. The dynamics of nonlinear collective excitations and peculiarities of soliton motion in molecular systems with high dispersion were considered in Refs. [22–24].

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Many researchers use the nonlinear Schrödinger equation (NLSE) to describe nonlinear wave dynamics [12,13,18,19,25,26]. The NLSE describes the nonlinear surface shear acoustic waves in crystals [27], interaction of optic solitons with the interface between two nonlinear media [28], small-amplitude spin waves in multilayer magnetic materials [29,30], and spatial localization of nonlinear wave fluxes in systems of parallel optical waveguides [29,31]. We used the NLSE system to describe the coupled states localized near the interface between nonlinear media in two-level systems [32–34].

We note that the stationary NLSE is equivalent to the Ginsburg–Landau equation describing local electron phase transition [35,36]. The Gross–Pitaevskii equation describing excitons in a trap was also reduced to the NLSE [37] with local potential.

In this paper, we consider the field excitations of special nonsymmetrical kinds appearing near the interface between nonlinear focusing media. We suppose that the interface is a thin defect boundary layer with properties influencing the perturbations of the field distribution.

We use the NLSE with positive Kerr-type nonlinearity to describe the spatial distribution of such kinds of excitations in focusing media. Potential in the NLSE models the interface as a planar defect with inner properties in short-range (local) approximation of self-consistent fields. We take into account two interface characteristics: intensity of the interaction of the interface with excitations and intensity of the Kerr-type nonlinear response in the defect layer on excitation interactions.

Nonlinear potential in such a form was used in Refs. [38–40] to describe the spatial localization of nonlinear wave fluxes in a nonlinear medium containing a plane defect layer with nonlinear properties. Localized states near the interface with anharmonic properties between nonlinear media with different characteristics at opposite sides of the interface were analyzed in Ref. [41]. We also obtained the spatial periodic states forming the soliton lattice in a nonlinear focusing medium generated by a nonlinear planar defect in Ref. [42]. Periodic states near the plane defect with nonlinear response separating a nonlinear self-focusing medium and linear medium were considered in Ref. [43].

In the present paper we obtain new types of stationary states corresponding to the field blocking at a nonlinear interface separating nonlinear focusing media. The field blocking effect for the case of a simple interface without nonlinear response separating a nonlinear focusing medium and linear medium was described in Refs. [13,34]. The field blocking effect is the localization of a cnoidal wave during the transition from a half-space with a nonlinear medium to a half-space with a linear medium in which the wave function is monotonically damping from the interface [13]. Now we propose the theoretical analysis of the field blocking's existence on the boundary of two nonlinear media. We will find the wave functions describing the stationary states of such types. We will find the exact energy levels of such states and analyze the conditions of their existence.

The field blocking effects to be studied in nonlinear optic systems containing parallel optical waveguides are used as optical switches in existing devices [44,45].

2. The model formulation

We consider the propagation of a nonlinear electromagnetic wave along optical media with focusing nonlinearity containing the plane waveguide. The waveguide is formed by a defect layer characterized by the refractive index differing from that in the optical medium between them. The thin defect layer (interface) is located in the yz plane perpendicular to the x axis (perpendicular to the x coordinate). The interface thickness is much smaller than the characteristic width of excitation localization. We analyze the spatial distribution of nonlinear excitations propagating along the interface in the direction perpendicular to the direction of their propagation. Such a layered optical structure is similar to the so-called transverse Bragg waveguides created by thin-film multilayer structures [46] or the impurity band in a deep photonic bandgap [47].

For a plane-polarized wave propagating in nonmagnetic media along the interface, the electric field vector obeys the Maxwell equation:

$$n^2(x, \boldsymbol{E})\boldsymbol{E}''_{tt} = c^2 \Delta \boldsymbol{E},$$

with the refractive index dependent on coordinate x with Kerr-type nonlinearity in media and in thin defect layers:

$$n(x, \boldsymbol{E}) = n_0 + n_1 - \sigma\alpha|E|^2,$$

where n_0 and n_1 are the linear refractive indices in the wide layers and narrow layers of the waveguide, respectively; σ is a parameter that is -1 and $+1$ in the focusing medium and defocusing medium, respectively; and α is the nonlinearity coefficient of the medium [29].

We introduce the complex function $\psi = E_1 + iE_2$, where E_1 and E_2 are slowly varying functions of x and t related to the electric field strength by the following expression [29]:

$$\boldsymbol{E} = \boldsymbol{e}_y\{E_1(x,t)\cos(kz - \omega t) + E_2(x,t)\sin(kz - \omega t)\}.$$

This expression describes a monochromatic wave with wave vector and frequency $\omega = ck/n_0$.

In terms of the dimensionless time t in units of $2n_0/\alpha\omega$ and coordinate x in units $(n_0/\alpha)^{1/2}/k$, the function ψ at $n_1 \ll n_0$ and $\alpha|\psi|^2 \ll n_0$ satisfies the standard nonlinear Schrodinger equation (for more details, see Ref. [29]):

$$i\psi'_t = -\psi''_{xx} + 2\sigma\,|\psi|^2\,\psi + F(x, \psi),$$

with self-consistent nonlinear potential in the following form [42]:

$$F(x, \psi) = \sum \{U_0 + W_0\,|\psi|^2\}\delta(x - 2na)\psi,$$

where $U_0 = -2hn_1/n_0$, $W_0 = -a\beta/n_0$, h is the width of the waveguides, $2a$ is the distance between them, and β is the nonlinearity coefficient inside the waveguides [48,49].

We suppose that the width of waveguides is much smaller than the distance between them. Therefore, we consider a point interaction described by a Dirac delta function. In the case of weak coupling between plane parallel waveguides, the amplitude of the field in them is much larger than the average field amplitude in the entire crystal. For this reason, it was proposed to take into account nonlinear terms inside the waveguides [48]. Below we consider the optical media containing one plane waveguide corresponding to the interface between two nonlinear media.

Note that the nonlinear equation for ψ with a delta function term $F(x,\psi)$ with two parameters U_0 and W_0 was used in Ref. [49]. The model of an optical system in which periodic modulation of a linear refractive index is combined with a spatially inhomogeneous nonlinearity represented by a periodic Kronig–Penny lattice with a single nonlinear defect thin layer was described in Refs. [48,49].

Thus, we can use the model of an interface with nonlinear response based on the one-dimensional NLSE. The wave function ψ describes the excitation propagated along the interface and nonhomogeneously distributed perpendicular to the interface plane. We restrict our consideration to the case of spatially distributed stationary states and we will not analyze nonstationary phenomena. The energy of stationary state E plays the role of frequency ω of the monochromatic electric field.

The problem of finding stationary states with energy reduces to the stationary NLSE for wave function $\psi(x)$ in the following form:

$$E\psi = -\frac{1}{2m}\psi'' + \Omega\psi - \gamma(x)\left|\psi\right|^2\psi + U(x)\psi. \tag{1}$$

Here the values of energy range edge Ω and effective mass m of excitation are constant everywhere.

We assume that the nonlinearity of the medium is characterized by different absolute values of nonlinearity at the opposite sides of the interface:

$$\gamma(x) = \begin{cases} \gamma_1, \ x < 0; \\ \gamma_2, \ x > 0; \end{cases}$$

where the values of $\gamma_{1,2}$ are constant everywhere. In the present paper we consider the contact of self-focusing media described by the positive nonlinearity at both sides of the interface $\gamma_{1,2}>0$.

The interaction of excitations with nonlinear interface can be described by the short-range potential with Dirac delta function since the perturbation of the medium parameters is local. Following Ref. [38–43], we use nonlinear self-consistent potential of the following form:

$$U(x) = \{U_0 + W_0\left|\psi\right|^2\}\delta(x). \tag{2}$$

The potential given in Eq. (2) describes the thin layer (interface) with nonlinear properties (plane defect with nonlinear response). In Eq. (2) $\delta(x)$ is the Dirac delta function and U_0 is the intensity of the interaction of the defect with excitation (the "power" of the defect). For $U_0>0$ excitation is repelled from the defect and for $U_0<0$ the excitation is attracted to the defect. Parameter W_0 is the intensity of the nonlinear defect response. A positive value of W_0 corresponds to the defocusing inside the thin defect layer, and a negative value of W_0 corresponds to self-focusing.

Note that the case of a defect without nonlinear response when $W_0=0$ and $U_0\neq 0$ in nonlinear medium was analyzed in Ref. [27]. The case of $U_0=0$ and $W_0\neq 0$ was considered in Refs. [39–41]. The nonlinear defect with both nonzero defect parameters ($U_0\neq 0, W_0\neq 0$) was considered in Ref. [38] in the case of a linear medium, in [42] in the case of the contact of two nonlinear media with different characteristics, and in the case of the contact of nonlinear focusing and linear media in Ref. [43].

We introduce the two wave functions describing the excitation distributions on both sides of the interface in the following form:

$$\psi(x) = \begin{cases} \psi_1(x), \ x < 0; \\ \psi_2(x), \ x > 0; \end{cases}$$

The NLSE of Eq. (1) with the potential of Eq. (2) reduces to solving two NLSEs:

$$\frac{1}{2m}\psi_j''(x) + (E-\Omega)\psi_j(x) + \gamma_j\left|\psi_j(x)\right|^2\psi_j(x) = 0. \tag{3}$$

Here and subsequently, $j=1$ corresponds to the left side of the interface ($x<0$) and $j=2$ corresponds to the right side of the interface ($x>0$).

The wave functions must satisfy the conjugation boundary conditions on the interface at the plane $x=0$:

$$\psi_1 = \psi_2 = \psi_0, \tag{4}$$

$$\psi_2' - \psi_1' = 2m\psi_0\{U_0 + W_0\,|\psi_0|^2\}. \tag{5}$$

Here ψ_0 is the amplitude of the interface oscillations.

From Eq. (5), in the case of $W_0 = 0$, we can obtain the well-known standard boundary condition corresponding to the simple defect (short-range delta function potential) [50].

3. Exact solutions and dispersion equations

3.1. Localization of the first type of state on half space

As is well known, the types of solutions of the NLSE are determined by the sign of nonlinearity. In focusing nonlinear medium the NLSE of Eq. (3) has two types of periodic solutions in self-focusing media with energy in the range of $E<\Omega$.

The first type of state on the left side of the interface for $x<0$ for the positive nonlinearity γ_1 in the energy range of $E<\Omega$ is described by the periodic NLSE solution:

$$\psi_1(x) = A_c \mathrm{cn}(q_c(x - x_1), k), \tag{6}$$

$$q_c^2 = 2m(\Omega - E)/(2k^2 - 1), \tag{7}$$

$$A_c^2 = k^2 q_c^2/(m\gamma). \tag{8}$$

Here, k is the modulus of elliptical function cn, $1/2<k^2<1$. The function of Eq. (6) describes a nonlinear periodic state (nonlinear wave) of the first type. Wave number q_c of the wave of Eq. (6) is determined by Eq. (7). The amplitude A_c of the wave of Eq. (6) is determined by Eq. (8).

The first type of state on the right side of the interface for $x>0$ for the positive nonlinearity γ_2 in the energy range of $E<\Omega$ is described by the following equations:

$$\psi_2(x) = A/\cosh\left(q(x - x_2)\right), \tag{9}$$

$$q^2 = 2m(\Omega - E) = (2k^2 - 1)q_c^2, \tag{10}$$

$$A^2 = q^2/(m\gamma). \tag{11}$$

Eq. (9) is a well-known NLSE soliton for the case of positive nonlinearity. Eq. (9) describes a nonlinear localized state of the right side of the interface. Wave number q of the soliton of Eq. (9) is determined by Eq. (10). Soliton amplitude A is determined by Eq. (11). Parameter x_2 characterizes the positions of the soliton "center" position.

The wave functions of Eqs. (6) and (9) coupled by the boundary conditions of Eqs. (4) and (5) on the interface are the interface state of the first type. We obtain the following dispersion equations after substituting of Eqs. (6) and (9) into of Eqs. (4) and (5):

$$\eta k \mathrm{cn}(q_c x_1, k) = (2k^2 - 1)^{1/2}/\cosh(qx_2), \tag{12}$$

$$q\{\tanh(qx_2) - (2k^2-1)^{-1/2}\mathrm{sn}(q_c x_1,k)\mathrm{dn}(q_c x_1,k)/\mathrm{cn}(q_c x_1,k)\} = F(U_0,V_0), \tag{13}$$

where $\eta = (\gamma_2/\gamma_1)^{1/2}$, $V_0 = W_0/\gamma_2$, and $F(U_0,V_0) = 2\{mU_0 + V_0 q_c^2/\cosh^2(qx_2)\}$.

The energy of the first type of state is determined from dispersion relations, Eqs. (12) and (13).

3.2. Localization of the second type of state on half space

The second type of state on the left side of the interface for $x<0$ is described by the periodic NLSE solution for the positive nonlinearity γ_1 in the energy range of $E<\Omega$:

$$\psi_1(x) = A_d \mathrm{dn}(q_d(x-x_1),k), \tag{14}$$

$$q_d^2 = 2m(\Omega - E)/(2-k^2), \tag{15}$$

$$A_d^2 = q_d^2/(m\gamma). \tag{16}$$

Here, k is the modulus of elliptical function dn, $0<k^2<1$. Eq. (14) describes the nonlinear periodic state (nonlinear wave) of the second type. Wave number q_d of the wave of Eq. (14) is determined by Eq. (15). Amplitude A_d of the wave of Eq. (14) is determined by Eq. (16).

The second type of state on the right side of the interface for $x>0$ is described by Eqs. (9)–(11). We note that $q = q_d(2-k^2)^{1/2}$.

The wave functions of Eqs. (9) and (14) coupled with the boundary conditions of Eqs. (4) and (5) on the interface are the stationary interface state of the second type. We obtain the following dispersion equations after substituting Eqs. (9) and (14) into the boundary conditions of Eqs. (4) and (5):

$$\eta \mathrm{dn}(q_d x_1,k) = (2-k^2)^{1/2}/cosh(qx_2), \tag{17}$$

$$q\{\tanh(qx_2) - k^2(2k^2-1)^{-1/2}\mathrm{sn}(q_d x_1,k)\mathrm{cn}(q_d x_1,k)/\mathrm{dn}(q_d x_1,k)\} = F(U_0,V_0). \tag{18}$$

Here the function $F(U_0,V_0)$ is defined by the same equation from Section 3.1.

We can exclude, for example, x_2 from Eq. (13) (or Eq. (18)) using Eq. (12) (or Eq. (17)). Thus, from the dispersion relations, we can obtain the energy as a function of the parameters: $E = E(m,U_0,V_0,\eta,k,x_1)$. Therefore, the coupled interface states of both types are the NLSE solutions with two free parameters k and x_1.

The stationary interface states of the first type defined by Eqs. (6) and (9) and the second type defined by Eqs. (9) and (14) describe the field blocking effect. The periodic nonlinear waves of Eqs. (6) or (14) forming the soliton lattice on one side of the interface transform into the localized state of Eq. (9) damping monotonically on another side of the interface.

4. Energy of field blocking

4.1. Energy of field blocking of the first type of states

We can find the exact solutions of the dispersion equations (Eqs. (12) and (13)) for the three following cases.

Case A: $x_1 = 0$ and $x_2 = 0$.

From Eq. (12) we derive the elliptic function module:

$$k = (2 - \eta^2)^{-1/2}. \tag{19}$$

From Eq. (13) we obtain the equation $F(U_0, V_0) = 0$. We find the wave number:

$$q = (-mU_0/V_0)^{1/2}. \tag{20}$$

Using Eq. (19), from Eq. (10) we derive the exact energy level of field blocking:

$$E = \Omega + (U_0/2V_0). \tag{21}$$

Using Eq. (19) we also find the amplitude of the interface oscillations:

$$\psi_0 = (-U_0/W_0)^{1/2}. \tag{22}$$

From Eq. (19) we see that $0 < \eta^2 < 1$. In such a case, the nonlinearity parameter of the media where the field fades away (on the right side of the interface) should be less than the nonlinearity parameter of the media where the soliton lattice is formed (on the left side of the interface): $\gamma_2 < \gamma_1$.

We also see that the state with energy described by Eq. (21) is possible only at opposite signs of the interface parameters. Therefore, the field blocking effect appears for the energy described by Eq. (21) in the case of the defocusing nonlinearity of the defect ($W_0 > 0$) and the attractive defect ($U_0 < 0$) or in the case of the focusing nonlinearity of the defect ($W_0 < 0$) and the repulsive defect ($U_0 > 0$).

Moreover, the existence of the field blocking effect for the energy described by Eq. (21) is exclusively due to the nonlinear properties of the defect because it does not appear at $W_0 = 0$.

Case B: $x_1 = 0$ and $x_2 \neq 0$.

From the dispersion equations (Eqs. (12) and (13)), we find the wave number:

$$q = q_{c0}\{1 \pm (1 - q_{ca}/q_{c0})^{1/2}\}, \tag{23}$$

where $q_{c0} = (2k^2 - 1)^{1/2}\{(2 - \eta^2)k^2 - 1\}^{1/2}/4V_0\eta^2k^2$ and

$$q_{ca} = 4mU_0(2k^2 - 1)^{1/2}/\{(2 - \eta^2)k^2 - 1\}^{1/2}.$$

From Eq. (12) we obtain the soliton "center" position with the substituting q defined by Eq. (23):

$$x_2 = \frac{1}{q}\cosh^{-1}\left\{\frac{(2k^2 - 1)^{1/2}}{\eta k}\right\}. \tag{24}$$

Using Eq. (23), from Eq. (10) we derive the exact energy level of the field blocking:

$$E = \Omega - \Omega_{c0}\{1 \pm (1 - q_{ca}/q_{c0})^{1/2}\}^2, \tag{25}$$

where $\Omega_{c0} = q_{c0}^2/2m$.

We see that the field blocking effect for energy described by Eq. (25) can exist under conditions of $U_0 < \{(2-\eta^2)k^2-1\}/16mV_0\eta^2k^2$ and $\eta^2 < 2-k^{-2}$. The second one leads to $0 < \eta^2 < 1$ $(\gamma_2 < \gamma_1)$.

We also see that the state with energy described by Eq. (25) is possible only at the same signs of the interface parameters. Therefore, the field blocking effect appears for the energy described by Eq. (25) in the case of the defocusing nonlinearity of the defect ($W_0 > 0$) and the repulsive defect ($U_0 > 0$) or in the case of the focusing nonlinearity of the defect ($W_0 < 0$) and the attractive defect ($U_0 < 0$).

In the case of an interface without nonlinear response, when $W_0 = 0$ and $U_0 \neq 0$ we obtain $q = q_{ca}/2$ and the following energy:

$$E = \Omega - 2mU_0^2(2k^2-1)/\{(2-\eta^2)k^2-1\}. \tag{26}$$

We derive that in the case of an interface without nonlinear response the characteristic width of excitation localization is inversely proportional to the "power" of the interface: $l \sim 1/q \sim 1/|U_0|$.

In the case of the nonlinear interface, when $U_0 = 0$ and $W_0 \neq 0$ we obtain wave number $q = 2q_{c0}$ and the following energy:

$$E = \Omega - (2k^2-1)\{(2-\eta^2)k^2-1\}/8mV_0^2\eta^4k^4. \tag{27}$$

We find that in the case of a nonlinear interface the characteristic width of excitation localization is proportional to the interface nonlinearity: $l \sim 1/q \sim |W_0|$.

Case C: $x_1 \neq 0$ and $x_2 = 0$.

From the dispersion equations (Eqs. (12) and (13)) we find the wave number:

$$q = q_{c1}\{-1 \pm (1 - q_{cb}/q_{c1})^{1/2}\}, \tag{28}$$

where $q_{c1} = a_c(\eta,k)/4V_0$, $q_{cb} = 4mU_0/a_c(\eta,k)$, and

$$a_c(\eta,k) = \{(2-\eta^2)k^2-1\}^{1/2}\{1-\eta^2-(2-\eta^2)k^2\}^{1/2}/\eta(2k^2-1).$$

Using Eq. (28), from Eq. (10) we derive the exact energy level of field blocking:

$$E = \Omega - \Omega_{c1}\{-1 \pm (1 - q_{cb}/q_{c1})^{1/2}\}^2, \tag{29}$$

where $\Omega_{c1} = q_{c1}^2/2m$.

We see that the field blocking effect for energy described by Eq. (29) can exist under conditions of $U_0 < a_c^2(\eta,k)/16mV_0$ and $\eta^2 < 2-k^{-2}$. Also, we see that the state with energy described by Eq. (29) is possible only at the same signs of the interface parameters.

In the case of an interface without nonlinear response, when $W_0 = 0$ and $U_0 \neq 0$ we obtain $q = -q_{cb}/2$ and the following energy:

$$E = \Omega - 2mU_0^2/a_c^2(\eta,k). \tag{30}$$

The characteristic width of excitation localization is inversely proportional to the "power" of the interface $l \sim 1/|U_0|$ as we have obtained in case B for interface without nonlinear response.

In the case of a nonlinear interface, when $U_0 = 0$ and $W_0 \neq 0$ we obtain wave number $q = -2q_{c1}$ and the following energy:

$$E = \Omega - a_c^2(\eta,k)/8mV_0^2. \tag{31}$$

The characteristic width of excitation localization is proportional to the interface nonlinearity $l \sim |W_0|$ as we have obtained in case B for nonlinear interface.

As is well known, the types of solutions of the NLSE are determined by the sign of nonlinearity. In the focusing nonlinear medium the NLSE of Eq. (3) has two types of periodic solutions in self-focusing media with energy in the range of $E<\Omega$.

4.2. The energy of field blocking of the second type of states

We can find the exact solutions in the dispersion equations (Eqs. (17) and (18)) for three following cases.

Case A: $x_1 = 0$ and $x_2 = 0$.

From Eq. (17) we define the elliptic function module:

$$k = (2 - \eta^2)^{1/2}. \tag{32}$$

We note that the product of elliptic modules specified by Eqs. (19) and (32) is equal to unity.

From Eq. (32) we derive that $1 < \eta^2 < 2$. In such a case, the nonlinearity parameter of the media where the field fades away (on the right side of the interface) is fulfilled by the condition $\gamma_1 < \gamma_2 < 2\gamma_1$.

From Eq. (18), we obtain the same wave number of Eq. (20) and the exact energy level of the field blocking of Eq. (21). Thus, the coupled states of both types in case A have the same energy.

Case B: $x_1 = 0$ and $x_2 \neq 0$.

From the dispersion equations (Eq. (17) and (18)), we find the following wave number:

$$q = q_{d0}\{1 \pm (1 - q_{da}/q_{d0})^{1/2}\}, \tag{33}$$

where $q_{d0} = (2 - k^2)^{1/2}(2 - k^2 - \eta^2)^{1/2}/4V_0\eta^2$ and $q_{da} = 4mU_0(2 - k^2)^{1/2}/(2 - k^2 - \eta^2)^{1/2}$.

We obtain the soliton "center" position from the dispersion equation (Eq. (17)) substituting q as defined by Eq. (33):

$$x_2 = \frac{1}{q}\cosh^{-1}\left\{\frac{(2 - k^2)^{1/2}}{\eta}\right\}. \tag{34}$$

Using Eq. (33), from Eq. (10) we derive the exact energy level of field blocking:

$$E = \Omega - \Omega_{d0}\{1 \pm (1 - q_{da}/q_{d0})^{1/2}\}^2, \tag{35}$$

where $\Omega_{d0} = q_{d0}^2/2m$.

We derive that the field blocking effect for energy described by Eq. (35) can exist under conditions of $U_0 < (2 - k^2 - \eta^2)/16mV_0\eta^2$ and $\eta^2 < 2 - k^2$. The second one leads to $1 < \eta^2 < 2$ $(\gamma_1 < \gamma_2 < 2\gamma_1)$.

In the case of an interface without nonlinear response, when $W_0 = 0$ and $U_0 \neq 0$ we obtain $q = q_{da}/2$ and the following energy:

$$E = \Omega - 2mU_0^2(2 - k^2)/(2 - k^2 - \eta^2). \tag{36}$$

We derive that in the case of an interface without nonlinear response the characteristic width of the second type of excitation localization is inversely proportional to the "power" of the interface $l \sim 1/|U_0|$ as for the first type of state.

In the case of a nonlinear interface, when $U_0 = 0$ and $W_0 \neq 0$ we obtain wave number $q = 2q_{d0}$ and the following energy:

$$E = \Omega - (2-k^2)(2-k^2-\eta^2)/8mV_0^2\eta^4. \tag{37}$$

We find that in the case of a nonlinear interface the characteristic width of the second type of excitation localization is proportional to the interface nonlinearity $l \sim |W_0|$ as for the first type of state.

Case C: $x_1 \neq 0$ and $x_2 = 0$.

From the dispersion equations (Eqs. (17) and (18)) we find the wave number of the second type of excitation:

$$q = q_{d1}\{-1 \pm (1 - q_{db}/q_{d1})^{1/2}\}, \tag{38}$$

where $q_{d1} = a_d(\eta,k)/4V_0$, $q_{db} = 4mU_0/a_d(\eta,k)$,

and $a_d(\eta,k) = (\eta^2 - 2 + k^2)^{1/2}\{2 - k^2 - \eta^2(1-k^2)\}^{1/2}/\eta(2-k^2)$.

Using Eq. (38), from Eq. (10) we derive the exact energy level of field blocking for the second type of state:

$$E = \Omega - \Omega_{d1}\{-1 \pm (1 - q_{db}/q_{d1})^{1/2}\}^2, \tag{39}$$

where $\Omega_{d1} = q_{d1}^2/2m$.

We obtain that the field blocking effect for the energy described by Eq. (39) can exist under the condition $U_0 < a_d^2(\eta,k)/16mV_0$. Also, we see that the state with energy described by Eq. (39) is possible only for the same signs of the interface parameters.

In the case of an interface without nonlinear response, when $W_0 = 0$ and $U_0 \neq 0$ we obtain $q = -q_{db}/2$ and the following energy:

$$E = \Omega - 2mU_0^2/a_d^2(\eta,k). \tag{40}$$

The characteristic width of the second type of excitation localization is inversely proportional to the "power" of the interface $l \sim 1/|U_0|$ as we have obtained in case B for interface without nonlinear response.

In the case of a nonlinear interface, when $U_0 = 0$ and $W_0 \neq 0$ we obtain wave number $q = -2q_{d0}$ and the following energy:

$$E = \Omega - a_d^2(\eta,k)/8mV_0^2. \tag{41}$$

The characteristic width of the second type of excitation localization is proportional to the interface nonlinearity $l \sim |W_0|$ as we have obtained in case B for nonlinear interface.

5. Excitation localization at both sides of the interface

It is known that elliptic functions transform into hyperbolic ones in the limit $k \to 1$. This circumstance allows us to describe in the framework of the proposed model the states localized at both sides of the interface of nonlinear media.

The periodic state on the right side of the interface of the first type described by Eq. (6) in the limit $k \to 1$ transforms into the localized state described by Eq. (9), since $\mathrm{cn}(z) \to 1/\cosh(z)$, and from Eqs. (7) and (8) $q_c^2 \to q^2 = 2m(\Omega - E)$ and from Eqs. (8) and (11) $A_c \to A = q(m\gamma)^{-1/2}$.

The periodic state on the right side of the interface of the second type described by Eq. (14) in the limit $k \to 1$ transforms into the same localized state described by Eq. (9), since $\mathrm{dn}(z) \to 1/\mathrm{ch}(z)$, and from Eq. (15)

$q_d \to q$, and from Eq. (15) $A_d \to A$. Thus, two periodic states degenerate into one localized state in the limit $k \to 1$.

The wave functions of the localized state have the following form:

$$\psi_j(x) = A/\cosh(q(x - x_j), k). \tag{42}$$

Here, wave number q is defined by Eq. (10) and the amplitude of nonlinear localized state A is defined by Eq. (11).

Substituting into Eq. (12) or Eq. (17) $k = 1$, we obtain the following:

$$\eta/\cosh(qx_1) = 1/\cosh(qx_2). \tag{43}$$

Substituting into Eq. (18) or Eq. (22) $k = 1$, we obtain the dispersion relation localized on both sides from defect states:

$$q\{\tanh(qx_2) - \tanh(qx_1)\} = 2\{mU_0 + V_0q^2/\cosh^2(qx_2)\}. \tag{44}$$

We analyze the solutions of the dispersion equations (Eqs. (43) and (44)) in the four following cases.

Case A: $x_2 = -x_1 = x_0$.

From Eq. (43) we find $\eta = 1$. In the case considered the nonlinearity parameters are equal to each other: $\gamma_1 = \gamma_2 = \gamma$.

The amplitude of interface oscillation has the following form:

$$\psi_0 = q/(m\gamma)^{1/2}\cosh^2(qx_0). \tag{45}$$

The dispersion equation (Eq. (44)) takes the following form:

$$q\tanh(qx_0) = mU_0 + V_0q^2/\cosh^2(qx_2). \tag{46}$$

From Eq. (46) for $V_0 = 0$ and $U_0 \neq 0$ we obtain the dispersion equation analyzed in Ref. [26] in the case of defect without nonlinear response in focusing medium.

We can find solution of the dispersion equation (Eq. (46)) for the long-wave approximation when $qx_0 \ll 1$. The long-wave approximation means that the excitation energy is close to the edge of the spectrum under the condition $|\Omega - E| \ll 1/2mx_0^2$.

From the dispersion equation (Eq. (26)) we obtain the wave number:

$$q^2 = mU_0/(x_0 - V_0), \tag{47}$$

and the energy of long-wave excitations:

$$E = \Omega - U_0/2(x_0 - V_0). \tag{48}$$

Localized long-wave states with energy of Eq. (48) can exist when one of the following two pairs of conditions is fulfilled: 1) $U_0 > 0$ and $W_0 < \gamma x_0$, 2) $U_0 < 0$ and $W_0 > \gamma x_0$.

If x_0 is positive then the interface must be characterized by defocusing nonlinearity ($W_0 > 0$) and the excitation has been attracted to the interface ($U_0 < 0$). If x_0 is negative then the interface must be characterized by focusing nonlinearity ($W_0 < 0$) and the excitation has been repelled from interface ($U_0 > 0$).

Case B: $x_1 = x_2 = x_0$.

From Eq. (43) we find the same as in the case A result: $\gamma_1 = \gamma_2$.

The dispersion equation (Eq. (44)) takes the following form:

$$q^2/\cosh^2(qx_0) = -mU_0/V_0. \tag{49}$$

From Eq. (49) we see that the localized states in the case considered can exist only for the opposite signs of interface parameters independent of the sign of x_0.

From Eq. (49) we obtain the energy of long-wave excitations (qx_0 «1):

$$E = \Omega - \{1 \pm (1 + 4mx_0^2U_0/V_0)^{1/2}\}/4mx_0^2. \tag{50}$$

Localized long-wave states with the energy of Eq. (50) can exist under the condition $U_0 > -W_0/4mx_0^2\gamma$.

We see that the solution of the dispersion equations (Eqs. (46) and (49)) gives the nonlinear localized state energy as the functions of one free (control) parameter, x_0.

Case C: $x_1 = 0$ and $x_2 \neq 0$.

In the limit $k = 1$ we obtain the wave number in the form of Eq. (23) or (33) with

$$q_{c0} = q_{d0} = (1 - \eta^2)^{1/2}/4V_0\eta^2, \tag{51}$$

$$q_{ca} = q_{da} = 4mU_0/(1 - \eta^2)^{1/2}. \tag{52}$$

We obtain the soliton “center” position from Eq. (34):

$$x_2 = \frac{1}{q}\cosh^{-1}(1/\eta). \tag{53}$$

The exact energy localized level has the form of Eq. (28) or (38) with parameters defined by Eqs. (51) and (52).

Case D: $x_1 \neq 0$ and $x_2 = 0$.

In the limit $k = 1$ we obtain the wave number in the form of Eq. (23) or (33) with

$$q_{c1} = q_{d1} = (\eta^2 - 1)^{1/2}/4V_0\eta, \tag{54}$$

$$q_{db} = 4mU_0\eta/(\eta^2 - 1)^{1/2}. \tag{55}$$

We obtain the soliton “center” position:

$$x_1 = \frac{1}{q}\cosh^{-1}(\eta). \tag{56}$$

The exact energy localized level has the form of Eq. (29) or (39) with parameters defined by Eqs. (54) and (55).

We see that the localized state of case C differs from the localized state of case D in the following. For case C the nonlinearity parameter on the right side of the interface should be less than the nonlinearity parameter on the left side of the interface ($\gamma_2 < \gamma_1$), but for case D the nonlinearity parameter on the right side of the interface should be greater than the nonlinearity parameter on the left side of the interface ($\gamma_2 > \gamma_1$).

6. Discussion

If it is possible to measure the amplitude of the defect layer oscillations in the experimental situation then we can choose ψ_0 as a free parameter. We show below that we can find the exact solution of the dispersion equation (Eq. (46)) to obtain the localized state with $x_2 = -x_1 = x_0$ without using long-wave approximation.

If we now choose ψ_0 as a free parameter, then we derive from Eq. (46) the dispersion equation in the following form:

$$(q^2 - m\gamma\psi_0^2)^{1/2} = m(U_0 + W_0\psi_0^2). \tag{57}$$

We find the wave number as the exact solution of Eq. (57):

$$q^2(\psi_0) = m\{\gamma\psi_0^2 + m(U_0 + W_0\psi_0^2)^2\}. \tag{58}$$

Using Eq. (58) we obtain the exact energy level of the localized state in the following form:

$$E(\psi_0) = \Omega - \{\gamma\psi_0^2 + m(U_0 + W_0\psi_0^2)^2\}/2. \tag{59}$$

From Eq. (38) we can find the soliton "center" position:

$$x_0(\psi_0) = \frac{\cosh^{-1}\{[m\{\gamma\psi_0^2 + m(U_0 + W_0\psi_0^2)^2\}]^{1/2}/\psi_0(mg)^{1/2}\}}{[m\{\gamma\psi_0^2 + m(U_0 + W_0\psi_0^2)^2\}]^{1/2}}. \tag{60}$$

Eqs. (58)–(60) totally define the parameters of the localized state described by functions:

$$\psi_j(x) = A/\cosh(q(|x| + x_0)). \tag{61}$$

We can simplify Eqs. (58) and (59) for the case of small-amplitude oscillations when $\psi_0^2 << |U_0/W_0|$. We obtain from Eq. (58) the wave number of small-amplitude localized oscillations in the following form:

$$q(\psi_0) = q_{L0} + \alpha\psi_0^2,$$

where $q_{L0} = m\,|U_0|$ and $\alpha = (2mU_0W_0 + \gamma)/2\,|U_0|$.

We obtain from Eq. (59) the energy of small-amplitude localized oscillations in the following form:

$$E(\psi_0) = E_{L0} - \alpha\,|U_0|\,\psi_0^2,$$

where $E_{L0} = \Omega - q_{L0}^2/2m$.

We see that the energy of small-amplitude nonlinear localized oscillations depends on the square of the amplitude of the interface oscillations, as expected.

It is interesting to note that in the case of linear medium ($\gamma = 0$) containing a simple defect without nonlinear response ($U_0 \neq 0, W_0 = 0$) we obtain the classical solution for the linear Schrodinger equation with delta function potential corresponding to the attractive defect ($U_0 < 0$).

In this case, the wave function of Eq. (61) takes the well-known form (see Ref. [50]):

$$\psi(x) = \psi_0 \exp(-q\,|x|).$$

From Eq. (58) we derive the damping space factor localized in linear media states:

$$q = q_{L0} = -mU_0,$$

and from Eq. (64) we derive the energy:

$$E = E_{L0} = \Omega - mU_0^2/2.$$

Thus, the model that we propose contains in the limit case the well-known results for the linear Schrodinger equation with delta function potential modeling an attractive defect [50].

7. Conclusions

In the present paper we derived new types of stationary states appearing near a nonlinear focusing media interface with nonlinear properties. Such states are generated by coupling on interface periodic and soliton solutions of the nonlinear Schrodinger equation with positive Kerr-type nonlinearity. We used the nonlinear potential with cubic nonlinearity for mathematical formulation of the model to describe the defect with nonlinear properties. As a result, we came to the contact boundary-value problem for the NLSE with nonlinear boundary conditions.

We found the solutions of the formulated problem and derived the dispersion equations to define the nonlinear excitation energy of new two types. The exact periodic NLSE solutions in the form of Jacobi elliptic functions of Eqs. (6) and (14) are well known (see, for example, Refs. [51,52]). The novelty of this paper lies in the fact that new types of stationary states have been found in the form of coupled periodic and soliton NLSE solutions due to the presence of a nonlinear defect. We considered excitations having energy values in the range below the continuous (bulk) spectrum.

In the present paper we obtained new types of states describing the blocking of the field as it passes through the media interface. The field blocking effect is manifested in the formation of a soliton lattice at one side of the interface and in the damping of the field at the other side of the interface. We found the solutions of the coupled NLSE soliton at one side of the interface with periodic NLSE solutions of two types. The two types of periodic solutions of the NLSE are described by elliptic functions cn() and dn(), respectively.

We analyzed the conditions of the existence of stationary states describing the field blocking effect. We found exact energy levels of excitations of the special kinds. Also, we analyzed the long-wave excitations of the types considered.

We obtained that, for the field blocking effect existence described by the first type of stationary states, the nonlinearity parameter of the media where the field fades away should be less than the nonlinearity parameter of the media where a soliton lattice is formed ($\gamma_2 < \gamma_1$). Also, we found that for the field blocking effect existence described by the second type of stationary states the nonlinearity parameter of the media where the field decays fulfills the following condition: $\gamma_1 < \gamma_2 < 2\gamma_1$. Thus, the relationship between the nonlinearity parameters on different sides of the interface determines the type of state corresponding to the blocking of the field.

We obtained that in the case of interface without nonlinear response the characteristic width of excitation localization is inversely proportional to the "power" of the interface. Also, we derived that in the case of a nonlinear interface the characteristic width of excitation localization is proportional to the interface nonlinearity.

We found that the field blocking effect is possible only at opposite signs of the interface parameters in one special case (when $x_1 = 0$ and $x_2 = 0$) and only at the same signs of the interface parameters in other special cases (when $x_1 = 0$ and $x_2 \neq 0$ or $x_1 \neq 0$ and $x_2 = 0$). The existence of the field blocking effect in the first special case is exclusively due to the nonlinear properties of the interface.

The model that we propose allows describing the field localization at both sides of the interface in the limit $k \to 1$ when elliptic functions transform into hyperbolic ones. In such a limit two stationary states degenerate into one localized state. The localized state has the form of NLSE symmetric solitons at both sides of the interface. We obtained the exact energy level of the symmetric localized state choosing the amplitude of interface oscillations as a free parameter. We derived that the energy of small-amplitude nonlinear localized excitations depends on the square of the amplitude of the plane defect oscillations. Also, we have noted that in the case of linear medium containing a simple defect without nonlinear response our model can give the classical solution for the linear Schrodinger equation with delta function potential modeling an attractive defect.

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