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Nature of entropy in a scalar-tensor theory of gravity

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Abstract: This work is devoted to studying the generalized nature of the entropy function in a specific form of the scalar-tensor-vector (STeVe) theory of gravity for the Friedmann–Lemaitre–Robertson–Walker (FLRW) spacetime in the state of thermal equilibrium. For this purpose, we define an equilibrium picture of thermodynamics by making use of a suitable set of the dark energy density and dark pressure descriptions, which defines an energy-momentum tensor obeying the local energy conservation law. Furthermore, we also discuss our results numerically in order to get additional implications.

Key words: Thermodynamics, entropy, modified gravity, cosmology

1. Introduction

Galactic thermodynamics is one of the noteworthy puzzles in cosmology today. In 1995, Jacobson [1] introduced a gravity–thermodynamics relation. Making use of the first thermodynamic law in black hole physics and the Bekenstein–Hawking entropy-area formulation [2,3]:

$$S = \frac{A}{4G_0}, \tag{1}$$

where A describes the area of horizon while G_0 indicates the gravitational constant. Jacobson [4] rewrote the general relativistic field equations (GRFEs) on the local Rindler horizons. Later, starting from the GRFE for a static spherical symmetric line-element, Padmanabhan [5] formulated the first law on the apparent horizon. After this pioneering work, this interesting issue has been generalized to the cosmological point of view. Cai and Kim [6], in 2005, showed that the Friedmann equation written in the general theory of relativity (FEGR) can be obtained from the first law:

$$-dE = T_H dS_h \tag{2}$$

Note that dE, T_H, $S_h = \frac{\pi R_h^2}{G_0}$, and R_h indicate the energy flux, Hawking temperature observed inside the horizon, horizon entropy, and dynamical apparent horizon, respectively. Subsequently, in 2007, Akbar and Cai [7] derived another form of the FEGR by considering the dynamical apparent horizon:

$$dE + T_H dS_h = W dV, \tag{3}$$

where V describes the volume of the system while W denotes the work density, which is given as:

$$W = \frac{1}{2}(\rho - p), \tag{4}$$

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with the energy density ρ and the pressure p. The problem of finding a relation between gravitational effects and thermodynamics was studied in several gravity theories in later papers [7–19].

On the other hand, recent cosmological evidence [19–23] has shown that our universe entered a speedy expansion phase. The first clue about this mysterious expansionary nature was given by the Supernovae Cosmology Project Collaboration (SCPC) [19–21] and subsequent attempts [22,23] also favored this significant phenomenon. This important issue is one of the well-established notions in modern cosmology and it is generally accepted [19–23] that entering the speedy expansion era is raised by two exotic dark contents: dark matter and dark energy. The matter in our universe is dominated by 68.3% dark energy and 26.8% dark matter [23]. Many proposals have been introduced to understand the dark nature of the universe, but the mysterious behavior is still completely unknown [24]. The scalar and vector fields, modified gravity theories, extra dimensions, and intergalactic gas formulations are possible theoretical candidates proposed to investigate the dark part of our universe. Using modified theories of gravity in order to study the accelerated expansionary feature leads to very significant results at solar and galactic systems scales. However, there is no meaningful reason to adopt one of them as a suitable point of view for all scales [25].

The above conclusions motivated us to discuss whether an equilibrium picture can be defined in a specific framework of the STeVe cosmology. Our investigation may yield a few specific conclusions that would discriminate the STeVe cosmology from various modified gravity theories. To reach this goal, we assume that our universe is bounded by the dynamical apparent horizon, because considering the cosmological event horizon gives some significant difficulties. For instance, the event horizon does not exist in the standard big-bang model [17]. In 2006, Wang et al. [26] found that the laws of generalized thermodynamics are not valid at the event horizon. In addition to this, the existence of the cosmological event horizon yields a nonstatic space-time model and we get more complicated relations for thermodynamic quantities than those defined in the static universe models [17]. Throughout this work, we use the following convention. The Greek alphabet (μ, ν, ...=0, 1, 2, 3) represents space-time while the Latin alphabet (a, b, ...=0, 1, 2, 3) denotes the tangent space. Additionally, we choose $c=1$ and adopt the metric signature $\eta_{ij} = diag(-1, +1, +1, +1)$, where η_{ij} is the Minkowski space-time metric.

2. Preliminary relations: the STeVe cosmology

Adopting modified theories of gravity yields very interesting results at the cosmological, galactic, and solar system scales. Both the nonsymmetric theory of gravity (NSTG) [27] and the metric-skew-tensor theory (MSTT) [28] have been introduced in order to investigate cosmology without dark contents of the universe and rotational velocity curves of galaxies [29]. The STeVe theory of gravity leads to better fits for many galaxies [30] and galaxy clusters [31]. In this work, we investigate a specific limit of the STeVe cosmology via the first law of thermodynamics.

We start with the STeVe theory of gravity, which is represented by the following action [29]:

$$S_{steve} = S_{grav} + S_{matter} + S_{scalar} + S_{vector}, \tag{5}$$

where:

$$S_{grav} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\frac{R+2\Lambda}{G}\right), \tag{6}$$

$$S_{matter} = -\frac{1}{2}\int d^4x\sqrt{-g}L_m, \tag{7}$$

$$S_{vector} = -\frac{1}{4}\int d^4x\sqrt{-g}\,[w\,(B^{\mu\nu}B_{\mu\nu} + 4V(\phi))], \tag{8}$$

$$S_{scalar} = \int d^4x\sqrt{-g}\,[L_1 + L_2 + L_3], \tag{9}$$

with

$$L_1 = \frac{1}{G^3}\left(\frac{g^{\mu\nu}}{2}\nabla_\mu G\nabla_\nu G - V\,(G)\right), \tag{10}$$

$$L_2 = \frac{1}{G}\left(\frac{g^{\mu\nu}}{2}\nabla_\mu w\nabla_\nu w - V\,(w)\right), \tag{11}$$

$$L_3 = \frac{1}{G\gamma^2}\left(\frac{g^{\mu\nu}}{2}\nabla_\mu \gamma\nabla_\nu \gamma - V\,(\gamma)\right), \tag{12}$$

$$B_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu. \tag{13}$$

In the above definitions, ∇_μ shows the covariant derivative with respect to the metric tensor $g_{\mu\nu}$, R denotes the Ricci curvature scalar, Λ is known as the cosmological constant, L_m defines the matter part of total Lagrangian density, and ϕ_μ is a vector field. Next, $V(\phi)$ shows a vector field potential while $V\,(G)$, $V\,(w)$, and $V\,(\gamma)$ describe potentials for the scalar fields $G(x)$, $w(x)$, and $\gamma(x)$, respectively. It is significant to emphasize here that $w(x)$ is a dimensionless scalar field.

Next, variation of the action given in Eq. (5) with respect to the metric tensor gives the following field equation for the STeVe theory of gravity [29]:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\,(R + 2\Lambda) + Q_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{14}$$

where:

$$R_{\mu\nu} = \partial_\sigma\Gamma^\sigma_{\mu\nu} - \partial_\nu\Gamma^\sigma_{\mu\sigma} + \Gamma^\sigma_{\mu\nu}\Gamma^\rho_{\sigma\rho} - \Gamma^\sigma_{\mu\rho}\Gamma^\rho_{\nu\sigma}, \tag{15}$$

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}), \tag{16}$$

$$Q_{\mu\nu} = G(\nabla^\sigma\nabla_\sigma\frac{g_{\mu\nu}}{G} - \nabla_\mu\nabla_\nu\frac{1}{G}). \tag{17}$$

It is seen from Eq. (14) that G takes a role as the effective gravitational "constant".

In this step, we focus on the homogeneous and isotropic FLRW space-time model, which is described by the following line element:

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right], \tag{18}$$

where $k = -1, 0, +1$ stands for the open, flat, and closed universe types, respectively. In the shadow of this background model, for a specific limit of the STeVe theory, we can assume $\phi_0 \neq 0$, $\phi_i = 0$, and $B_{\mu\nu} = 0$ [29] for simplicity.

Moreover, a perfect fluid can be described by the following energy-momentum tensor:

$$T^{\mu\nu} = (\rho_m + p_m)\, u^\mu u^\nu - p_m g^{\mu\nu}, \tag{19}$$

where u^μ is the four-velocity vector, ρ_m displays the energy density of ordinary matter, and p_m denotes pressure. Next, we also have

$$g_{\mu\nu} u^\mu u^\nu = 1. \tag{20}$$

Hence, using the FLRW metric in the field equation of Eq. (14) gives the following extended Friedmann equations (EFE) [29]:

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho_m + H\frac{\dot{G}}{G} + \frac{\Lambda}{3}, \tag{21}$$

$$\dot{H} + H^2 = -\frac{4\pi G}{3}(\rho_m + 3p_m) + \frac{1}{2}\left(\frac{\ddot{G}}{G} - \frac{\dot{G}^2}{G^2} + 2H\frac{\dot{G}}{G}\right) + \frac{\Lambda}{3}, \tag{22}$$

where $H = \frac{\dot{a}}{a}$ is the cosmic Hubble parameter.

3. Nature of galactic entropy function

We now consider both the nonequilibrium picture of thermodynamics (nEPT) and the equilibrium picture of thermodynamics (EPT) to investigate the nature of the entropy function. It is known that unlike the first law of thermodynamics, the generalized form of the second law has a universal form, which is why the validity of this law does not depend on whether the system is in equilibrium or not. In this section, we try to define an EPT by discussing the first law of thermodynamics. Ergo, we focus on the first thermodynamics law and the EFE.

3.1. The nEPT

As a starting point, we focus on the following definitions of dark content quantities:

$$\rho_d = \frac{3}{8\pi G_0}(H\frac{\dot{G}}{G} + \frac{\Lambda}{3}), \tag{23}$$

$$p_d = -\frac{1}{8\pi G_0}\left(\frac{\ddot{G}}{G} - \frac{\dot{G}^2}{G^2} + 3H\frac{\dot{G}}{G} + \Lambda\right). \tag{24}$$

It is important to mention here that taking $G(t) = G_0$ and $\Lambda = 0$ gives $\rho_d = p_d = 0$, and therefore the STeVe gravity is reduced to the general theory of relativity. The above descriptions turn the EFE into the form of the FEGR:

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_0}{3}\rho_t, \tag{25}$$

$$3H^2 + 2\dot{H} + \frac{k}{a^2} = -8\pi G_0 p_t, \tag{26}$$

where ρ_t and p_t denote total energy density and total pressure, respectively. Hence, we have

$$\rho_t = \frac{G}{G_0}\rho_m + \rho_d, \tag{27}$$

$$p_t = \frac{G}{G_0}p_m + p_d. \tag{28}$$

Consequently, using Eqs. (25) and (26), we get

$$\dot{H} - \frac{k}{a^2} = -4\pi G_0(\rho_t + p_t). \tag{29}$$

Next, considering Eqs. (25) and (29), one can obtain the following conservation relation:

$$\dot{\rho}_t + 3H\left(\rho_t + p_t\right) = 0. \tag{30}$$

From this point of view, using Eqs. (27) and (28) in Eq. (30) gives

$$\dot{\rho}_m + 3H\left(\rho_m + p_m\right) = 0, \tag{31}$$

$$\dot{\rho}_d + 3H\left(\rho_d + p_d\right) = -\frac{\dot{G}}{G_0}\rho_m. \tag{32}$$

We see that the dark energy density and the pressure due to the space-time contribution do not satisfy the conservation law. That is why we call this scenario the nonequilibrium picture. Additionally, using Eqs. (23)–(25) and (29), we get

$$\rho_m + p_m = \frac{1}{4\pi G}\left[\left(\frac{k}{a^2} - \dot{H}\right) - \frac{1}{2}(\frac{\dot{G}^2}{G^2} - \frac{\ddot{G}}{G})\right], \tag{33}$$

$$\rho_d + p_d = \frac{1}{8\pi G_0}(\frac{\dot{G}^2}{G^2} - \frac{\ddot{G}}{G}). \tag{34}$$

Now we are in a position to discuss the first thermodynamic law. The dynamical apparent horizon is given as [32–34]

$$R_h = \left[H^2 + \frac{k}{a^2}\right]^{-\frac{1}{2}}, \tag{35}$$

which can be reduced to the Hubble horizon, i.e. $R_h = \frac{1}{H}$, for the flat FLRW space-time case. Next, the Hawking temperature on the dynamical apparent horizon is defined by [6]

$$T_H = \frac{1}{2\pi R_h}\frac{2HR_h - \dot{R}_h}{2HR_h}, \tag{36}$$

where $2HR_h > \dot{R}_h$ provides a positive Hawking temperature. In further calculations, we will also need the following auxiliary relations [35]:

$$\dot{R}_h = HR_h^3\left[\frac{k}{a^2} - \dot{H}\right], \tag{37}$$

$$HR_h - \dot{R}_h = HR_h^3(H^2 + \dot{H}), \tag{38}$$

$$2HR_h - \dot{R}_h = HR_h^3(2H^2 + \dot{H} + \frac{k}{a^2}). \tag{39}$$

At this point, we can focus on the Bekenstein–Hawking entropy-area formulation given in Eq. (1). It leads to

$$\frac{1}{2\pi R_h}\frac{dS_h}{dt} = 4\pi HR_h^3(\rho_t + p_t). \tag{40}$$

After multiplying each side of the above result by a factor of $\frac{2HR_h - \dot{R}_h}{2HR_h}$, one can obtain the following equality:

$$T_H\frac{dS_h}{dt} = 4\pi HR_h^3\left(\rho_t + p_t\right) + \frac{R_h^2\dot{R}_h}{2G_0}\left[\dot{H} - \frac{k}{a^2}\right]. \tag{41}$$

It is generally known that the total energy density is defined by $\rho_t = \frac{E}{V}$, where $V = \frac{4\pi}{3}R_h^3$. Ergo, taking the time derivative of the total energy yields

$$\frac{dE}{dt} = \dot{\rho}_t V + \rho_t \dot{V}. \tag{42}$$

Making use of Eq. (30) and the definition of volume, we find that

$$\frac{dE}{dt} = -4\pi HR_h^3\left(\rho_t + p_t\right) + \frac{3}{2G_0}\left[H^2 + \frac{k}{a^2}\right]. \tag{43}$$

Moreover, adding Eqs. (41) and (43) gives

$$\frac{dE}{dt} + T_H\frac{dS_h}{dt} = 2\pi HR_h^5(3H^2 + \dot{H} + \frac{k}{a^2})\left(\rho_t + p_t\right). \tag{44}$$

Subsequently, we can rewrite this relation in the following form:

$$-dE = T_H dS_h + T_H d\bar{S}, \tag{45}$$

where

$$T_H d\bar{S} = -2\pi HR_h^5(3H^2 + \dot{H} + \frac{k}{a^2})\left(\rho_t + p_t\right)dt. \tag{46}$$

The additional $d\bar{S}$ term raised in Eq. (45) violates the first law of thermodynamics in the specific limit of STeVe cosmology. Normally, all matter fields see the same apparent horizon and feel the same Hawking temperature, but it seems that some gravitational quantities in this cosmological scenario see a different horizon and feel a different Hawking temperature [36]. It is known that black holes having such conditions cannot be in equilibrium [36]. Dubovski and Sibiryakov [37], in 2006, obtained a similar result after investigation of the effects of spontaneous Lorentz invariance breaking in black hole thermodynamics. Here, this extra term may be interpreted as an entropy contribution that comes from the nonequilibrium framework in the specific limit of the STeVe theory. Furthermore, we can rearrange $T_H d\bar{S}$ in another form as

$$W = \frac{1}{2}\left(\rho_t - p_t\right) = \frac{1}{8\pi G_0}(3H^2 + \dot{H} + \frac{2k}{a^2}), \tag{47}$$

$$dV = 4\pi R_h^2 \dot{R}_h dt. \tag{48}$$

Thus, we can rewrite Eq. (46) in a new form:

$$T_H d\bar{S} = -W dV. \tag{49}$$

This relation transforms Eq. (44) into the following form:

$$-dE = T_H dS_h - W dV, \tag{50}$$

which is the same as the new form of the first law of gravitational thermodynamics obtained by Akbar and Cai [7] in 2007.

On the other hand, the additional entropy term that comes from the geometrical effects in curved space-time can be analyzed numerically. For this purpose, we consider the following relation obtained by Moffat [29]:

$$G\left(t\right) = G_0 \left(1 + \alpha e^{-\frac{t}{\beta}} \left[1 + \frac{t}{\beta}\right]\right), \tag{51}$$

where α and β are two auxiliary constants. In the limiting cases, we have [29]

$$t \gg \beta \rightarrow G\left(t\right) = G_0(1+\alpha), \tag{52}$$

$$t \ll \beta \rightarrow G\left(t\right) = G_0. \tag{53}$$

In the Figure, one can see that the additional entropy term will never be equal to zero. That is why we can say that the new form of the first law for the specific limit of the STeVe theory is always valid throughout the history of the universe.

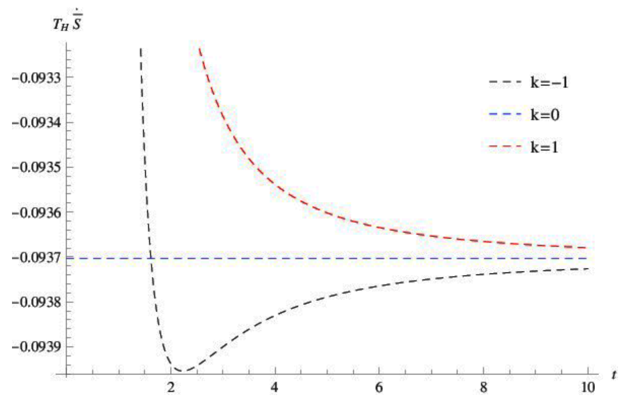

Figure. Here we consider a specific choice describing the late time acceleration phase for the scale factor, i.e. $a = t^2$, in order to plot the evolution of an additional entropy term. Auxiliary parameters are chosen as $\alpha = \beta = 1$ and $G_0 = 6.67$.

3.2. The EPT

We have concluded, in the specific limit of the STeVe cosmology, that there is an entropy production term $d\bar{S}$ in the first law of thermodynamics that affects the galactic entropy. So far, it has been shown in the literature that an EPT can be obtained in modified theories of gravity [10,14,16,38–40]. We can rewrite the EFE in another form by redefining the dark energy density and the pressure. This new form of the Friedmann equations can be obtained as

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_0}{3}\breve{\rho}_t, \tag{54}$$

$$3H^2 + 2\dot{H} + \frac{k}{a^2} = -8\pi G_0 \breve{p}_t, \tag{55}$$

where

$$\breve{\rho}_t = \rho_m + \breve{\rho}_d, \tag{56}$$

$$\breve{p}_t = p_m + \breve{p}_d, \tag{57}$$

$$\breve{\rho}_d = \frac{3}{8\pi}\left[\left(H^2 + \frac{k}{a^2}\right)\left(\frac{1}{G} - \frac{1}{G_0}\right) + \frac{1}{G}(H\frac{\dot{G}}{G} + \frac{\Lambda}{3})\right], \tag{58}$$

$$\breve{p}_d = \frac{1}{8\pi}\left[\left(3H^2 + 2\dot{H} + \frac{k}{a^2}\right)\left(\frac{1}{G} - \frac{1}{G_0}\right) + \frac{1}{G}(\frac{\dot{G}^2}{G^2} - \frac{\ddot{G}}{G} - 3H\frac{\dot{G}}{G} - \frac{\Lambda}{3})\right]. \tag{59}$$

Thus, these new dark energy density and pressure definitions obey the following continuity equation:

$$\dot{\breve{\rho}}_d + 3H\left(\breve{\rho}_d + \breve{p}_d\right) = 0. \tag{60}$$

On the other hand, total energy density and total pressure of the universe also satisfy the continuity relation given below:

$$\dot{\breve{\rho}}_t + 3H\left(\breve{\rho}_t + \breve{p}_t\right) = 0. \tag{61}$$

Hence, we can discuss the EPT similarly to the approach taken for the GRFE.

Making use of these new definitions, we get the following conclusions:

$$d\breve{E} = -4\pi H R_h^3\left(\breve{\rho}_t + \breve{p}_t\right) + \breve{\rho}_t dV, \tag{62}$$

$$T_H d\breve{S}_h = 4\pi H R_h^3\left(\breve{\rho}_t + \breve{p}_t\right) - \frac{1}{2}\left(\breve{\rho}_t + \breve{p}_t\right)dV. \tag{63}$$

As a result, after adding these equations to each other, we find

$$d\breve{E} + T_H d\breve{S}_h = \frac{1}{2}(\breve{\rho}_t - \breve{p}_t)dV. \tag{64}$$

It is known that $\breve{W} = \frac{1}{2}(\breve{\rho}_t - \breve{p}_t)$ defines the work density; hence, we get

$$d\breve{E} + T_H d\breve{S}_h = \breve{W}dV, \tag{65}$$

which is known as the new form of the first law of thermodynamics introduced by Akbar and Cai [7]. This conclusion shows that an EPT can be achieved in in the specific limit of the STeVe cosmology by defining a suitable dark energy density and pressure.

4. Conclusion

We have mainly analyzed the specific limit of STeVe cosmology thermodynamically on the dynamical apparent horizon $\boldsymbol{R_h}$ with the area $\boldsymbol{4\pi R_h^2}$ in the FLRW-type universe model to discuss the galactic nature of entropy. With this purpose, two different pictures (nEPT and EPT) of gravitational thermodynamics have been discussed. We have shown that an EPT can be achieved in the specific limit of STeVe gravity by redefining the dark components of the universe. We have concluded that there is an additional entropy term that comes from geometrical contributions. Moreover, the mathematical redefinition of dark energy-pressure quantities yields more meaningful results than those obtained by using the dark content components written in the nEPT by making use of the usual method followed by many researchers in the literature.

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