

Super inflation mechanism and dark energy from a cosmological entropy perspective

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Abstract: Various approaches are developed that provide a unification of the early inflationary epoch of the universe and dark energy (cosmic accelerating expansion case). In one respect, we express this unification in a universal thermodynamic framework within the context of $F(T, T_G)$ gravity, where T represents the torsion invariant and T_G is the teleparallel equivalent of the Gauss–Bonnet term. For a Friedmann–Robertson–Walker universe model bounded by the dynamical apparent horizon, we explore the generalized second law of gravitational thermodynamics (GSLT). Finally, we discuss the super inflation mechanism, the deceleration case (radiation and dust regions), and the late time cosmic acceleration of the universe into the validity of the GSLT that we obtain.

Key words: $F(T, T_G)$ gravity, thermodynamics, inflation, dark energy

1. Introduction

The current observational data show that the universe is in the accelerating expansion phase. This case has been tried to be explained by the dark energy concept. This concept is dealt with by different approaches like modified gravity theories or scalar fields. In this study we take into account modified gravity theories, which are modifications of Einstein’s field equations. Some of these theories are $f(R)$ gravity [1–3], modified Gauss–Bonnet gravity $R + f(G)$ [4], torsion gravity $f(T)$ [5], and $f(R, G)$ gravity [6], where R , G , T are the Ricci scalar, Gauss–Bonnet invariant, and torsion scalar, respectively. The extra terms coming from the field equations of these gravity theories produce a large negative pressure in the equation of the state parameter (EOS) $w = \frac{p}{\rho}$, where p , ρ represent the pressure and the energy density, respectively. According to the different values of this parameter, it can be said that the universe is in different phases. That is:

- $w = \frac{1}{3}$ and $w = 0$ describe the radiation and the dust regions, respectively.
- $-1 < w < \frac{-1}{3}$, $w = -1$, and $w < -1$, which show the large negative pressure and describe the quintessence, de Sitter, and phantom phases, respectively.

On the other hand, in the early time inflationary universe it is possible to see the quintessential inflation ($-1 < w < \frac{-1}{3}$) [7], quasi-de Sitter vacuum inflation $w \cong -1$ [8,9], or a unification of both types of the expansion [10] in $F(T, T_G)$ gravity theory [11–13]. However, the early time inflation and the late time acceleration of the universe in a unified form could be observed in the literature [14–16]. In order to verify the early or the current

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acceleration of the universe from the thermodynamics aspect, many studies have been done by researchers. In this direction, the connection between gravity dynamics and horizon thermodynamics was first established by Bekenstein, Hawking, Gibbons, Unruh [17–21], etc. Accordingly, Einstein’s field equations and the field equations in modified gravity theories can be derived from thermodynamics properties [22–25]. However, in the Friedmann–Robertson–Walker (FRW) universe, Cai and Cao obtained an entropy expression for the apparent horizon by using the unified first law of thermodynamics method [26,27]. They also showed that the Lovelock gravity and the scalar-tensor theories could be achieved from the unified first law. The method is associated with the projection of the unified first law along the direction of a tangent vector ξ . In this approach, the unified first law can be written as $\langle A\psi\xi \rangle = \frac{\kappa}{8\pi G} \langle dA\xi \rangle$, where A is the surface area of the horizon, and ψ is energy-supply vector that indicates energy flux, which is defined by the energy-momentum tensor of the matter. It is seen that this equation is similar to the Clausius relation $\delta Q = TdS$ for a dynamical black hole. Recently, Mitra et al. [28–30] examined the unified first law (equation above) for different gravity theories in a FRW universe model that is bounded by an apparent horizon or event horizon. However, Zubair and Jawad [31] obtained the generalized second law of gravitational thermodynamics (GSLT) inequality for $F(T, T_G)$ gravity theory. In the present study, we have used the unified first law of thermodynamics method, which was proposed by Cai and Cao [27], to obtain the entropy expression on the apparent horizon of the FRW universe model for $F(T, T_G)$ gravity. Then we obtain a GSLT inequality that differs from that obtained in [31].

In this paper, we study the theory of $F(T, T_G)$ gravity, and for this gravity we derive the entropy of the apparent horizon by using the unified first law of thermodynamics method in the context of the FRW universe. Next, the GSLT inequality is obtained. The super inflation mechanism, which was proposed by Keskin [10], and dark energy are discussed in terms of the validity of the GSLT frame that we obtain. It is observed that the results coming from the GSLT analysis verify these cases (i.e. super inflation mechanism and dark energy cases) in a unified picture. The paper is organized as follows: in Section 2, we briefly mention the unified solutions of the field equations of $F(T, T_G)$ gravity, and in Section 3, we obtain the GSLT inequality for the gravity theory and investigate the solutions corresponding to the field equations in the GSLT frame. Section 4 gives a summary of our findings.

2. The field equations and unified solutions

The action integral of this gravity is given by [11–13]:

$$S = \frac{1}{2k^2} \int d^4x e F(T, T_G) + S_m, \tag{1}$$

where $k^2 = 8\pi\tilde{G}$, $S_m = \int d^4x \sqrt{-g} L_m$, and $e = \det(e_\mu^\alpha) = \sqrt{|g|}$ are the dynamical vielbein fields, $e_\alpha(x^\mu)$, in the teleparallel gravity theory. T and T_G represent the torsion scalar and the teleparallel equivalent of the Gauss–Bonnet term, respectively. In this study, we take the flat-FRW geometry given by

$$ds^2 = dt^2 - a^2(t) [dx^2 + dy^2 + dz^2]. \tag{2}$$

The variation of the action of Eq. (1) gives the Friedmann equations as follows [11,12]:

$$-12H^2 F_T - T_G F_{T_G} + F(T, T_G) + 24H^3 \dot{F}_{T_G} = 2k^2 \rho, \tag{3}$$

$$F(T, T_G) - 4H\dot{F}_T - T_G F_{T_G} + \frac{2}{3H} T_G \dot{F}_{T_G} + 8H^2 \ddot{F}_{T_G} - 4(\dot{H} + 3H^2) F_T = -2k^2 p, \quad (4)$$

where $F_{T_G} = \frac{\partial F(T, T_G)}{\partial T_G}$, $F_T = \frac{\partial F(T, T_G)}{\partial T}$, and $H = \frac{\dot{a}}{a}$ is the Hubble parameter. The over dot denotes the time derivative that is given as follows:

$$\dot{F}_T = F_{TT} \dot{T} + F_{TT_G} \dot{T}_G, \quad (5)$$

$$\dot{F}_{T_G} = F_{TT_G} \dot{T} + F_{T_G T_G} \dot{T}_G, \quad (6)$$

$$\ddot{F}_{T_G} = F_{TTT_G} \dot{T}^2 + 2F_{TT_G T_G} \dot{T} \dot{T}_G + F_{T_G T_G T_G} \dot{T}_G^2 + F_{TT_G} \ddot{T} + F_{T_G T_G} \ddot{T}_G. \quad (7)$$

For the geometry of Eq. (2), T and T_G are defined by [11,12]:

$$T = 6H^2 \quad (8)$$

$$T_G = 24H^2(\dot{H} + H^2). \quad (9)$$

Eqs. (3) and (4) can be written in the form of the standard Einstein equations:

$$H^2 = \frac{8\pi\tilde{G}}{3}(\rho + \rho_e), \quad (10)$$

$$\dot{H} = -4\pi\tilde{G}(\rho + p + \rho_e + p_e). \quad (11)$$

Herein, the effective energy density and the pressure are defined by [11,12]:

$$\rho_e = \frac{1}{16\pi\tilde{G}}(6H^2 + T_G F_{T_G} - F(T, T_G) + 12H^2 F_T - 24H^3 \dot{F}_{T_G}), \quad (12)$$

$$p_e = \frac{1}{16\pi\tilde{G}}(-2(2\dot{H} + 3H^2) + F(T, T_G) - 4(\dot{H} + 3H^2)F_T - 4H\dot{F}_T - T_G F_{T_G} + \frac{2}{3H} T_G \dot{F}_{T_G} + 8H^2 \ddot{F}_{T_G}), \quad (13)$$

respectively. The sum of these expressions is:

$$\rho_e + p_e = \frac{1}{8\pi\tilde{G}} \left[4H^2 \ddot{F}_{T_G} + 8H\dot{H}\dot{F}_{T_G} - 4H^3 \dot{F}_{T_G} - 2\dot{H} - 2\dot{H}F_T - 2H\dot{F}_T \right]. \quad (14)$$

The conservation law satisfies the following relations:

$$\dot{\rho} + 3H(\rho + p) = 0, \quad \dot{\rho}_e + 3H(\rho_e + p_e) = 0. \quad (15)$$

Now, we briefly show the unified solutions Eqs. (10) and (11), which are discussed in ref. [10]. For this, we use the relationship $M = 8\pi\tilde{G}(p_e - w\rho_e) + 2\dot{H} + 3H^2(1+w)$ that satisfies the following equation:

$$4H^2 \ddot{F}_{T_G} + 8\dot{H}H\dot{F}_{T_G} + 4H^3 \dot{F}_{T_G} (2+3w) - 12(\dot{H}H^2 + H^4)(1+w)F_{T_G}$$

$$+\frac{F(T, T_G)}{2}(1+w) - 6H^2 F_T(1+w) - 2H\dot{F}_T - 2\dot{H}F_T = 0, \quad (16)$$

with $w = w_e = \frac{p_e}{\rho_e}$ and $w = -1 - \frac{2\dot{H}}{3H^2}$. Here, $w = \frac{p}{\rho}$ is the equation of the state parameter (EOS) for the normal matter, which equals its effective value, i.e. w_e . The model $F(T, T_G) = f_1(T_G) + f_2(T)$ is investigated in Eq. (16), where $f_2(T) = -T + \alpha T^2$, with a small α parameter. Substituting the model into Eq. (16), we have

$$4H^2 \ddot{f}_{1T_G} + 8\dot{H}H \dot{f}_{1T_G} + 4H^3 \dot{f}_{1T_G} (2+3w) - 12(\dot{H}H^2 + H^4)(1+w) f_{1T_G} + (\frac{f_1(T_G)}{2} + \frac{f_2(T)}{2})(1+w) - 6H^2 f_{2T}(1+w) - 2H \dot{f}_{2T} - 2\dot{H} f_{2T} = 0. \quad (17)$$

This equation can be written as

$$4H^2 \ddot{f}_{1T_G} + 8\dot{H}H \dot{f}_{1T_G} + 4H^3 \dot{f}_{1T_G} (2+3w) - 12(\dot{H}H^2 + H^4)(1+w) f_{1T_G} + \frac{f_1(T_G)}{2}(1+w) = 0, \quad (18)$$

$$-2H \dot{f}_{2T} - 2\dot{H} f_{2T} - 6H^2 f_{2T}(1+w) + \frac{f_2(T)}{2}(1+w) = 0. \quad (19)$$

When using Eqs. (6), (7), and (15) and the following solutions,

$$a(t) = a_0 t^h, \quad H = \frac{h}{t}, \quad \dot{H} = \frac{-h}{t^2}, \quad (20)$$

Eq. (18) can be written in terms of T_G :

$$4T_G F_{T_G T_G T_G} + (7+h) F_{T_G T_G} = \frac{-k^2 \rho_0 (1+w) (\gamma)^{\frac{-3h(1+w)}{4}}}{16h^2} T_G^{\frac{3h(1+w)-8}{4}} + \frac{3(1+w)}{16} \gamma^{\frac{1}{2}} T_G^{\frac{-3}{2}} - \frac{54\alpha(1+W)h^2}{16} T_G^{-1} \quad (21)$$

The solution of this differential equation is

$$f_1(T_G) = A \left(\frac{T_G}{\gamma} \right)^{\frac{3h(1+w)}{4}} + B(T_G)^{\frac{1}{2}} + DT_G (\ln T_G - 1) + \frac{4c_1}{(h+3)(h-1)} T_G^{\frac{-h+1}{4}} + c_2 T_G, \quad (22)$$

where $A = \frac{-k^2 \rho_0 (1+w) \gamma}{h^2 (-3h(1+w) - h + 1) (-3h(1+w) + 4) (3h(1+w))}$, $B = -\frac{3(1+w) \gamma^{\frac{1}{2}}}{4(h+1)}$, $D = -\frac{54h^2(1+w)\alpha}{16(h+3)}$, and c_1, c_2 are integration constants where $c_2 = 0$. Next, using $f_2(T) = -T + \alpha T^2$ and Eqs. (8) and (19), the following equation is obtained:

$$6\alpha h^3 [4 - 3h(1+w)] t^{-4} - h [2 - 3h(1+w)] t^{-2} = 0. \quad (23)$$

This equation produces the super accelerated $h_1 = \frac{4}{3(1+w)}$ and the FRW $h_2 = \frac{2}{3(1+w)}$ solutions due to $t > 0$.

The real value of the Lagrangian function can be written as follows [10]:

$$F(T, T_G) = DT_G (\ln T_G - 1) + FT_G^n - T + \alpha T^2, \quad (24)$$

where $F = \frac{36(1+w)^2 c_1}{(1-3w)(9w+13)}$, $n = \frac{-h+1}{4} = \frac{3w+1}{12(1+w)}$. This Lagrangian describes the evolutionary stages of the universe in a unified form:

- 1- The three regions in the inflationary stage: the first is the region where there is a de Sitter type inflation (vacuum state), which is described by the term $-T + \alpha T^2$. The second is the super accelerated phase, which is composed of two regions, i.e. quintessential and matter creation regions. This phase is described by the $DT_G (\ln T_G - 1)$ term.
- 2- The radiation and dust regions are expressed by the $-T$ term.
- 3- The late time cosmic acceleration (dark energy) case is explained by the FT_G^n term, with $n < 0$.
- 4- de Sitter expansion in the late time universe is shown by $-T + \alpha T^2$.

In this study we discuss the first, the second, and the third cases in the GSLT frame.

3. The dynamical entropy term for the apparent horizon

In this section, we will derive the entropy of the apparent horizon for the gravity theory. With this purpose, we follow the method proposed by Cai and Cao [27]. They showed that the unified first law in Einstein's gravity can be written as follows:

$$\delta Q = TdS = \langle A\psi_m \xi \rangle = \frac{\kappa}{8\pi G} \langle dA\xi \rangle - \langle A\psi_e \xi \rangle, \quad (25)$$

where:

- $\delta Q = TdS$ is the Clausius relation and the symbol \langle , \rangle denotes the inner product.
- The energy-supply vector $\psi_\alpha = T_\alpha^\beta \partial_\beta R_A + W \partial_\alpha R_A$, with radius of apparent horizon $R_A = ar$, which is defined as $R_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}$.
- The work term $W = -\frac{1}{2} T^{\alpha\beta} h_{\alpha\beta}$, with the metric of two-dimensional space $h_{\alpha\beta} = \text{diag}(-1, \frac{a^2}{1-kr^2})$.
- $T_{\alpha\beta} = \text{diag}(\rho_m, \frac{p_m a^2}{1-kr^2})$, $T_\alpha^\beta = \text{diag}(-\rho_m, p_m)$.
- ξ shows a tangent vector to the trapping horizon and κ is the surface gravity, respectively defined by

$$\xi = \left[\frac{\partial}{\partial t} - (1 - 2\varepsilon) Hr \frac{\partial}{\partial r} \right] \kappa = -\frac{(1 - \varepsilon)}{R_A}. \quad (26)$$

The energy-supply vector and the work term can be written as follows:

$$W = W_m + W_e = \frac{1}{2} (\rho - p) + \frac{1}{2} (\rho_e - p_e), \quad (27)$$

$$\psi = \psi_m + \psi_e = -\frac{1}{2} (\rho + p + \rho_e + p_e) H R_A dt + \frac{1}{2} (\rho + p + \rho_e + p_e) a dr, \quad (28)$$

respectively. Herein, $\psi_m = -\frac{1}{2}(\rho + p)HR_A dt + \frac{1}{2}(\rho + p)adr$ and $\psi_e = -\frac{1}{2}(\rho_e + p_e)HR_A dt + \frac{1}{2}(\rho_e + p_e)adr$. Using Eqs. (14) and (26), the term $\langle A\psi_e\xi \rangle$ given in Eq. (25) is found as follows:

$$\langle A\psi_e\xi \rangle = -\frac{AHR_A(1-\varepsilon)}{8\pi\tilde{G}} \left[4H^2\ddot{F}_{T_G} + 8H\dot{H}\dot{F}_{T_G} - 4H^3\dot{F}_{T_G} - 2\dot{H} - 2\dot{H}F_T - 2H\dot{F}_T \right]. \quad (29)$$

Using the expressions given by Eq. (26) and $dA = 8\pi R_A(HR_A dt + adr)$, the following equality is obtained:

$$\frac{\kappa}{8\pi\tilde{G}} \langle dA\xi \rangle = -\frac{(1-\varepsilon)2\varepsilon HR_A}{\tilde{G}}. \quad (30)$$

We assume the Hawking temperature on the apparent horizon, i.e. $T = \frac{|\kappa|}{2\pi}$. Then the unified first law can be written as:

$$\delta Q = -\frac{(1-\varepsilon)2\varepsilon HR_A}{\tilde{G}} - \frac{AHR_A(1-\varepsilon)}{8\pi\tilde{G}} (4H^2\ddot{F}_{T_G} + 8H\dot{H}\dot{F}_{T_G} - 4H^3\dot{F}_{T_G} - 2\dot{H} - 2\dot{H}F_T - 2H\dot{F}_T) \quad (31)$$

or ($\tilde{G} = 1$),

$$\delta Q = T \langle dS_1 + dS_2, \xi \rangle, \quad (32)$$

where $dS_1 = -2\pi H^{-3}\dot{H}dt$, $dS_2 = -\pi R_A^4 H(4H^2\ddot{F}_{T_G} + 8H\dot{H}\dot{F}_{T_G} - 4H^3\dot{F}_{T_G} - 2\dot{H} - 2\dot{H}F_T - 2H\dot{F}_T)dt$. Taking into account Eq. (32) and the Clausius relation $\delta Q = TdS$, the dynamical entropy term, \dot{S}_A , takes the following form:

$$\dot{S}_A = -\pi H^{-3} \left[4H^2\ddot{F}_{T_G} + 8H\dot{H}\dot{F}_{T_G} - 4H^3\dot{F}_{T_G} - 2\dot{H}F_T - 2H\dot{F}_T \right], \quad (33)$$

where $\dot{S}_A = \dot{S}_1 + \dot{S}_2$.

3.1. Generalized second law of thermodynamics

Now we will derive the GSLT inequality for this gravity theory. It is known that the GSLT always satisfies the following relation:

$$GSLT = \dot{S}_A + \dot{S}_m \geq 0, \quad (34)$$

where \dot{S}_m is the dynamics of matter entropy that flows inside the horizon. To find \dot{S}_m one can use the first law of thermodynamics, $dE + pdV = T_m dS_m$, where T_m is the matter temperature inside the horizon. Using $E = mc^2 = \rho V$ ($c = 1$), we have:

$$Vd\rho + (\rho + p)dV = T_m dS_m. \quad (35)$$

Also, when using $V = \frac{4}{3}\pi R_A^3$, the dynamic of matter entropy is obtained as follows:

$$\dot{S}_m = -8\pi^2 H^{-5} (\dot{H} + H^2) (\rho + p). \quad (36)$$

Here, we assume that the matter temperature inside the horizon is in thermal equilibrium with the temperature in the area of the horizon (i.e. $T_m = T$). As a result, the GSLT given by Eq. (34) is constructed as:

$$-\pi H^{-3} \left[4H^2\ddot{F}_{T_G} + 8H\dot{H}\dot{F}_{T_G} - 4H^3\dot{F}_{T_G} - 2\dot{H}F_T - 2H\dot{F}_T \right] - 8\pi^2 H^{-5} (\dot{H} + H^2) (\rho + p) \geq 0. \quad (37)$$

Using Eq. (3) and (4), the inequality of Eq. (37) can be rewritten in a useful form:

$$\pi \dot{H} H^{-5} \left[4H^2 \ddot{F}_{TG} + 8H \dot{H} \dot{F}_{TG} - 4H^3 \dot{F}_{TG} - 2\dot{H} \dot{F}_T - 2H \ddot{F}_T \right] \geq 0. \quad (38)$$

Now we investigate the solution of Eq. (24) in the validity of the GSLT frame. Substituting Eq. (24) into the inequality of Eq. (38), with the solutions of Eq. (20), the GSLT is obtained as:

$$18\alpha h [-4 + 3h(1 + w)] \geq 16n(n - 1) F \beta^{n-1} (4n - 1 + h) t^{-4n+4} - \frac{2}{h} t^2. \quad (39)$$

The super inflation mechanism predicts the three regions for the early time universe as we explained before and provides a unification of the two types of the scale factor (i.e. exponential and power-law types). The first region, which is a vacuum state, is described by the exponential form of the scale factor. Next, the super accelerated period, which is composed of quintessential and matter creation regions, is described by the power-law form of the scale factor. Accordingly:

1. When $n \rightarrow \infty$ [10] the EOS parameter goes to $w = -1$ (due to $\dot{H} = 0$), which shows de Sitter type inflation. The general expression of the GSLT inequality given by Eq. (38) due to $\dot{H} = 0$ is equal to zero. Namely, the universe should be initially started with zero entropy. After this point, we must take the value of n as $n < 0$. Otherwise, the universe does not enter a new phase [10].
2. When $n < 0$ the inflation process continues with the first term given by Eq. (24). This case can be seen in Eq. (39). For instance, the second term (i.e. $\sim t^2$) in the right side of the inequality of Eq. (39) dominates the first term (i.e. $\sim t^{-4n+4}$) for very early time universe. When using the super accelerated solution $h = \frac{4}{3(1+w)}$, it is observed that there are two regions:
 - (a) For $h > 2$ the equation of the state parameter is in the range $-1 < w < \frac{-1}{3}$, which describes the quintessence type of expansion.
 - (b) For $1 < h < 2$ the equation of the state parameter is in the range $\frac{-1}{3} < w < \frac{1}{3}$, which shows the ordinary matter creation region.

The GSLT inequality given by Eq. (39) for the super accelerated period is as follows:

$$\frac{3(1+w)}{2} t^2 \geq 0, \quad (40)$$

with the super accelerated solution $h = \frac{4}{3(1+w)}$. The GSLT is protected for $-1 < w < \frac{1}{3}$, and conditions a) and b) are naturally provided.

In the Figure it is seen that the GSLT is increasing with cosmic time after the vacuum state, where the universe has zero entropy.

After the inflation stage the universe enters the decelerated case (i.e. the radiation and the dust regions), and then the quintessence type acceleration is observed until the de Sitter point. Therefore, the standard FRW solutions $h = \frac{2}{3(1+w)}$ are valid in these stages. The inequality of Eq. (39) can be written as:

$$t^2 \geq \frac{8\alpha}{(1+w)^2}, \quad (41)$$

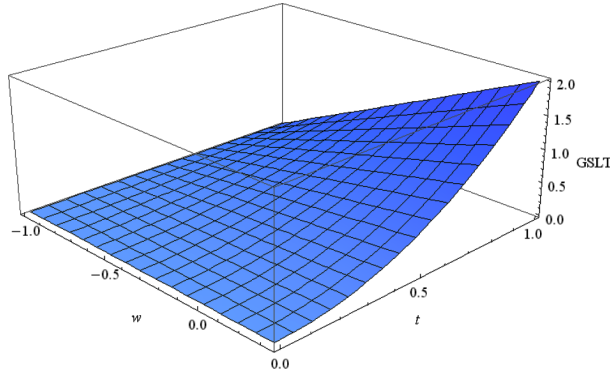


Figure. Evolution of the GSLT versus t and w given in the ranges of $10^{-3} \leq t \leq 1$ and $-1 < w < \frac{1}{3}$, respectively. The universe initially has zero entropy and then the entropy of the universe is increasing as expected.

where we use $n = \frac{-h+1}{4} = \frac{3w+1}{12(1+w)}$ (see Eq. (24)). When the universe is in the radiation $w = \frac{1}{3}$ and the dust $w = 0$ regions the GSLT due to $\alpha < 0$ [10] is protected. However, in the case of $n < 0$ we have the quintessence type of dark energy. Namely, for all the cases the GSLT of Eq. (41) is valid.

Hence, we show the super inflation mechanism and dark energy in the validity of the GSLT frame given by Eq. (39) in a unified form besides the radiation and the dust regions. In other words, it is observed that the cosmological entropy law clarifies the super inflation mechanism with the quintessence type of dark energy from the aspect of the horizon thermodynamics principle.

4. Conclusion

In this study, we have examined the validity of the GSLT in $F(T, T_G)$ gravity for the flat FRW universe model. After deriving the dynamical entropy expression of the apparent horizon by using the unified first law of thermodynamics method we have constructed the GSLT inequality given by Eq. (38), in which the Hawking temperature is assumed on the horizons. Next, by applying the solutions in the GSLT frame, the super inflation mechanism, deceleration, and dark energy cases have been shown in a unified form in the GSLT frame given by Eq. (39).

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