Turkish Journal of Physics

http://journals.tubitak.gov.tr/physics/

Research Article

Turk J Phys
(2018) 42: 198 – 209
© TÜBİTAK
doi:10.3906/fiz-1710-23

Thermodynamics of cosmological models with generalized G and Λ

M. Saratchandra SINGH, S. Surendra SINGH*
Department of Mathematics, National Institute of Technology, Manipur, Imphal, India

Received: 20.10.2017 • **Accepted/Published Online:** 10.01.2018 • **Final Version:** 26.04.2018

Abstract: Here we investigate spatially flat Friedmann–Robertson–Walker space-time models of the universe to study the thermodynamic behaviors in light of variables G and Λ. A particular method to find the solutions of Einstein's field equations and mathematical expressions for temperature and entropy of the perfect fluid cosmological models is introduced by using both the equation of state and the assumption $G = G_0\,a^n$. It is observed that the phantom field of $(\omega < -1)$ is thermodynamically not viable when the temperature increases with the evolution of the universe. The second law of thermodynamics is also constrained by cosmological models with time-dependent G and Λ. The mathematical expressions for the look-back time, proper distance, luminosity distance, and angular diameter distances are obtained for an isotropic stiff-fluid model and their significances are investigated. The most interesting part of the investigation is that the dynamical and physical behaviors of the models are less constrained by the second law of thermodynamics.

Key words: Cosmology, cosmological constant, thermodynamics, singularities

1. Introduction

Recent observations of the cosmological data [1–4] and cosmic microwave background (CMB) data [5] show that the universe is expanding in an accelerated regime. The exact explanation for the cause of the present scenario of the universe is still an unknown problem for all cosmologists. It requires the existence of negative-pressure energy, termed dark energy, to make a possible explanation of the observed acceleration of the universe or modified gravity. Dark energy is still an unknown problem for cosmologists in this modern era. In recent years, cosmologists are also more interested in modeling cosmological models for an alternative theory of gravitation to accommodate the present scenario of the universe. According to the established laws of physics, many theories are developed, but they are still inaccurate to explain the mystery of the driving force that causes the accelerating of the universe. As an alternative to modify the general theory of relativity, many cosmologists have studied the role of variables G and Λ to explain the present scenario of the accelerating universe. One of the possibilities is that Newton's constant G can be treated as a function of time or the scale factor. Following the earlier work of Dirac, to incorporate a time-dependent G in cosmological models, some authors [6–8] studied a theory of gravitation using G and Λ as nonconstant coupling scalars. A perfect fluid cosmological model and an equation of state (EoS) are used to investigate the universe in observational and relativistic cosmology. In the general theory of gravity, the gravitational constant G plays the role of a coupling constant between the geometry of space and matter content in Einstein's field equations. The cosmological constant (Λ) arises naturally in general-relativistic quantum field theory, where it is expressed in terms of the vacuum energy density. They are also treated as fundamental constants. A cosmological model with a variable cosmological constant that is free of

*Correspondence: ssuren.mu@gmail.com

the cosmological problems was developed after finding a possible solution to the cosmological constant problem [9]. These derived models and analyses give a major boost in the investigation of the universe's singularity and its relationship with particle fields. Authors [10] also studied the model with coupling scalars G and Λ as a simple generalization of Einstein's field equations with the usual conservation laws for a matter-dominated epoch. They also derived an acceptable general approach to solve Einstein's field equations and calculated the exact solutions for dust models in the background of variable G and Λ. In other work [11,12], false vacuum models were studied with variables G and Λ, which is in agreement with observations.

In this article, the thermodynamic aspects of cosmological models where G and Λ are time-dependent variables will be the main focus of investigation. As we know, the second law of thermodynamics bound the entire system of the universe so that it maintains the temperature law in its original state. Thermodynamics are also applied for investigation of the CMB radiation and black holes. The significant amount of entropy present in the universe, which is in the form of black-body radiation, can determine the evolution of our universe. A wide range of CMB data show that matter was in a state in which the same temperature was present in all parts of a system. From the isotropy of the CMB data, it is also observed that the universe is homogeneous in its early phase. Thus, thermodynamics can also be applied to investigate the early behaviors of the early universe. Dark energy is given by parameter $\omega = -1$ in the EoS ($p = \omega\rho$) where p and ρ denote the pressure and energy density, respectively. The simplest form of dark energy is given by cosmological constant Λ at $\omega = -1$, whereas a phantom energy is described by $\omega < -1$. Though the cosmological constant Λ is used to explain the dynamics of the universe, there is also a difference between the observed and the theoretically calculated values of Λ. The quintessence field also plays roles that are responsible for the acceleration of the universe with the help of an acceptable form of potential, but it is still not confirmed for theoretical support of scalar potential. Reconstruction [13] is one of the most important attempts for investigating a quintessence field to construct the cosmological model from the observational data. There are some authors [14–16] who reconstructed a quintessence potential to explain the present acceleration of the universe. Another group of authors [17–19] also investigated the phantom energy-dominated universe as an alternative finding to suggest the present universe. Since the 1970s, the discoveries of black-hole thermodynamics [20–22] have revolutionized the relation between gravity theories and thermodynamics. We investigate the first law and second law of thermodynamics for the study of the universe on the background of thermodynamic approaches. Thermodynamics may also play some important roles in explaining the origin and the subsequent evolution of the universe. The thermodynamics approach to the description of dark energy and the related properties of matter is at present under active investigation among cosmologists. The thermodynamic properties of dark energy in the presence of perfect fluid were analyzed [23] for different values of ω. Yun [24] found the expression of pressure and energy density in terms of scale factor a. In the literature [25,26], the general equation was obtained by observing that the universe should fulfill adiabatic conditions from a thermodynamic concept. The total entropy evolution as a function of time also indicates that the second law of thermodynamics is in agreement with the dynamic dark-energy cosmological model. As we shall see, particle creation [27] in the dark-energy universe is also one of the possible candidates to explain the accelerating universe, which obeys the second law of thermodynamics. There is a possibility of annihilation of baryonic and/or dark matter particles [28] and the model is validated by the second law of thermodynamics. It is also observed that the developed model also proposes a particle interchange with dark energy.

Because of motivation by the aforementioned works, we are interested in investigating the compatibility of the second law of thermodynamics for spatially flat Friedmann–Robertson–Walker (FRW) universe on the background of generalized G and Λ. We are going to investigate the constraints in which the universe follows the generalized second law of thermodynamics in such a way that the sum of entropies of the underlying system including that of the background are nonnegative. The paper is organized as follows. In Section 2, we derive the Friedmann equations on a thermodynamic background. In Section 3, we study the solution of models by assuming time variable $G = G_0 a^n$ and some observational parameters in Section 4. We conclude the article in Section 5 with an interpretation of some dynamical and physical behaviors from the solution.

2. Field equations and thermodynamics

Recent observations do not support the stability of fundamental constants and the equivalence principle of general relativity. Accelerated expansion of the universe being due to the dark energy, which is the energy density stored in the vacuum state of all existing fields in the universe, is also one of the many possible alternatives. The variable cosmological constant [29,30] is one of the alternatives to explain the dark-energy problem, compatible with observations. A small and positive value of the effective cosmological constant is predicted by observations in the present epoch. Several authors [31,32] also studied time-dependent cosmological constants in different contexts. Dirac [33,34] was the first to introduce the time-dependent gravitational constant G in a large number of hypotheses. In the context of cosmological models, according to the cosmological principle, these varying constants are only time-dependent. In this work, we develop a cosmological model based on the homogeneous and isotropic spatially flat FRW line element in the framework of time-dependent G and Λ. Two Friedmann equations based on the FRW metric are as follows:

$$H^2 = \frac{8\pi G}{3}\left(\rho + \frac{\Lambda}{8\pi G}\right) \tag{1}$$

$$\dot{H} = -4\pi G\left(\rho + p\right) \tag{2}$$

Here, $H = \frac{\dot{a}}{a}$ is the Hubble parameter and the overdot denotes the derivative w.r.t time (t). Differentiating Eq. (1) and using Eq. (2), we have:

$$\dot{\rho} + 3H(\rho + p) = -\left(\frac{\rho\dot{G}}{G} + \frac{\dot{\Lambda}}{8\pi G}\right) \tag{3}$$

It is observed that matter production emerges in a gradual process as long as the cosmological term decays. In this model, the effective cosmological constant is a time-dependent parameter. We also assume G to be a function of time. Canuto et al. [35,36] also studied G-varying cosmologies. Cosmological models with time-dependent G were also studied in some works [37–39]. As we know, the conservation of the energy momentum law is also an essential part of modeling the universe and it yields:

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{4}$$

From the combination of Eqs. (4) and (3), we obtain:

$$8\pi\dot{G}\rho + \dot{\Lambda} = 0 \tag{5}$$

From Eqs. (1) and (2), we get:

$$8\pi G\rho = 3H^2 - \Lambda \tag{6}$$

$$8\pi Gp = -2\dot{H} - 3H^2 + \Lambda \tag{7}$$

Using the value of Hubble parameter $H = \frac{\dot{a}}{a}$ in Eq. (4), we get:

$$\frac{d}{dt}(\rho a^3) + p\frac{d}{dt}a^3 = 0 \tag{8}$$

To study the thermodynamics of variables G and Λ, we assume that the universe satisfies the first law of thermodynamics. For thermodynamic study [24,25], the union of the first law and second law of thermodynamics is used to a co-moving volume element of unit coordinate volume. Thus, physical volume $V = a^3$ becomes:

$$\begin{aligned} TdS = d(\rho V) + p\,dV &= d\left[(\rho + p)\,V\right] - V\,dp \\ &= \tfrac{(\rho+p)}{T}dV + \tfrac{V}{T}\tfrac{d\rho}{dT}dT \end{aligned} \tag{9}$$

Here, $S = S\,(V,T)$ is the entropy density of the universe within volume V at temperature T and $\rho = \rho(T)$ and $p = p(T)$ are the energy density and the pressure of the matter in the universe. Here $\left(\frac{\partial S}{\partial V}\right)_T = \frac{\rho+p}{T}$ and $\left(\frac{\partial S}{\partial T}\right)_V = \frac{V}{T}\frac{d\rho}{dT}$. To define a thermodynamic system, it is necessary to use the integrability condition [41] in the FRW universe. It is expressed as:

$$\begin{aligned} \frac{\delta^2 S}{\delta T \delta V} &= \frac{\delta^2 S}{\delta V \delta T} \\ &\Rightarrow \frac{1}{T}\left(\frac{d\rho}{dT}\right) + \frac{1}{T}\left(\frac{dp}{dT}\right) - \frac{1}{T^2}(p+\rho) = \frac{1}{T}\frac{d\rho}{dT} \end{aligned} \tag{10}$$

This leads to the relation between the pressure, energy density, and temperature as follows:

$$dp = \frac{\rho + p}{T}dT \tag{11}$$

This is also found directly from the equilibrium expression for the pressure and energy density. Using Eq. (11) in Eq. (9), we get a different entropy definition:

$$d\,S = \frac{1}{T}d\left[(\,\rho{+}\,p)\,V\,\right] - (\,\rho{+}\,p)\,V\frac{dT}{T^2} = d\,\left[\frac{(\rho+p)\,V}{T}\right] \tag{12}$$

In this model, the entropy is defined in terms of co-moving volume $V = a^3$. Integration of the above equation then gives:

$$S = \frac{(\rho + p)V}{T} \tag{13}$$

Here, the constant of integration is taken to be zero. On the other hand, Eq. (4) along with the EoS, $p = \omega\rho$, gives the energy density as:

$$\rho(a) = \rho_0 a^{-3\,(1+\omega)} \tag{14}$$

By considering the equivalent relation of $d(\rho V) + pdV = 0$, the conservation law corresponds to $dS = 0$, or the equivalent. Eq. (12) leads to:

$$d\left[\frac{(\rho + p)\,V}{T}\right] = 0 \tag{15}$$

The above equation suggests that the FRW universe satisfying the conservation law is expanding adiabatically. The same definition of entropy is also derived from the energy conservation law $d(\rho V) + p\,dV = 0$, which can be written as:

$$d\left[(\rho + p)\,V\right] = V dp \tag{16}$$

Eq. (15) can be obtained by using Eq. (11) in Eq. (16). With the help of the EoS, $p = \omega\,\rho$, where $-1 \leq \omega < 1$, we obtain the expression of entropy from Eq. (13) as:

$$S = \frac{(1+\omega)\rho V}{T} \tag{17}$$

The above equation gives the relation of entropy in terms of ρ and T. Using Eq. (14) and co-moving volume $V = a^3$, Eq. (17) gives the relation of the temperature in terms of the scale factor and entropy as:

$$T = \frac{(1+\omega)\rho_0}{S} a^{-3\omega} \tag{18}$$

At this point, Eq. (18) may be used to make the application of the concept of negative temperature for a phantom field to accommodate positive entropy or vice versa. On the other hand, in order to keep the phantom regime validated, the concept of negative temperature is accepted for $\omega < -1$. Scaling law $T \sim a^{-3\omega}$ is always true for $\omega \geq -1$. Thus, this idea of negative temperature is discarded because the temperature of a fluid should be positive-definite. Another possible way is the establishment of a negative potential to the fluid [42] to validate the hypothesis of the phantom. However, the generalized second law of thermodynamics postulates that the total entropy S of the universe should increase with time during cosmological evolution, i.e. $dS \geq 0$, which implies that the total entropy of the system is positive, i.e. $S \geq 0$. In order to show the validity of the second law of thermodynamics for the derived model, we consider $\omega > -1$ and $T > 0$, which are in agreement with the present observations. From the integrability condition of Eq. (11), we have:

$$\frac{dp}{p} = \left[\frac{1+\omega}{\omega}\right]\frac{dT}{T} \tag{19}$$

Integration of the above equation gives:

$$\ln p = \left[\frac{1+\omega}{\omega}\right]\ln T + \ln C(\omega) \tag{20}$$

From the above equation, we observe that the relation of pressure and temperature is obtained as:

$$p(T) = C(\omega) T^{\frac{1+\omega}{\omega}} \tag{21}$$

Here, $C(\omega)$ is an unknown function. Also, the integrability condition, i.e. $\frac{d\rho}{\rho} = \left[\frac{1+\omega}{\omega}\right]\frac{dT}{T}$, shows that the energy density is related to the temperature as:

$$\rho(T) = \tilde{C}(\omega) T^{\frac{1+\omega}{\omega}} \tag{22}$$

Here, $\tilde{C}(\omega)$ is also an unknown function. The pressure and density are of the same form, which has the same factor, $T^{\frac{1+\omega}{\omega}}$, and one can easily obtain the important relation between p, ω, and ρ if we allot $C(\omega) \sim \omega$ and

$\tilde{C}(\omega) \sim 1$. Using Eqs. (18),(21), and (22) leads to expression of ρ and p associated with entropy as:

$$p(T) = \omega(1+\omega)^{\frac{1+\omega}{\omega}} \left[\frac{\rho_0}{S}\right]^{\frac{1+\omega}{\omega}} a^{-3\,(1+\omega)} \tag{23}$$

$$\rho(T) = (1+\omega)^{\frac{1+\omega}{\omega}} \left[\frac{\rho_0}{S}\right]^{\frac{1+\omega}{\omega}} a^{-3\,(1+\omega)} \tag{24}$$

Here the term $(1+\omega)^{\frac{1+\omega}{\omega}}$ does not exist for $\omega \leq -1$ and it is also undefined for $\omega = -1$. It is observed that when the second law of thermodynamics ($S > 0$) is satisfied, $\rho(T)$ is positive and is a decreasing function of $a(t)$ for all $\omega > -1$ and $T > 0$. On the other hand, the model becomes unphysical when the second law of thermodynamics is not valid. The present model does not exist for $\omega = -1$, which is the dark-energy epoch.

3. Models for power law form of $G = G_0 a^n$.

With substitution of the EoS, $p = \omega\,\rho$, in Eq. (8), we obtain:

$$\frac{1}{\Psi}\frac{d\,\Psi}{d\,a} + \frac{3\omega}{a} = 0 \tag{25}$$

Here,

$$\Psi = \rho a^3 \tag{26}$$

Ψ can be solved as in the following equation:

$$\Psi = \Psi_0 a^{-3\omega} \tag{27}$$

If Ψ is solved, then we can also determine ω as in the following relation:

$$\omega = -\frac{a}{3\Psi}\frac{d\Psi}{da} \tag{28}$$

With the application of $\frac{d}{d\,t} = \dot{a}\frac{d}{d\,a}$, from Eqs. (5) and (26), we obtain:

$$8\pi\frac{d\,G}{da} + \frac{a^3}{\Psi}\frac{d\Lambda}{da} = 0 \tag{29}$$

If $G = G(a)$ is determined, then the above differential equation gives $\Lambda = \Lambda(a)$ and $a = a(t)$. Hence, $p(T)$ and $\rho(T)$ are expressed as a function of time and entropy. On the other hand, if $\Lambda = \Lambda(a)$ is known, then we can also find the value of $G = G(a)$ from Eq. (29) and the value of $a(t)$ is obtained by integrating Eq. (6) after inserting the known values of Λ, G, and ρ given by Eq. (14). Since we are looking for physically acceptable models of the universe consistent with observations, we consider the following:

$$G = G_0 a^n \tag{30}$$

Here, n is a positive integer. Using the value of Ψ from Eq. (27) and the assumed value of G in Eq. (29), we obtain:

$$\Lambda = \begin{cases} -8\pi G_0 \Psi_0 \left(\frac{n}{n-3(1+\omega)}\right) a^{n-3(1+\omega)}, & \text{if} \quad n \neq 3(1+\omega); \\ -8\pi G_0 \Psi_0 \left[3(1+\omega)\ln a\right], & \text{if} \quad n = 3(1+\omega). \end{cases} \tag{31}$$

Here, the integrating constants are taken to be zero to find a viable model. Putting the values of G, Λ, and Ψ in Eq. (6), we have:

$$3H^2 = \begin{cases} 8\pi G_0\Psi_0 \left(1 - \frac{n}{n-3(1+\omega)}\right) a^{n-3(1+\omega)}, & \text{if} \quad n \neq 3(1+\omega); \\ 8\pi G_0\Psi_0 \left[1 - 3(1+\omega)\ln a\right], & \text{if} \quad n = 3(1+\omega). \end{cases} \tag{32}$$

3.1. Models with $n = 3(1+\omega)$

To find the value of $a(t)$, we consider the case of $n = 3(1+\omega)$. The solution of the differential equation given by Eq. (32) gives the expression of:

$$a(t) = \Lambda_1 + \exp\left[\frac{1}{3(1+\omega)} - (1+\omega)A_0 t^2\right] \tag{33}$$

Here, $A_0 = 2\pi G_0\Psi_0$. Here $A_0 > 0$ as the scale factor is positive. Also, we get $G_0 > 0$ or otherwise A_0 cannot be positive. It is also observed that $a(t) > 0$ when $0 < t < \frac{1}{\sqrt{3(1+\omega)^2 A_0}}$. Since $H < 0$, we observe that the universe is contracting, which is inconsistent with present observations. Thus, the derived model is not considered for our investigation.

3.2. Models with $n \neq 3(1+\omega)$

In this model, the differential equation given by Eq. (32) leads to the expression of $a(t)$ as:

$$a(t) = \left[\frac{3(1+\omega)-n}{2}\right]^{\frac{2}{3(1+\omega)-n}} \left(\left[\frac{8\pi G_0\Psi_0}{3}\right]\left[1 - \frac{n}{n-3(1+\omega)}\right]\right)^{\frac{1}{3(1+\omega)-n}} t^{\frac{2}{3(1+\omega)-n}} \tag{34}$$

Here we see that the scale factor is still a function of ω. Since the scale factor has to be positive, i.e. $a(t) > 0$, we have $[3(1+\omega)-n] > 0 \Rightarrow \omega > \frac{n}{3} - 1$. From Eqs. (23) and (24), we observed that $(1+\omega)^{\frac{1+\omega}{\omega}}$ is not available for $\omega \leq -1$ and is also undetermined for $\omega = 0$, and we found n in the range of $3 < n < 6$ for permissible values of equation of state ω. For simplicity of our calculation, we assume $n = 5$ to reduce Eq. (34) in simple form. We discuss the stiff-fluid model $(\omega = 1)$ and we obtain $p = \rho$. Then Eq. (34) leads to the expression of the scale factor as:

$$a(t) = a_0\, t^2 \tag{35}$$

Here, $a_0 = 4\pi G_0\Psi_0$. It is observed that the universe evolves with power law expansion. Here $G_0 > 0$ as the scale factor is positive, or otherwise a_0 cannot be positive. The expansion scalar factor, $\Theta = 3\frac{\dot{a}}{a} = \frac{6}{t}$, is infinite at $t = 0$. The universe starts from big-bang singularity with an infinite rate of expansion. The deceleration parameter $q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{1}{2} < 0$ shows that the present universe is accelerating, which is in agreement with the observations. It is also observed that the universe shows an infinite expansion as $t \to \infty$. With the help of Eq. (35), Eqs. (30), (31), (23), and (24) give:

$$G = G_0 a_0^5 t^{10} \tag{36}$$

$$\Lambda(t) = 2t^{-2} \tag{37}$$

and

$$p(T) = \rho(T) = A\left[\frac{\rho_0}{S}\right]^2 t^{-12} \tag{38}$$

Here, $A = \frac{4}{a_o^6}$. It is observed that pressure and energy density are expressed as a function of entropy (S) and vice versa. Since $A > 0$, $\rho_0 > 0$ and $S > 0$, $\rho > 0$ is a decreasing function of time, which is consistent with observations. From Eq. (8), we get $\frac{d(\rho a^3)}{dt} = 0$, which gives $\rho \propto a^{-3}$. Since the parameter $\frac{1+\omega}{\omega}$ is undefined for the dust model $(\omega = 0)$, Eq. (24) shows that the energy density is not defined for the dust model although we know that energy density is inversely proportional to a^3. In the false vacuum model $(\omega = -1)$, the limiting value of $(1+\omega)^{\frac{1+\omega}{\omega}}$ is 1, but it is not defined at $\omega = -1$. In the case of zero entropy, Eq. (18) shows that the temperature is badly defined. This suggests that the thermodynamic interpretation for the vacuum model is still ambiguous. From Eq. (36), it is observed that G is a rapidly increasing function of time. In most of the variable G cosmologies [43], G is a decreasing function of time, but choice of an increasing G has also been executed by several authors [44,45]. This alteration in G also keeps Einstein's field equations unchanged by allowing variation of G to be accompanied by a time-dependent Λ. This enables us to find the solutions of many unsolved cosmological problems, such as the cosmological constant problem or the inflationary scenario [46].

4. Some observational parameters

In this section, we investigate the observational parameters for the stiff-fluid model $\omega = 1$ and $n = 5$, which gives scale factor $a = a_0 t^2$. We also study physical parameters [47] such as red-shift, look-back time, proper distance, luminosity distance, and angular diameter distance for our derived model.

4.1. Look-back time

Look-back time is defined as the time in the past at which the light we receive from a distant object was emitted. The dynamics of the universe give the look-back time. For any red-shift z, the regime of emission of light is given mathematically by:

$$\frac{a_\theta}{a} = 1 + z \tag{39}$$

Here a_θ is the present-day scale factor. The radiation travel time $(t_\theta - t)$ for a photon emitted by a source at instant t and received at t_θ is given by:

$$t_\theta - t = \int_a^{a_\theta} \frac{da}{\dot{a}} \tag{40}$$

Here, from Eq. (35), we obtain:

$$\frac{a_\theta}{a} = 1 + z = \left(\frac{t_\theta}{t}\right)^2 \tag{41}$$

And $H_\theta = \frac{2}{t_\theta}$ (42) Here, $H_\theta(kms^{-1}Mpc^{-1})$ is Hubble's constant at present, which is in the range $(50 \leq H_\theta \leq 100)$ km s^{-1} Mpc^{-1}. Therefore:

$$t = t_\theta\sqrt{\frac{a}{a_\theta}} \tag{42}$$

Eq. (40) gives:

$$t_\theta - t = \frac{2}{H_\theta}\left[1 - (1+z)^{-\frac{1}{2}}\right] \tag{43}$$

For small z, we have:

$$H_\theta(t_\theta - t) = 2\left[\frac{1}{2}z + \frac{1}{8}z^2 + ...\right] \tag{44}$$

$$H_\theta(t_\theta - t) \approx z \tag{45}$$

Taking the limit $z \to \infty$ in Eq. (43), the present age of the universe (the extrapolated time back to the big-bang) is given by:

$$t_\theta = 2H_\theta^{-1} \tag{46}$$

This present age of the universe is consistent with the observations.

4.2. Proper distance

Proper distance indicates the distance where a distant object would be at a particular moment of cosmological time, which may change with time due to the expansion of the universe. Co-moving distance measures the expansion of the universe, which allows for a distance that remains unaltered over time due to the expansion of space coordinates. Thus, the proper distance $d(z)$ is defined in terms of the distance between a cosmic source emitting light at $t = t_1$ positioned at $r = r_1$ with red-shift z and an observer located at $t = t_\theta$ and $r = 0$ gathering the light from the cosmic source emitted, i.e.:

$d(z) = r_1\, a_\theta$ (48) Here the distance is:

$$r_1 = \int_{t_1}^{t_\theta} \frac{dt}{a} = \frac{2}{H_\theta a_\theta}(\sqrt{1+z} - 1) \tag{47}$$

For the discussed model, we have the proper distance as:

$$d(z) = 2H_\theta^{-1}(\sqrt{1+z} - 1) \tag{48}$$

It is observed that the proper distance $d(z)$ is a function of red-shift z. The distance $d(z = \infty)$ is always infinite.

4.3. Luminosity distance

The luminosity distance is a way of magnifying the amount of light received from a distant object. Assuming the inverse square law for the reduction of light intensity with distance holds, it will be a distance in which the object seems to hold. As the inverse square law is not valid in the universe, the luminosity distance is also not the real distance to the object. As the universe is expanding and it may not be flat geometry, both are destroyed. In other words, it is defined as the generalization of the inverse square law of the brightness in the static Euclidean space to an expanding curved space [48]. If the luminosity distance to the object is denoted by d_L, then:

$$d_L = \left(\frac{L}{4\pi l}\right)^{\frac{1}{2}} \tag{49}$$

Here L denotes the total energy emitted per unit time by the source, and the apparent luminosity of the object is denoted by l. Therefore, we can obtain:

$$d_L = d(z)(1+z) \tag{50}$$

Using Eq. (48) in Eq. (50), we have the relation for luminosity distance as:

$$H_\theta d_L = 2(1+z)(\sqrt{1+z}-1) \tag{51}$$

Here, the luminosity distance increases with red-shift z. The luminosity distance is constructed upon the examined cosmological model and hence can be used as a tool to predict the standard cosmological model that describes our universe. On the other hand, the radiation flux density received from an object is also an observable quantity and this can also be defined in terms of luminosity distance if the total energy emitted by the source per unit time of the object is observed.

4.4. Angular diameter

The ratio of an object's physical transverse size to its angular size is given by the angular diameter distance (d_A). It is used to change the angular separations in images of telescope into proper separations at the sources. The formulation of the angular diameter (d_A) of a light source where proper distance is denoted by (d) is given by:

$$d_A = d(z)(1+z)^{-1} \tag{52}$$

With the help of Eq. (48) in Eq. (52), we obtain:

$$H_\theta d_A = 2(1+z)^{-1}(\sqrt{1+z}-1) \tag{53}$$

Here, d_A has a critical or extreme point at $z=-1$. Here d_A tends to 0 as $z \to \infty$. Hence, objects look very dim at large red-shifts, which is consistent with present observations. Thus, the distance parameters such as look-back time, proper distance, and luminosity distances are consistent with the present observed universe

5. Remarks

In this work, we investigated the FRW universe with variables G and Λ in light of the second law of thermodynamics. We introduced an acceptable form of the general method to solve Einstein's field equations by using the global EoS and assuming $G = G_0 a^n$. The mathematical expressions for temperature and entropy of the perfect fluid cosmological model were obtained for the derived model. In the model with $n = 3(1+\omega)$, the scale factor $a(t)$ is inversely proportional to $\exp[A_0(1+\omega)t^2]$. However, this model gives a contracting universe, which is inconsistent with the observation since $H < 0$. We discuss the evolution of the stiff-fluid model with $n \neq 3(1+\omega)\omega = 1$ when $n = 5$. In this model, temperature is taken to be positive. A phantom-dominated universe does not exist to validate the second law of thermodynamics in this derived model, whereas the phantom epoch survives in [49] with the validity of the same law. We observed the power-law expansion of the stiff-fluid universe for $1 < n < 6$. The model always exists when the second law of thermodynamics $(S > 0)$ is always obeyed, assuming $A > 0$ and $\rho_0 > 0$ and vice versa. Thus, the derived model is constrained by the second law of thermodynamics. In this model, we see that the energy density is a decreasing function of time while pressure is increasing with the evolution of the universe for nonzero entropy. Since $\Theta > 0$ and $q = -\frac{1}{2}$, the solutions predict an ever-expanding universe with the initial singularity $a(0) = 0$ for open-space geometry as $t \to \infty$. The universe expands with an infinite rate of expansion as $t \to \infty$. Dark energy $(\omega = -1)$ and matter-dominated $\omega = 0$ cosmological models are not feasible in this thermodynamic setup, whereas [50] observed the dark-energy epoch in viscous cosmology. We also investigate some familiar observational phenomena like look-back time, the proper distance, the luminosity distance, and the angular diameter distance versus red-shift for the model with time-dependent G and Λ to ensure that the second law of thermodynamics remains unviolated. For the derived model, the square of the age of the universe corresponding to red-shift z is proportional to $2H_\theta^{-1}$. The angular diameter has an extreme point at $z = -1$. It has been observed that such models are compatible with present observations. Cosmological constant Λ is very large in the early universe and then it gradually decreases as the universe evolves, whereas G increases with time. Thus, the present model, in spite of its simplicity, is in agreement with the second law of thermodynamics to some extent.

Acknowledgement

We would like to thank the DST, SERB, India, Grant ECR/2016/001485, for financial support.

References

[1] Riess, A. G.; Filippenko, A. V.; Challis, P.; Clocchiatti, A.; Diercks, A.; Garnavich, P. M.; Gilliland, R. L.; Hogan, C. J.; Jha, S.; Kirshner, R. P. et al. *Astron. J.* **1998**, *116*, 1009-1038.

[2] Perlmutter, S.; Aldering, G.; Goldhaber, G.; Knop, R. A.; Nugent, P.; Castro, P. G.; Deustua, S.; Fabbro, S.; Goobar, A.; Groom, D. E. et al. *Astrophys. J.* **1999**, *517*, 565-586.

[3] Bennett, C. L.; Halpern, M.; Hinshaw, G.; Jarosik, N.; Kogut, A.; Limon, M.; Meyer, S. S.; Page, L.; Spergel, D. N.; Tucker G. S. et al. *Astrophys. J. Suppl. Ser.* **2003**, *148*, 1-27.

[4] Tegmark, M.; Strauss, M.; Blanton, M.; Abazajian, K.; Dodelson, S.; Sandvik, H.; Wang, X.; Weinberg, D.; Zehavi, I.; Bachall, N. et al. *Phys. Rev. D* **2004**, *69*, 103501.

[5] Spergel, D. N.; Verde, L.; Peiris, H. V.; Komatsu, E.; Nolta, M. R.; Bennet, C. L.; Halpern, M.; Hinshaw, G.; Jarosik, N.; Kogut, A. et al. *Astrophys. J. Suppl. Ser.* **2003**, *148*, 175-194.

[6] Page, L.; Nolta, M. R.; Barnes, C.; Bennett, C. L.; Halpern, M.; Hinshaw, G.; Jarosik, N.; Kogut, A.; Limon, M.; Meyer, S. S. et al. *Astrophys. J. Suppl. Ser.* **2003**, *148*, 233-241.

[7] Beesham, A. *Nuovo Cimento B* **1986**, *96*, 17-20.

[8] Beesham, A. *Int. J. Theor. Phys.* **1986**, *25*, 1295-1298.

[9] Abdel-Rahman, A. M. M. *Gen. Rel. Grav.* **1990**, *22*, 655-663.

[10] Ozer, M.; Taha, M. O. *Nucl. Phys. B* **1987**, *287*, 776-796.

[11] Sistero, R. F. *Gen. Relativ. Gravit.* **1991**, *23*, 1265-1277.

[12] Singh, N. I.; Singh, S. S.; Devi, S. R. *Astrophys. Space Sci.* **2009**, *324*, 67-70.

[13] Singh, N. I.; Chanu, S. K.; Singh, S. S. *Chin. Phys. Lett.* **2009**, *26*, 060403.

[14] Ellis, G. F. R.; Madsen, M.S. *Class. Quantum Grav.* **1991**, *8*, 667-676.

[15] Starobinsky, A. A. *JETP Lett.* **1998**, *68*, 757-763.

[16] Sahni, V.; Starobinsky, A. A. *Int. J. Mod. Phys. D* **2006**, *15*, 2105-2132.

[17] Mukherjee, A.; Banerjee, N. *Eur. Phys. J. Plus* **2015**, *130*, 201.

[18] Gonzalez-Diaz, P. F.; Siguenza, C. L. *Nucl. Phys. B* **2004**, *697*, 363-386.

[19] Caldwell, R. R.; Kamionkowski, M.; Weinberg, N. N. *Phys. Rev. Lett.* **2003**, *91*, 071301.

[20] Nojiri, S.; Odintsov, S. D. *Phys. Rev. D* **2004**, *70*, 103522.

[21] Bekenstein, D. *Phys. Rev. D* **1973**, *7*, 2333-2346.

[22] Hawking, S. W. *Commun. Math. Phys.* **1975**, *43*, 199-220.

[23] Bardeen, J. M.; Carter, B.; Hawking, S. W. *Commun. Math. Phys.* **1973**, *31*, 161-170.

[24] Pereira, S. H.; Lima, J. A. S. *Phys. Lett. B* **2008**, *669*, 266-270.

[25] Yun, S. M. *Phys. Lett. B* **2009**, *671*, 216-218.

[26] Silva, H. H. B.; Silva, R.; Gonalves, R. S.; Zhu, Z.; Alcaniz, J. S. *Phys. Rev. D* **2013**, *88*, 127302.

[27] Chattopadhyay, S.; Debnath, U.; Bhattacharya, S. *Int. J. Theor. Phys.* **2012**, *51*, 3168-3185.

[28] Singh, S. S. *Int. J. Theor. Phys.* **2014**, *53*, 1743-1751.

[29] Balfagon, A. C. *Gen. Relativ. Gravit.* **2015**, *47*, 111.

[30] Peebles, P. J. E.; Ratra, B. *Rev. Mod. Phys.* **2003**, *75*, 559-606.

[31] Padmanabhan, T. *Gen. Rel. Gravitation* **2008**, *40*, 529-564.

[32] Singh, C. P.; Kumar, S.; Pradhan, A. *Class. Quantum Grav.* **2007**, *24*, 455-474.

[33] Pradhan, A.; Jotania, K. *Indian J. Phys.* **2011**, *85*, 497-514.

[34] Dirac, P. A. M. *Nature* **1937**, *139*, 323-323.

[35] Dirac, P. A. M. *Nature* **1937**, *139*, 1001-1002.

[36] Canuto, V. M.; Adams, P. J.; Hseih, S. H.; Tsing, E. *Phys. Rev. D* **1977**, *16*, 1643-1663.

[37] Canuto, V. M.; Hsieh, S. H.; Owen, J. R. *Astrophys. J. Suppl. S.* **1979**, *41*, 263-300.

[38] Belinchon, J. A. *Astrophys. Space Sci.* 2014, *349*, 501-527.

[39] Vishawakarma, R. G. *Class. Quantum Grav.* **2001**, *18*, 1159-1172.

[40] Chand, H.; Srianand, R.; Petitjean P.; Aracil, B. *Astron. Astrophys.* **2004**, *417*, 853-871.

[41] Kolb, E. W.; Turner, M. S. *The Early Universe*; Addison-Wesley: New York, NY, USA, 1990.

[42] Lima, J. A. S.; Pereira, S. H. *Phys. Rev. D* **2008**, *78*, 083504.

[43] Krori, K. D.; Mukherjee, A. *Gen. Relativ. Gravit.* **2000**, *32*, 1429-1438.

[44] Weinberg, S. *Gravitation and Cosmology*; Wiley: New York, NY, USA, 1972.

[45] Singh, T.; Beesham, A.; Mbokazi, W. S. *Gen. Relativ. Gravit.* **1998**, *30*, 573-581.

[46] Pradhan, A.; Singh, A. K.; Otarod Roman, S. *Rom. J. Phys.* **2007**, *52*, 445-458.

[47] Bishi, B. K.; Pacif, S. K. J.; Sahoo, P. K.; Singh, G. P. *Int. J. Geom. Methods M.* **2017**, *14*, 175015.

[48] Waga, I. *Astrophys. J.* **1993**, *414*, 436-448.

[49] Wu, S. F.; Wang, B.; Yang, G. H.; Zhang, P. M. *Class. Quantum Grav.* **2008**, *25*, 235018.

[50] Davis, P. C. W. *Class. Quantum Grav.* **1987**, *4*, L225-L228.