

Phenomenological potential calculations for cluster decays

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Received: 31.10.2016

Accepted/Published Online: 24.02.2017

Final Version: 13.06.2017

Abstract: The half-lives of cluster decays from various nuclei having experimentally measured values have been calculated by using the Wentzel–Kramers–Brillouin method together with the Bohr–Sommerfeld quantization condition for three different types of phenomenological nuclear interactions: Woods–Saxon (WS), squared Woods–Saxon (WS-2), and mixed Woods–Saxon (mWS) potentials. By comparing the results with experimental values, rms values have been calculated. The half-lives of ^{14}C , ^{15}N , ^{16}O , ^{17}O , ^{18}O , ^{20}O , ^{22}O , ^{23}F , ^{22}Ne , ^{24}Ne , ^{25}Ne , ^{26}Ne , ^{28}Mg , ^{30}Mg , and ^{34}Si decays from various isotopes whose experimental half-lives are unknown have also been calculated by using the WS, WS-2, and mWS potentials. The obtained results have been compared with results of the liquid drop model, the Christensen–Winther potential form, and the Coulomb and proximity potential model. Even if the logarithmic values of the half-lives have similar behaviors, there are differences in size between the values. The present results provide useful information and extra theoretical data for future measurements of the unknown half-lives of possible cluster emissions from various nuclei.

Key words: Wentzel–Kramers–Brillouin method, cluster decay, half-lives, phenomenological potential

1. Introduction

The investigation of the alpha decays of heavy, super heavy, and light nuclei is an attractive topic in nuclear studies since this sort of research gives useful knowledge about nuclear structures. Moreover, the α -decay of nuclei has become a useful tool for studies on the new super heavy elements being produced at accelerator centers around the world [1–3]. The alpha particle already exists in the parent nucleus before the decay and it can be tunneled through an effective potential between the α -particle and core nuclei in terms of quantum tunneling, which is a quantum mechanical model of the α -decay mechanism [4,5]. In this way, a many-body system can be reduced to a two-body system and the problem can be easily solved. After Gamow wrote a closed formula between the logarithmic values of half-lives and the Q-values, this relation and formula were also obtained by Geiger and Nuttall [6]. In recent years studies considering deformations on the α -decay of heavy and super heavy nuclei have become interesting and popular research avenues [7–25].

On the other hand, a new research field in the nuclear world appeared when the cluster radioactivity of nuclei was uncovered experimentally by Rose and Jones [26]. ^{14}C exotic cluster decay from ^{223}Ra was shown and then the experimental results were established [27,28]. The first description of heavier cluster decay of nuclei was done theoretically by Sandulescu et al. [29]. The theoretical calculations related to cluster decays in terms of quantum tunneling can be performed by fission-like or alpha-like theory. Numerous papers in which authors have used different methods have been published on finding half-lives of these cluster emissions with

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and without deformation [30–38]. More recently, intensive works have been continuing in order to get closed analytical formulas like the Geiger and Nuttall rule for the cluster decays of isotopes [39–44].

In theoretical studies, the most important topic is finding out the nuclear potential form as the only unknown term is the nuclear potential between the cluster and core nuclei in the effective potential. Therefore, there are many works in the literature that aim to describe nuclear interactions. Heavier cluster decays using a microscopic double folding cluster model were investigated by Ren et al., the cluster emission half-lives with a new type of formula were also obtained [45,46], and the liquid drop model (GLDM) [47] was used in the calculations. The heavy cluster decays of some nuclei were studied by using the nuclear Coulomb and proximity potential model (CPPM) [48–51]. More recently, Nazarzadeh and Mohebali also used the Christensen–Winter potential form (CW76) in order to model nuclear interaction. They obtained good agreement with experimental data by considering the deformation in the calculations [38]. Soylu et al. applied phenomenological potentials, which were the deformed Woods–Saxon (WS), the squared Woods–Saxon (WS-2), and Cosh, to find out the half-lives of the cluster decays from various isotopes [31] and radium isotopes [37] with and without the deformation cases. Buck et al. proposed the mixed Woods–Saxon (mWS) form to calculate the cluster states of heavy nuclei [52]. They used phenomenologically chosen parameters inside the potential in the calculations.

In this study, the cluster decay half-lives of various isotopes have been computed phenomenologically with the Wentzel–Kramers–Brillouin (WKB) method and a preformed cluster model by considering the Bohr–Sommerfeld condition. To see how the nuclear potentials affect half-lives, 3 different types of potential, WS, WS-2, and mWS, are used for obtaining half-lives of isotopes that have experimentally measured values. After calculating rms values for the half-lives, the 3 different potentials have also been used in order to calculate half-lives of all possible cluster decays that do not have experimental values yet. The half-lives of ^{14}C , ^{15}N , ^{16}O , ^{17}O , ^{18}O , ^{20}O , ^{22}O , ^{23}F , ^{22}Ne , ^{24}Ne , ^{25}Ne , ^{26}Ne , ^{28}Mg , ^{30}Mg , and ^{34}Si decays from various nuclei have been obtained and compared with the results obtained with the GLDM, CPPM, and CW76 potential models.

In Section 2, we introduce basic formulations required to get the cluster emission half-lives in the WKB method. The obtained numerical results can be found in Section 3. Section 4 is devoted to a discussion.

2. The model

The effective interaction potential between the cluster and daughter is given by:

$$V_{eff}(r) = V_C(r) + V_N(r) + V_L(r) \quad (1)$$

where V_N is the nuclear potential, V_C the Coulomb potential, and V_L is the centrifugal potential. In semiclassical calculations, V_L , the centrifugal potential, is used as the Langer modified centrifugal barrier potential as follows:

$$V_L = \frac{(l + 1/2)^2 \hbar^2}{2\mu r^2}, \quad (2)$$

where l is angular momentum for the cluster and μ is the reduced mass [53]. The repulsive Coulomb potential is given by:

$$V_C(r) = 1.43 \left(\frac{Z_c Z_d}{2R_C} \right) \left(3 - \frac{r^2}{R_C^2} \right), \quad r \leq R_C \quad (3a)$$

$$= 1.43 \left(\frac{Z_c Z_d}{r} \right), \quad r > R_C, \quad (3b)$$

where Z_c and Z_d denote the proton numbers of the cluster and daughter nuclei, respectively, and R_C is the Coulomb radius. The following approximation potential form can be used to solve the discontinuity in the Coulomb potential at $r = R_C$ [54]:

$$\begin{aligned} V_{Cf}(r) &= 1.43 \frac{Z_c Z_d}{r} \left(1 - e^{-\varphi r - \frac{1}{2}(\varphi r)^2 - 0.35(\varphi r)^3} \right) \\ \varphi R_C &= 3/2. \end{aligned} \quad (4)$$

By choosing $\varphi R_C = 3/2$, one finds that Eq. (4) gives a potential that is identical to the Coulomb potential to overcome the discontinuity. To obtain the parameters in the effective potential, one uses the Bohr–Sommerfeld quantization condition as follows:

$$\int_{r_1}^{r_2} \sqrt{\frac{2\mu}{\hbar^2} (Q - V_{eff}(r))} dr = (2n + 1) \frac{\pi}{2} = (G_c - l + 1) \frac{\pi}{2}, \quad (5)$$

where Q is the Q -value for the decay of the cluster, n is the number of nodes, and l is the angular momentum carried by the cluster nuclei. We use $G_c = A_c G_\alpha / 4$ as in the literature for the cluster emissions [55,56]. The value of G is obtained with the Wildermuth rule. In this study, $G_\alpha = 18$ is used for nuclei that have neutron number $N < 82$, $G_\alpha = 20$ is used for nuclei that have neutron number $82 \leq N \leq 126$, and $G_\alpha = 22$ is used for nuclei that have neutron number $N > 126$ [31,37,52]. Moreover, there are three classical turning points, r_1 , r_2 , and r_3 , for the effective interaction potential. They might be achieved by solving the equation of $V_{eff}(r) = Q$ numerically.

According to the semiclassical method, the cluster decay width is given by [31]:

$$\Gamma = PF \frac{\hbar^2}{4\mu} S, \quad (6)$$

where P is the preformation factor probability, F is the normalization factor, and S is the transition probability of the cluster nuclei. In the present calculations, the following numbers are used for the preformation factor: $P = 1$ for even–even nuclei, $P = 0.6$ for odd- A nuclei, and $P = 0.35$ for odd–odd nuclei [57].

The normalization factor, F , is obtained by

$$F \int_{r_1}^{r_2} dr \frac{1}{k(r)} \cos^2 \left(\int_{r_1}^r dr' k(r') \frac{\pi}{4} \right) = 1, \quad (7)$$

so that

$$F \int_{r_1}^{r_2} dr \frac{1}{2k(r)} = 1, \quad (8)$$

with the wave number $k(r) = \sqrt{\frac{2\mu}{\hbar^2} (Q - V_{eff}(r))}$. After the turning points are calculated, F can be obtained as in Eq. (8). Transition factor S is given as follows [31,37,52]:

$$S = \exp \left[-2 \int_{r_2}^{r_3} dr \kappa(r) \right] \quad (9)$$

The wave number between r_2 and r_3 in the effective potential is $\kappa(r) = \sqrt{\frac{2\mu}{\hbar^2} |Q - V_{eff}(r)|}$. Then, by using the formulas above, the cluster decay width can be calculated. From the relation between the decay width and half-life, one can write the following equation [34,35,37,52]:

$$T_{1/2} = \hbar \frac{\ln 2}{\Gamma} \quad (10)$$

Even if the forms of the other potentials are known very well, the nuclear potential shape in Eq. (1) is the only unknown term. As one does not determine the nuclear potential between the cluster and the core nuclei clearly, one should use potential models such as phenomenological or microscopic models to describe nuclear interactions. Buck et al. used realistic nuclear potentials extensively, in particular the mWS potential [52,58,59], in order to study alpha and cluster radioactivity.

In this study, three different forms are used for the nuclear term in the effective potential: the WS, the WS-2, and the mWS forms, respectively:

$$\begin{aligned} V_{WS}(r) &= -\lambda \frac{V_0}{\left[1 + e^{\frac{r-R}{a}}\right]}, \\ V_{WS-2}(r) &= -\lambda \frac{V_0}{\left[1 + e^{\frac{r-R}{a}}\right]^2}, \\ V_{mWS}(r) &= -\lambda V_0 \left\{ \frac{\alpha}{1 + \exp\left[\frac{r-R}{a}\right]} + \frac{1-\alpha}{\left(1 + \exp\left[\frac{r-R}{3a}\right]\right)^3} \right\}, \end{aligned} \quad (11)$$

where λ , the renormalization factor, is obtained by the Bohr–Sommerfeld condition. λ is not an adjustable parameter but it is determined separately for each decay by applying the Bohr–Sommerfeld quantization condition. For each decay, one first obtains the turning points and uses them with Eq. (5) in order to get λ renormalization values for each nuclear interaction potential. $R = r_0 A_c^{1/3} + r_0 A_d^{1/3}$ is used in the calculations, where A_c and A_d show the mass numbers of cluster and daughter nuclei, respectively. In order to find out the parameters V_0 , a , r_0 , and α in the potentials, initially ^{22}Ne decay from ^{230}U is focused on and we adjust the potential parameters by reproducing the logarithmic experimental value, which is 19.57 for each potential. The fixed potential parameters are then used for all calculations. In the present calculations, $V_0 = 200$ MeV is used for both the WS and WS-2 potentials. For the mWS potential, $V_0 = 56.6 A_c$ has been used [59]. The parameters in nuclear potential are investigated phenomenologically, and $r_0 = 0.90$ and $a = 0.40$ fm for WS, $r_0 = 0.95$ and $a = 0.52$ fm for WS-2, and $r_0 = 0.80$, $a = 0.72$ fm, and $\alpha = 0.33$ for mWS are used in all calculations.

3. Results

The $\log_{10} T_{1/2}$ values of the cluster decays from various nuclei are computed by using different types of phenomenological nuclear interaction potentials in the WKB method in order to see how the shapes of nuclear potentials affect the cluster half-lives of isotopes.

In Table 1, while the first column presents the parent nucleus, the second one gives the cluster nuclei and the third one shows the Q-values taken from Belgya et al. [60]. The Exp. column presents the experimental $\log_{10} T_{1/2}$ for the related decays taken from Bonetti and Guglielmetti [61] and l_m shows the maximum angular

momentum of the cluster. In Table 1, the $\log_{10} T_{1/2}$ values of the present work are compared with experimental values, the results of the GLDM [47], the results of Zheng et al. [49] considering nuclei as spherical, and the results of Nazarzadeh and Mohebbali [38] using CW76. The WS column shows the results of the calculations with WS potential, the WS-2 column shows the results of calculations with WS-2 potential, and the mWS column shows the results of calculations with mWS potential.

In order to see which potentials produce more reasonable $\log_{10} T_{1/2}$ values for the cluster emission half-lives, the rms errors of calculated half-lives are calculated for 3 potentials values by:

$$\delta = \sqrt{\frac{1}{N-1} \sum_{k=1}^N [\log_{10} (T_{k,\frac{1}{2}}^{theo}) - \log_{10} (T_{k,\frac{1}{2}}^{exp})]^2}, \quad (12)$$

where N denotes the number of the nuclei investigated. $\delta = 2.34, 2.33,$ and 2.40 have been obtained for the WS, WS-2, and mWS potentials, respectively.

The WS, WS-2, and mWS potentials have also been applied to obtain the $\log_{10} T_{1/2}$ of the cluster decays from various parent nuclei that do not have known experimental values. All calculations are listed with results with 3 phenomenological potentials in Table 2. As they do not have measured experimental half-lives yet, the results have been compared with the results obtained in other studies. In Table 2, $\log_{10} T_{1/2}$ values with the CPPM for barium and cerium isotopes are taken from Santhosh and Joseph [62,63], while all others are from Santhosh et al. [50].

As seen from these tables, the results in this study are in very good agreement with the results from previous calculations.

4. Discussion

In order to find out the influence of the nuclear phenomenological potential types on cluster emission, many calculations have been done for cluster decays of different nuclei with three different types of potentials. The calculations were performed with the WKB method and the Bohr-Sommerfeld condition. The WS, WS-2, and mWS potentials have been applied to find out the half-lives of cluster decays. It should be noted that changing types of phenomenological potentials does not change the results in a distinguishable manner. Calculations with 3 different potentials have also been performed for obtaining ^{14}C , ^{15}N , $^{16,17,18,20,22}\text{O}$, ^{23}F , $^{22,24,25,26}\text{Ne}$, $^{28,30}\text{Mg}$, and ^{34}Si decay half-lives of different isotopes for comparison of the results with other models, which are the GLDM, CPPM, and CW76 form.

For ^{16}O decays at small neutron numbers, while the GLDM and CW76 models agree with each other, our results are smaller than the results obtained by other models. However, they have similar behaviors according to the neutron numbers of parent nuclei. As for larger neutron numbers, our results are close to the results obtained with CW76 potential. Moreover, our results are very close to the results of CW76 for $^{17,18}\text{O}$ decays. Even if all results obtained by the 3 models have similar behaviors, they are different in magnitude in terms of logarithmic values. As seen in the tables, our results are much closer to the results of the CW76 model, but they are a bit different from the results of the GLDM and CPPM model, as seen in Table 2.

In conclusion, although the deformation effects have not been considered in the calculations, good agreement has been obtained with the results of CW76 potential given by Nazarzadeh and Mohebbali [38], who took into account deformation effects as well. The calculations presented in this paper could be important for providing extra theoretical data for the unknown cluster decay half-lives. In addition, the results obtained

Table 1. The obtained $\log_{10} T_{1/2}$ values with 3 type potentials for various the cluster decays from various nuclei together with the experimental values and the results in different theoretical studies.

Parent	Cluster	Q (MeV)	Exp. [61]	l_m	GLDM [47]	Ref. [49]	CW76 [38]	CPPM [36]	WS	WS2	mWS
^{221}Fr	^{14}C	31.28	14.52	3	13.68	14.40	-	13.88	14.58	14.71	14.43
^{221}Ra	^{14}C	32.39	13.39	3	12.17	13.36	-	12.56	13.45	13.57	13.29
^{222}Ra	^{14}C	33.05	11.01	0	10.59	11.81	-	11.07	12.06	12.19	11.87
^{223}Ra	^{14}C	31.85	15.20	4	13.44	14.37	-	13.74	14.14	14.32	13.68
^{224}Ra	^{14}C	30.53	15.68	0	16.59	16.55	-	16.72	16.49	16.66	16.07
^{225}Ac	^{14}C	30.48	17.16	4	17.81	17.89	-	18.03	17.82	17.98	17.46
^{226}Ra	^{14}C	28.21	21.19	0	22.50	21.54	-	22.58	21.52	21.67	21.20
^{226}Th	^{14}C	30.67	> 15.30	0	18.78	18.21	-	19.03	18.17	18.33	17.83
^{223}Ac	^{15}N	39.47	> 14.76	0	14.46	-	-	15.06	14.28	14.38	14.26
^{226}Th	^{18}O	45.73	> 16.8	0	18.95	18.21	17.55	19.29	18.13	18.22	18.10
^{228}Th	^{20}O	44.72	20.72	0	21.61	21.90	21.14	21.66	21.09	21.18	20.93
^{230}U	^{22}Ne	61.39	19.57	0	21.40	20.26	21.05	22.60	19.54	19.58	19.56
^{231}Pa	^{23}F	51.86	26.02	1	24.26	24.52	24.59	24.25	23.05	23.12	22.85
^{230}Th	^{24}Ne	57.76	24.61	0	25.45	24.99	25.21	26.00	23.19	23.26	23.03
^{230}U	^{24}Ne	61.35	> 18.2	0	21.97	22.32	22.57	22.37	20.56	20.63	20.38
^{231}Pa	^{24}Ne	60.41	22.88	1	21.93	22.29	22.59	22.56	20.52	20.58	20.32
^{232}Th	^{24}Ne	54.66	> 29.20	0	-	28.09	30.07	30.36	27.71	27.81	28.23
^{232}U	^{24}Ne	62.31	21.08	0	19.99	20.75	20.88	20.72	19.08	19.13	18.87
^{233}U	^{24}Ne	60.49	24.83	2	23.36	23.32	23.68	24.15	21.21	21.34	21.62
^{234}U	^{24}Ne	58.83	25.92	0	26.54	25.76	25.80	27.39	23.67	23.78	24.17
^{235}U	^{24}Ne	57.36	27.42	1	29.40	28.02	27.91	30.37	26.00	26.10	26.59
^{236}U	^{24}Ne	55.95	> 25.90	0	32.18	30.33	30.27	33.30	28.31	28.39	28.98
^{233}U	^{25}Ne	60.73	24.83	2	23.15	24.03	22.95	23.44	21.83	21.88	21.61
^{235}U	^{25}Ne	57.71	27.42	3	29.08	28.69	27.39	29.50	26.02	26.14	26.51
^{232}Th	^{26}Ne	55.91	> 29.20	0	29.72	29.91	28.77	29.54	30.27	30.19	30.15
^{234}U	^{26}Ne	59.41	25.92	0	25.91	26.83	25.62	25.88	24.26	24.30	24.01
^{236}U	^{26}Ne	56.69	> 25.90	0	31.48	31.20	29.79	31.57	28.14	28.26	28.60
^{232}U	^{28}Mg	74.32	> 22.26	0	25.74	25.43	26.57	27.41	22.68	22.70	22.47
^{233}U	^{28}Mg	74.23	> 27.59	3	25.78	25.58	25.65	27.45	22.81	22.82	22.63
^{234}U	^{28}Mg	74.11	25.14	0	25.90	25.48	25.35	27.55	22.85	22.86	22.68
^{235}U	^{28}Mg	72.43	> 28.09	1	29.26	28.17	27.73	31.13	24.55	24.66	25.07
^{236}U	^{28}Mg	70.73	27.58	0	-	28.51	29.58	32.01	26.95	27.03	27.57
^{236}Pu	^{28}Mg	79.67	21.67	0	20.00	21.05	21.08	21.73	18.58	18.59	18.34
^{238}Pu	^{28}Mg	75.91	25.7	0	26.34	25.69	25.28	28.31	22.73	22.81	23.30
^{236}U	^{30}Mg	72.27	27.58	0	29.28	29.43	30.47	30.03	26.09	26.09	25.83
^{237}Np	^{30}Mg	74.79	> 27.57	2	26.56	27.41	28.50	27.34	24.09	24.09	23.82
^{238}Pu	^{30}Mg	76.80	25.7	0	24.83	25.99	26.93	25.70	22.82	22.85	22.55
^{238}Pu	^{32}Si	91.19	25.27	0	25.73	24.91	24.84	28.68	22.13	22.10	21.93
^{240}Pu	^{34}Si	91.03	> 25.52	0	26.08	27.25	26.33	28.11	22.72	22.70	22.35
^{241}Am	^{34}Si	93.92	> 24.41	3	23.32	25.37	24.55	25.40	20.91	20.83	20.51
^{242}Cm	^{34}Si	96.51	23.15	0	21.11	23.70	22.82	23.20	19.48	19.40	19.01

Table 2. The obtained $\log_{10} T_{1/2}$ values with WS-2 type potential for various cluster decays from various nuclei and the results in different theoretical studies. In the present calculations, WS-2 potential with $V_0 = 200$ MeV, $r_0 = 0.95$, and $a = 0.52$ fm was used.

Parent	Cluster	Q (MeV)	WS	WS2	mWS	GLDM	CPPM [50]	CW76 [38]	Parent	Cluster	Q(MeV)	WS	WS2	mWS	GLDM	CPPM [50]	CW76 [38]
¹¹⁴ Ba	¹⁶ O	26.42	11.90	12.48	10.22	14.51	12.47	14.97	²²³ Th	¹⁸ O	43.93	21.30	21.38	21.31		23.56	21.09
¹¹⁵ Ba	¹⁶ O	25.56	13.67	14.24	12.04	15.86	16.54	16.24	²²⁴ Th	¹⁸ O	44.56	20.17	20.26	20.17	21.44	22.16	19.72
¹¹⁶ Ba	¹⁶ O	24.38	16.31	16.86	14.72	18.72	19.13	19.45	²²⁵ Th	¹⁸ O	45.54	18.48	18.57	18.44	19.36	20.05	18.13
¹¹⁷ Ba	¹⁶ O	23.19	19.17	19.72	17.63	21.83		22.34	²²⁷ Th	¹⁸ O	44.20	20.24	20.38	19.82	22.00	22.71	20.19
¹¹⁸ Ba	¹⁶ O	22.04	22.20	22.75	20.71	25.87	25.45	24.59	²²⁸ Th	¹⁸ O	42.28	23.65	23.78	23.33	26.04	26.84	23.25
¹¹⁹ Ce	¹⁶ O	27.70	12.10	12.65	10.54	14.88		15.13	²²⁹ Th	¹⁸ O	40.86	26.35	26.47	26.09	29.15	30.05	25.75
¹²⁰ Ce	¹⁶ O	26.62	14.30	14.83	12.78	17.43	16.99	17.20	²²⁶ Pa	¹⁸ O	45.68	19.39	19.47	19.41	20.45		19.20
¹²¹ Ce	¹⁶ O	25.29	17.21	17.74	15.75	20.08	17.88	19.84	²²⁷ Pa	¹⁸ O	45.87	19.04	19.11	19.06	19.99		18.55
¹²² Ce	¹⁶ O	24.32	19.50	20.02	18.08	22.85	22.39	22.27	²²⁷ U	¹⁸ O	46.17	19.73	19.8	19.79	20.79		19.18
¹²³ Ce	¹⁶ O	23.13	22.53	23.04	21.16	26.18	25.84	25.22	²²⁸ U	¹⁸ O	45.96	20.04	20.10	20.12	22.15		19.24
¹²⁴ Ce	¹⁶ O	21.95	25.81	26.31	24.49	29.82	29.92	27.55	²²⁷ Np	¹⁸ O	46.22	20.83	20.89	20.94	22.04		20.69
¹²⁴ Pr	¹⁶ O	24.12	21.60	22.11	20.26	25.53		23.25	²²³ Ra	²⁰ O	38.70	30.56	30.64	30.48		33.07	32.3
¹²⁵ Pr	¹⁶ O	23.08	24.33	24.83	23.03	28.48		25.66	²²⁴ Ra	²⁰ O	39.72	28.33	28.41	28.21		30.40	29.64
²²² Ac	¹⁶ O	43.61	19.13	19.21	19.30	21.04		18.41	²²⁵ Ra	²⁰ O	40.48	26.69	26.78	26.56	28.53	28.44	27.17
²²³ Ac	¹⁶ O	43.59	19.12	19.19	19.29	21.00		18.04	²²⁶ Ra	²⁰ O	40.81	25.97	26.06	25.84	27.67	27.55	26.17
²²⁴ Ac	¹⁶ O	41.72	21.90	22.03	21.73	24.68		21.45	²²⁷ Ra	²⁰ O	39.60	27.99	28.14	27.44		30.51	28.27
²²² Th	¹⁶ O	45.72	16.88	16.96	17.01	18.36		15.81	²²⁸ Ra	²⁰ O	38.24	30.92	31.06	30.43		33.94	31.01
²²³ Th	¹⁶ O	46.57	15.53	15.62	15.64	16.74		14.56	²²⁶ Ac	²⁰ O	42.76	23.49	23.58	23.32	24.66		24.06
²²⁴ Th	¹⁶ O	46.48	15.64	15.71	15.75	16.83		14.43	²²⁷ Ac	²⁰ O	43.08	22.84	22.93	22.67	23.87		23.11
²²⁵ Th	¹⁶ O	44.66	18.04	18.17	17.81	20.15		17.33	²²⁸ Ac	²⁰ O	41.84	24.72	24.87	24.14	26.67		25.65
²²⁶ Th	¹⁶ O	42.66	21.36	21.48	21.22	23.97		20.31	²²⁴ Th	²⁰ O	41.30	27.77	27.85	27.69		29.87	27.86
²²⁷ Th	¹⁶ O	41.03	24.27	24.37	24.20	27.26		23.02	²²⁵ Th	²⁰ O	42.28	25.78	25.86	25.67		27.46	26.18
²²⁸ Th	¹⁶ O	39.05	28.08	28.16	28.09	31.50		26.8	²²⁶ Th	²⁰ O	43.18	24.00	24.08	23.87	25.23	25.27	24.15
²²⁴ Pa	¹⁶ O	47.47	15.24	15.33	15.37	16.34		14.48	²²⁷ Th	²⁰ O	44.46	21.60	21.69	21.43	22.26	22.31	21.88
²²⁵ Pa	¹⁶ O	47.33	15.41	15.48	15.56	16.50		14.33	²²⁹ Th	²⁰ O	43.40	23.00	23.15	22.42	24.54	24.56	23.62
²²⁶ Pa	¹⁶ O	45.56	17.69	17.81	17.49	19.71		17.19	²³⁰ Th	²⁰ O	41.79	26.07	26.21	25.56	28.20	28.28	26.34
²²⁷ Pa	¹⁶ O	43.42	21.49	21.29	21.07	23.73		20.45	²³¹ Th	²⁰ O	40.51	28.66	28.79	28.21		31.37	28.72
²²⁵ U	¹⁶ O	48.48	14.82	14.89	14.97	15.73		13.74	²²⁸ Pa	²⁰ O	43.99	23.68	23.76	23.59	24.78		24.10
²²⁶ U	¹⁶ O	48.01	15.45	15.51	15.63	16.49		14.11	²²⁹ Pa	²⁰ O	44.35	22.97	23.04	22.81	23.89		23.08
²²⁷ U	¹⁶ O	46.19	17.78	17.9	17.61	19.76		16.95	²²⁶ U	²⁰ O	41.71	29.56	29.62	29.60		32.13	29.64
²²⁵ Np	¹⁶ O	49.20	14.84	14.91	15.01	15.73		14.19	²²⁷ U	²⁰ O	42.37	28.18	28.24	28.20		30.45	28.52
²²⁷ Np	¹⁶ O	48.94	15.13	15.19	15.33	16.04		13.64	²²⁸ U	²⁰ O	42.89	27.09	27.15	27.10		29.11	27.21
²²⁴ Ac	¹⁷ O	42.07	22.55	22.62	22.69	24.66		22.15	²²⁹ U	²⁰ O	43.78	25.34	25.4	25.32	26.73	26.97	25.33
²²³ Th	¹⁷ O	43.98	20.46	20.54	20.57	22.18		20.03	²³⁰ U	²⁰ O	43.77	25.31	25.37	25.31	26.69	26.92	25.05
²²⁴ Th	¹⁷ O	43.25	21.67	21.74	21.82	23.58		21.11	²³¹ U	²⁰ O	42.44	27.41	27.52	26.97		29.98	27.73
²²⁵ Th	¹⁷ O	44.86	18.89	18.96	18.98	20.30		18.33	²³² U	²⁰ O	41.18	29.94	30.05	29.58		33.01	29.94
²²⁶ Th	¹⁷ O	41.62	24.13	24.25	23.52	26.89		23.84	²³¹ Np	²⁰ O	43.47	27.14	27.19	27.22	28.83	28.83	26.61
²²⁷ Th	¹⁷ O	41.34	24.61	24.73	24.43	27.41		24.2	²²⁸ Ra	²² O	40.60	27.77	27.87	27.47	29.11		28.74
²²⁵ Pa	¹⁷ O	44.02	21.51	21.57	21.69	23.34		21.11	²²⁹ Ac	²² O	42.50	25.21	25.3	24.89	26.08		26.33
²²⁶ Pa	¹⁷ O	45.10	19.78	19.70	19.78	21.15		19.19	²²⁷ Th	²² O	40.29	31.34	31.42	31.15		32.99	32.4
²²⁷ Pa	¹⁷ O	42.43	23.83	23.94	23.64	26.48		23.72	²²⁸ Th	²² O	41.27	29.17	29.25	28.95	30.71	30.03	29.98
²²⁸ Pa	¹⁷ O	41.59	25.36	25.47	25.22	28.18		25.28	²²⁹ Th	²² O	42.75	26.07	26.16	25.79	26.88	26.25	27.02
²²⁷ U	¹⁷ O	45.78	19.63	19.69	19.81	21.08		18.89	²³⁰ Th	²² O	43.32	24.89	24.98	24.60	25.38	24.79	25.62
²²⁷ Np	¹⁷ O	45.34	21.54	21.59	21.79	23.26		20.98	²³¹ Th	²² O	42.14	26.73	26.89	26.01		27.62	28.08
²²³ Ra	¹⁸ O	40.30	25.52	25.60	25.58	27.80		25.28	²³² Th	²² O	40.89	29.33	29.48	28.67		30.75	30.32
²²⁴ Ac	¹⁸ O	43.27	21.21	21.29	21.20	22.75		21.10	²³³ Th	²² O	39.94	31.38	31.52	30.77		33.19	32.18
²²⁵ Ac	¹⁸ O	43.45	20.84	20.92	20.84	22.26		20.43	²³¹ Pa	²² O	42.40	28.07	28.14	27.89	29.30		28.83
²²⁶ Ac	¹⁸ O	41.84	23.29	23.43	22.91	25.65		23.50	²³¹ U	²² O	40.88	32.72	32.77	32.65	28.34		33.29

Table 2. Continued.

Parent	Cluster	Q (MeV)	WS	WS2	mWS	GLDM	CPPM [50]	CW76 [38]	Parent	Cluster	Q(MeV)	WS	WS2	mWS	GLDM	CPPM [50]	CW76 [38]
²¹⁹ Th	¹⁸ O	40.51	27.91	27.99	28.01		31.6	27.23	²²⁷ Th	²² Ne	56.11	24.84	24.88	24.91	28.00		26.64
²²⁰ Th	¹⁸ O	41.38	26.14	26.22	26.22		29.47	25.26	²²⁸ Th	²² Ne	55.74	25.39	25.43	25.49	28.68		26.85
²²¹ Th	¹⁸ O	42.50	23.97	24.05	24.01		26.84	23.21	²²⁷ Pa	²² Ne	58.67	22.28	22.33	22.31	24.9		24.16
²²² Th	¹⁸ O	43.09	22.85	22.93	22.88		25.47	22.35	²²⁸ Pa	²² Ne	59.20	21.45	21.49	21.47	23.88		23.25
²²⁹ Pa	²² Ne	58.95	21.76	21.80	21.80	24.23		23.63	²²³ U	²⁸ Mg	71.85	26.62	26.67	26.39		33.19	32.48
²³⁰ Pa	²² Ne	56.94	24.29	24.41	23.89	27.92		26.54	²²⁴ U	²⁸ Mg	72.56	25.56	25.61	25.32		31.71	31.16
²²⁰ U	²² Ne	57.10	26.48	26.54	26.56		32.26	27.90	²²⁵ U	²⁸ Mg	72.93	24.97	25.02	24.73		30.86	30.87
²²¹ U	²² Ne	57.84	25.26	25.32	25.31		30.67	26.81	²²⁶ U	²⁸ Mg	73.30	24.41	24.45	24.16		30.05	30.01
²²² U	²² Ne	58.57	24.09	24.15	24.13		29.14	25.57	²²⁷ U	²⁸ Mg	73.59	23.96	24.00	23.71		29.38	29.63
²²³ U	²² Ne	59.12	23.20	23.26	23.22		27.96	24.66	²²⁸ U	²⁸ Mg	73.75	23.68	23.72	23.44		28.95	29.02
²²⁴ U	²² Ne	59.67	22.32	22.38	22.34		26.80	23.6	²²⁹ U	²⁸ Mg	73.89	23.42	23.46	23.19		28.57	28.60
²²⁵ U	²² Ne	60.19	21.51	21.57	21.52		25.71	22.9	²³⁰ U	²⁸ Mg	73.98	23.25	23.28	23.03		28.28	27.85
²²⁶ U	²² Ne	60.46	21.06	21.12	21.07		25.09	22.74	²³¹ U	²⁸ Mg	74.10	23.04	23.06	22.83		27.96	27.44
²²⁷ U	²² Ne	60.82	20.51	20.56	20.51		24.33	22.46	²³⁴ Np	²⁸ Mg	77.23	20.27	20.29	20.04	22.41		23.07
²²⁸ U	²² Ne	61.04	20.15	20.19	20.15	22.18	23.83	21.83	²³⁵ Np	²⁸ Mg	77.10	20.38	20.39	20.17	22.57		23.28
²²⁹ U	²² Ne	61.69	19.17	19.22	19.16	20.94	22.52	20.97	²³⁶ Np	²⁸ Mg	75.15	22.31	22.42	21.57	25.91		25.64
²³¹ U	²² Ne	59.45	21.81	21.92	21.38	24.83	26.56	23.82	²²⁸ Pu	²⁸ Mg	77.35	21.97	22.01	21.73		26.90	24.37
²³² U	²² Ne	57.36	24.96	25.06	24.63		30.56	26.5	²²⁹ Pu	²⁸ Mg	77.68	21.49	21.52	21.25		26.18	23.98
²³¹ Np	²² Ne	61.91	20.10	20.13	20.17	22.04		21.67	²³⁰ Pu	²⁸ Mg	77.89	21.16	21.19	20.93		25.67	23.39
²³³ Np	²² Ne	57.83	25.59	25.68	25.32	29.36		27.43	²³¹ Pu	²⁸ Mg	78.09	20.84	20.87	20.61		25.17	23.14
²³² Pu	²² Ne	62.34	20.77	20.78	20.90	22.82		21.98	²³² Pu	²⁸ Mg	78.50	20.27	20.3	20.04		24.32	22.91
²³³ Pu	²² Ne	60.51	22.87	22.96	22.54	26.04		24.61	²³³ Pu	²⁸ Mg	78.84	19.78	19.8	19.54		23.57	22.73
²²⁸ Ac	²³ F	47.85	27.50	27.57	27.31	29.88		29.59	²³⁴ Pu	²⁸ Mg	79.15	19.33	19.35	19.09	21.08	22.88	22.00
²²⁹ Ac	²³ F	48.37	26.49	26.56	26.29	28.82		28.34	²³⁵ Pu	²⁸ Mg	79.66	18.65	18.67	18.40	20.11	21.24	21.43
²³⁰ Th	²³ F	48.57	27.53	27.58	27.39	29.86		29.37	²³⁷ Pu	²⁸ Mg	77.73	20.39	20.50	19.63		25.11	23.46
²²⁷ Pa	²³ F	48.61	29.04	29.10	28.93		31.82	30.71	²³⁹ Pu	²⁸ Mg	74.10	25.16	25.23	24.59		31.65	27.34
²²⁸ Pa	²³ F	49.36	27.59	27.65	27.45		29.99	29.59	²³⁷ Am	²⁸ Mg	79.85	19.78	19.78	19.64	21.63		22.27
²²⁹ Pa	²³ F	50.35	25.74	25.81	25.58	27.64	27.68	27.43	²³⁸ Am	²⁸ Mg	78.23	21.19	21.28	20.50	24.3		24.40
²³⁰ Pa	²³ F	51.30	24.05	24.11	23.85	25.52	25.52	25.84	²³⁸ Cm	²⁸ Mg	80.37	20.56	20.54	20.49	22.62		22.57
²³² Pa	²³ F	50.23	25.32	25.46	24.66	27.68	27.71	27.52	²³² U	³⁰ Mg	70.87	28.40	28.42	28.15		33.77	35.21
²³³ Pa	²³ F	48.89	27.73	27.86	27.14		30.7	29.56	²³³ U	³⁰ Mg	71.10	28.00	28.01	27.75		33.19	34.56
²³⁴ Pa	²³ F	47.50	30.34	30.45	29.83		33.9	32.01	²³⁴ U	³⁰ Mg	71.75	26.97	26.98	26.71		31.77	32.94
²²³ Th	²⁴ Ne	54.89	28.28	28.35	28.15		32.74	33.46	²³⁵ U	³⁰ Mg	72.12	26.37	26.38	26.11		30.92	31.16
²²⁴ Th	²⁴ Ne	55.45	27.26	27.33	27.12		31.42	32.1	²³⁷ U	³⁰ Mg	70.52	28.04	28.14	27.22		33.96	32.92
²²⁵ Th	²⁴ Ne	59.93	20.13	20.22	19.80		30.32	31.07	²³² Np	³⁰ Mg	72.26	28.00	28.01	27.77		33.38	33.63
²²⁶ Th	²⁴ Ne	56.50	25.42	25.49	25.26		29.02	29.5	²³³ Np	³⁰ Mg	72.62	27.40	27.41	27.17		32.52	32.65
²²⁷ Th	²⁴ Ne	57.03	24.51	24.57	24.34		27.81	28.4	²³⁴ Np	³⁰ Mg	73.21	26.48	26.48	26.23		31.23	32.07
²²⁸ Th	²⁴ Ne	57.41	23.84	23.9	23.66	26.26	26.93	27.2	²³⁵ Np	³⁰ Mg	73.78	25.61	25.61	25.35		29.99	30.82
²²⁹ Th	²⁴ Ne	57.83	23.13	23.19	22.95	25.38	26.00	25.50	²³⁶ Np	³⁰ Mg	74.54	24.46	23.08	24.18		27.53	29.3
²³¹ Th	²⁴ Ne	55.99	25.49	25.62	24.85	28.93	29.65	27.82	²³⁸ Np	³⁰ Mg	73.12	25.78	25.88	24.94		30.97	31.11
²²⁹ Pa	²⁴ Ne	59.67	21.72	21.78	21.52	23.54	24.22	24.05	²³⁹ Np	³⁰ Mg	71.86	27.58	27.67	26.81		33.4	32.45
²³⁰ Pa	²⁴ Ne	60.38	20.60	20.67	20.39	22.08	22.73	22.80	²³² Pu	³⁰ Mg	73.21	28.25	28.25	28.06		33.86	32.23
²²³ U	²⁴ Ne	57.02	27.68	27.74	27.59		32.23	29.61	²³³ Pu	³⁰ Mg	73.75	27.40	27.4	27.20		32.66	31.43
²²⁴ U	²⁴ Ne	57.91	26.17	26.23	26.05		30.26	28.03	²³⁴ Pu	³⁰ Mg	74.37	26.44	26.44	26.23		31.31	30.83
²²⁵ U	²⁴ Ne	58.59	25.03	25.09	24.90		28.76	26.96	²³⁵ Pu	³⁰ Mg	74.87	25.68	25.68	25.45		30.22	30.40
²²⁶ U	²⁴ Ne	59.22	23.99	24.05	23.84		27.38	25.66	²³⁶ Pu	³⁰ Mg	75.60	24.59	24.59	24.34		27.32	28.96
²²⁷ U	²⁴ Ne	59.76	23.10	23.16	22.94		26.19	24.83	²³⁷ Pu	³⁰ Mg	76.46	23.35	23.35	23.08		25.63	27.79
²²⁸ U	²⁴ Ne	60.29	22.24	22.3	22.08		25.04	24.28	²³⁹ Pu	³⁰ Mg	75.12	24.49	24.59	23.65		27.92	29.41
²²⁹ U	²⁴ Ne	60.93	21.23	21.29	21.04		23.67	23.55	²⁴⁰ Pu	³⁰ Mg	73.77	26.37	26.45	25.60		31.83	30.91
²³¹ U	²⁴ Ne	62.21	19.27	19.33	19.05		21.01	21.35	²³⁸ Am	³⁰ Mg	76.20	25.23	25.21	25.08		28.45	29.89
²³³ Np	²⁴ Ne	62.16	20.61	20.65	20.49	21.95		22.53	²³⁹ Am	³⁰ Mg	76.56	24.68	24.66	24.52		27.67	28.93
²³³ Pu	²⁴ Ne	62.37	21.68	21.71	21.62	23.3		23.54	²⁴⁰ Cm	³⁰ Mg	76.59	26.18	26.14	26.12		29.61	30.04
²³⁴ Pu	²⁴ Ne	62.26	21.80	21.82	21.75	23.45		23.38									

here suggest that these types of calculations would be helpful for the predictions of unknown experimental half-lives, as of yet unobserved, for the cluster emissions of isotopes.

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