Turkish Journal of Physics

http://journals.tubitak.gov.tr/physics/

Research Article

Turk J Phys
(2015) 39: 54 – 59

doi:10.3906/fiz-1404-1

Cosmological models filled with a perfect fluid source in the f(R,T) theory of gravity

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Received: 04.04.2014 • **Accepted:** 05.09.2014 • **Published Online:** 23.02.2015 • **Printed:** 20.03.2015

Abstract: We investigated the physical behavior of an LRS Bianchi type I cosmological model in the framework of the $f(R,T)$ theory of gravity in the presence of a perfect fluid, where R is the Ricci scalar and T is the trace of the stress energy tensor of the matter. In order to obtain a deterministic solution of the field equations we assumed the special law of variation of the Hubble parameter proposed by Berman that yields the constant deceleration parameter. Some physical properties of the models are discussed.

Key words: $f(R,T)$ Theory of gravity, perfect fluid, cosmological parameters

1. Introduction

The recent developments in terms of astrophysical observations in modern cosmology have indicated that the current universe is not only expanding but also accelerating. The mission of cosmologists is to determine the large-scale structure of the universe. Thus, there has been a lot of interest among researchers in modified theories of gravity in view of the direct evidence of late time accelerated expansion of the universe from high red shift supernovae experiments [1–4]. In general, it is predicted that the cosmic accelerated expansion of the universe is due to some kind of energy-matter with negative pressure called dark energy. The experimental observations such as cosmic microwave background radiation and large-scale structure provide an indirect proof for the late time accelerated expansion of the universe [5,6]. The unintelligible component of energy called dark energy is a prime candidate often introduced to explain the recent cosmic observations [7]. Two approaches have been used to investigate the issue of the current cosmic accelerating universe. In order to deal with such an issue in the framework of Einstein's theory of gravity, the first way is to use the concept of "exotic cosmic fluid", but this approach could not explain the empirical data completely [8,9]. Another way is to discuss the modified theories of gravity such as $f(R)$ and $f(R,T)$, where $R$and $T$are the Ricci scalar and the trace of the stress energy tensors, respectively [10–12]. The $f(R)$ and $f(R,T)$ theories of gravity are extensions of the general theory of relativity. There were no alternatives to the mysterious component called dark energy for the cosmic accelerated expansion of the universe up to a certain period but these theories are supposed to provide natural gravitational alternatives to dark energy. These theories provided the theoretical models in which Einstein–Hilbert action of general relativity with a general function $f(R)$ is considered, where $R$is the Ricci scalar replaced by an arbitrary function of the Ricci scalar. Eardley et al., Multamaki et al., Chiba et

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al., and Nojiri et al. [13–17] are some of the authors who have investigated different aspects of $f(R)$ gravity models showing the consistency of early time inflation and late time acceleration. Very recently, Harko et al. [12] proposed another extension of Einstein's theory of general relativity named the $f(R,T)$ theory of gravity in which the gravitational Lagrangian is given by an arbitrary function of Ricci scalar R and the trace of the stress energy tensor T. The $f(R,T)$ gravity model depends on a source term, which represents the variation in the matter energy tensors with respect to the metric. In order to describe the early universe, the $f(R,T)$ theory of gravity is considered a fundamental theory of gravitation. However, it is to be noted that the $f(R,T)$ theory of gravity is the generalization of $f(R)$ and $f(T)$ theories of gravity. Reddy et al. [18,19] investigated the higher dimensional Kaluza–Klein cosmological models as well as Bianchi type III cosmological models in the $f(R,T)$ theory of gravity. A new class of cosmological models using the special form of the average scale factor is derived by Abdussattar and Prajapati, and Pawar et al. [20,21]. Dark energy and dark energy models in the $f(R,T)$ theory of gravity have recently become an interesting subject of investigation for several authors (Samanta, Shriram et al., Chaubey and Shukla, Reddy et al., Shamir et al., Samanta et al., Sharif et al., Pradhan et al., Yadav and Yadav [22–31]). The present work is studied in the framework the $f(R,T)$ theory of gravity proposed by Hrko et al. (2011) in which $R$and T are the Ricci scalar and trace of the stress energy tensor, respectively, which is different from the $f(R,T)$ theory used by Myrzakulov [32] in which R is curvature scalar and T is torsion scalar. In the present paper we investigated the LRS Bianchi type I cosmological model used by Abdussattar et al. [20] by assuming the spatial law of variation of Hubble's parameter proposed by Berman [33] and Pawar et al. [34,35]. We obtained some physical parameters and discussed their physical behaviors.

2. Metric and field equations

We consider the LRS Bianchi Type I metric [20,21] given by

$$ds^2 = -dt^2 + a^2(t)\left\{dx^2 + dy^2 + \left(1 + \beta\int\frac{dt}{a^3}\right)^2 dz^2\right\}, \tag{2.1}$$

where $a(t)$ is a function of cosmic time t and β is $\oplus ve$ constant.

We have assumed the stress energy tensor of the matter as a perfect fluid, given by

$$T_{ij} = (\rho + p)\,u_i u_j - p g_{ij}. \tag{2.2}$$

Now assuming the arbitrary function $f(R,T)$ given by Harko et al. [12] as

$$f(R,T) = R + 2f(T), \tag{2.3}$$

where R is the Ricci scalar and T is the trace of stress energy tensor.

Thus the field equations in the framework of the $f(R,T)$ theory of gravity are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f' T_{ij} + [2pf'(T) + f(T)]\,g_{ij}, \tag{2.4}$$

where overhead prime denotes the differentiation with respect to the argument.

We choose the function $f(T)$ of the trace of the energy tensor of the matter source so that

$$f(T) = \lambda T, \tag{2.5}$$

where λ is constant.

By assuming the commoving coordinate system, the field equations (2.4) for the metric (2.1) with the equations (2.2), (2.3), and (2.5) can be written as

$$\frac{a_4^2}{a^2} + \frac{2a_{44}}{a} = (8\pi + 3\lambda)\,p - \lambda\rho,$$

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$$\frac{3a_4^2}{a^2} + \frac{2\beta a_4}{a^4\left(1 + \beta\int\frac{dt}{a^3}\right)} = \lambda p - (8\pi + 3\lambda)\,\rho, \tag{2.6}$$

where suffix 4 indicates the differentiation with respect to time.

The above system of the equations (2.6) reduces and takes the form

$$\frac{a_4^2}{a^2} + \frac{2a_{44}}{a} = (8\pi + 3\lambda)\,p - \lambda\rho, \tag{2.7}$$

$$\frac{3a_4^2}{a^2} + \frac{2\beta a_4}{a^4\left(1 + \beta\int\frac{dt}{a^3}\right)} = \lambda p - (8\pi + 3\lambda)\,\rho. \tag{2.8}$$

3. Solution of the field equations

Eqs. (2.7) and (2.8) contain 3 unknowns: a, p, ρ. Therefore, in order to obtain a deterministic solution we have to use one more additional condition. Here we assume the special law of variation of Hubble's parameter proposed by Berman [33] that yields the constant deceleration parameter given by the relation

$$q = -\frac{RR_{44}}{R_4^2}. \tag{3.1}$$

Here the sign of the constant deceleration parameter is supposed to be negative because for a negative constant deceleration parameter the universe undergoes acceleration. In Eq. (3.1) R is the overall average scale factor. From Eq. (2.1) of given metric R is given by

$$R = \left[a^3\left(1 + \beta\int\frac{dt}{a^3}\right)\right]^{\frac{1}{3}}. \tag{3.2}$$

Solving Eq. (3.1) for R we get

$$R = (ct + d)^{\frac{1}{(q+1)}}, q \neq -1, \tag{3.3}$$

where $c \neq 0, d$ are the constants of integrations. This equation indicates that the condition of expansion is $(1 + q) > 0$

Comparing Eqs. (3.2) and (3.3) we get

$$a = T^{\left(\frac{1}{q+1}\right)} e^{mT^{\left(\frac{q-2}{q+1}\right)}} \tag{3.4}$$

and

$$\left(1+\beta\int\frac{dt}{a^3}\right)=e^{-3mT^{\left(\frac{q-2}{q+1}\right)}}, \tag{3.5}$$

where

$$T=(ct+d)\Rightarrow R=T^{\frac{1}{(q+1)}}. \tag{3.6}$$

$$m=\frac{\beta(q+1)}{3c(2-q)}, q\neq 2 \tag{3.7}$$

Using Eqs. (3.4) and (3.5) the metric given by Eq. (2.1) takes the form

$$ds^2=-dT^2+T^{\left(\frac{2}{q+1}\right)}e^{2mT^{\left(\frac{q-2}{q+1}\right)}}\left(dx^2+dy^2\right)+T^{\left(\frac{2}{q+1}\right)}e^{(-4m)T^{\left(\frac{q-2}{q+1}\right)}}dz^2. \tag{3.8}$$

Eqs. (2.7) and (2.8) with the equations (3.4) and (3.5) become

$$\frac{\beta^2}{3c^2T^{\frac{6}{(q+1)}}}+\frac{(1-2q)}{(q+1)^2T^2}=(8\pi+3\lambda)p-\lambda\rho, \tag{3.9}$$

$$\frac{(1-2c)\beta^2}{3c^2}\frac{1}{T^{\frac{6}{(q+1)}}}+\frac{2(c-1)\beta}{c(q+1)}\frac{1}{T^{\left(\frac{q+4}{q+1}\right)}}+\frac{3}{(q+1)^2}\frac{1}{T^2}=\lambda p-(8\pi+3\lambda)\rho. \tag{3.10}$$

Solving Eqs. (3.9) and (3.10) simultaneously for p and ρ we get

$$\rho=\frac{\beta^2}{T^{\frac{6}{(q+1)}}}\left\{\frac{\lambda^2-(8\pi+3\lambda)(1-2c)}{3c^2\left[(8\pi+3\lambda)^2-\lambda^2\right]}\right\}-\frac{2\beta}{T^{\left(\frac{q+4}{q+1}\right)}}\left\{\frac{(8\pi+3\lambda)(c-1)}{c(q+1)\left[(8\pi+3\lambda)^2-\lambda^2\right]}\right\}$$

$$+\frac{1}{T^2}\left\{\frac{\lambda(1-2q)-3(8\pi+3\lambda)}{(q+1)^2\left[(8\pi+3\lambda)^2-\lambda^2\right]}\right\} \tag{3.11}$$

$$p=\frac{\beta^2}{T^{\frac{6}{(q+1)}}}\left\{\frac{(8\pi+3\lambda)-\lambda(1-2c)}{3c^2\left[(8\pi+3\lambda)^2-\lambda^2\right]}\right\}-\frac{\beta}{T^{\left(\frac{q+4}{q+1}\right)}}\left\{\frac{2\lambda(c-1)}{c(q+1)\left[(8\pi+3\lambda)^2-\lambda^2\right]}\right\}$$

$$+\frac{1}{T^2}\left\{\frac{(8\pi+3\lambda)(1-2q)-3\lambda}{(q+1)^2\left[(8\pi+3\lambda)^2-\lambda^2\right]}\right\} \tag{3.12}$$

3.1. Some physical parameters

Eq. (3.8) represents the LRS Bianchi Type I cosmological universe with perfect fluid source in the framework of the f(R, T) theory of gravity. The following physical parameters helped us to discuss the physical properties of the cosmological model (3.8).

The scalar expansion θ of the model is

$$\theta=\frac{3}{(q+1)T}. \tag{3.13}$$

The directional Hubble's parameters $H_x = H_y$, H_Z of the model are respectively given by

$$H_x = H_y = \frac{1}{(q+1)}\left[\frac{m(q-2)}{T^{\frac{3}{(q+1)}}} + \frac{1}{T}\right], \tag{3.14}$$

$$H_z = \frac{1}{(q+1)}\left[\frac{2m(2-q)}{T^{\frac{3}{(q+1)}}} + \frac{1}{T}\right]. \tag{3.15}$$

Eqs. (3.13) and (3.14) gives the mean generalized Hubble's parameter H as

$$H = \frac{1}{3}(H_x + H_y + H_z) = \frac{1}{(q+1)T}. \tag{3.16}$$

Thus from Eq. (3.13) and (3.14) we have

$$\theta = 3H. \tag{3.17}$$

The spatial volume of the model is

$$V = T^{\frac{1}{(q+1)}}. \tag{3.18}$$

The mean anisotropy parameter of the model is

$$A_m = \frac{2m^2(2-q)^2}{T^{\frac{2(2-q)}{(q+1)}}}. \tag{3.19}$$

The shear scalar of the model is

$$\sigma^2 = \frac{6m^2(q-2)^2}{(q+1)^2 T^{\frac{6}{(q+1)}}} \tag{3.20}$$

4. Discussion and conclusion

It can be observed from Eqs. (3.14) to (3.16) that the Hubble's parameters are the function of cosmic time T. All these parameters vanish for large value of T and become infinite at $T = 0$. The scalar expansion as well as shear scalar are extremely large at initial moment and are decreasing with increase in time. In the present model the mean anisotropy parameter A_m is a function of cosmic time. Since the mean anisotropy parameter $A_m \neq 0$ the models are anisotropic for $q \neq 2$ and if $q = 2, A_m = 0$ then the models become isotropic. The spatial volume of the model vanishes when time is zero and increases with increase in time. Moreover, since $(1+q) > 0$, the model represents an accelerating universe. Thus the universe starts evolving with big bang singularity at $T = 0$; hence it confirms that the universe is not only expanding but also accelerating and represents early stages evolution of the universe, which is in good agreement with recent observations. Similarly energy density and pressure are functions of time that vanish for large value of T and diverge when time is zero. The exact solutions of the field equations are obtained with suitable physical assumptions. Finally we conclude that our cosmological model in the $f(R,T)$ theory is consistent with the recent observations of Type-I_a supernovae and the solutions presented in this work may be one of the best findings to describe the observed universe.

Acknowledgement

Our grateful thanks are due to the referee for invaluable suggestions.

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