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Electric and magnetic field induced geometric phases for the 2D harmonic oscillator in noncommutative phase space

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Abstract: In this work, we point out the phenomenon whereby the noncommutative corrections to the geometric phases induced by an electric field and a magnetic field have different orders in terms of the noncommutative parameters. The first order correction is zero for the electric field induced geometric phase and it is nonzero for the magnetic field induced geometric phase. In our calculation, the system is in coherent states when the electric field is applied, so the corresponding geometric phase calculated is that of the coherent states. Considering that the noncommutative parameters are very small, it is better to use the magnetic field rather than the electric field for detecting the noncommutativity of spaces.

Key words: Noncommutative phase space, geometric phase, magnetic field, electric field

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1. Introduction

Theories on noncommutative spaces have been extensively studied [1–21]. The original motivation for introducing space-time noncommutativity was to overcome the difficulties of infinite energies in quantum field theory [1]. Renewed interest in such an idea is mainly due to the fact that space-time noncommutativity is naturally found in string/M theory [2,3]. In a 2D noncommutative phase space, while the coordinate–momentum commutation relations remain the same as in ordinary quantum mechanics, the coordinate–coordinate and momentum–momentum commutators are supposed to be nonzero.

$$[\hat{x}_1,\ \hat{x}_2] = i\mu,\ [\hat{p}_1,\ \hat{p}_2] = i\nu,\ [\hat{x}_j,\ \hat{p}_k] = i\hbar\delta_{j\ k}, \tag{1}$$

where μ and ν are the noncommutative parameters and $\delta_{j\ k}$ is the Kronecker δ function. In [6], it was pointed out that the bounds for the noncommutative parameters are $\mu \leq 4\times10^{-40}$ m^2 and $\nu \leq 1.76\times10^{-61}$ kg^{2}m^{2}s^{-2}, which are very small, as expected. To date, detecting the noncommutative effects directly is still difficult. In [13], using the electric field induced geometric phase [22–26] to detect the noncommutative effects was suggested. However, in the next section we will show that the first order correction to such a geometric phase is zero in terms of the noncommutative parameters. Considering that the noncommutative parameters are very small, such a suggestion is difficult to be realized. The geometric phase that we derived is actually that of the coherent states in noncommutative spaces. There are many discussions on the related problem in commutative spaces [27–31]. In the third section, we study the geometric phase by applying a magnetic field to a 2D harmonic oscillator in noncommutative phase space. The first order correction to the magnetic field induced geometric

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phase is nonzero. Thus, for detecting noncommutativity, it is easier to use a magnetic field than an electric field. The fourth section is the final section, where a summary is given.

2. Geometric phase induced by electric field

For a particle moving in 2D noncommutative phase space, we write the annihilation operators as

$$\hat{A}_1 = [\rho(\hat{x}_1 - i\hat{x}_2) + i(\hat{p}_1 - i\hat{p}_2)]/N_1, \tag{2a}$$

$$\hat{A}_2 = [\sigma(\hat{x}_2 - i\hat{x}_1) + i(\hat{p}_2 - i\hat{p}_1)]/N_2, \tag{2b}$$

where $N_1,\ N_2,\ \sigma,\ \rho$ are all constants to be determined. Demanding that $\hat{A}_1$ commutates with $\hat{A}_2$, we get

$$\hbar(\rho - \sigma) + \mu\sigma\rho = \nu. \tag{3}$$

In deriving Eq. (3), the commutation relations have been used. The commutation relations $[\hat{A}_1,\ \hat{A}_1^\dagger] = [\hat{A}_2,\ \hat{A}_2^\dagger] = 1$ result in

$$N_1 = \sqrt{2(\sigma+\rho)(\hbar - \mu\rho)},\ N_2 = \sqrt{2(\sigma+\rho)(\hbar+\mu\sigma)}. \tag{4}$$

Using the annihilation operators Eqs. (2a) and (2b) and the corresponding creation operators, the coordinate and momentum operators in 2D noncommutative phase space are expressed as follows.

$$\begin{aligned}
\hat{x}_1 &= \tfrac{1}{\sqrt{2}}[a(\hat{A}_1 + \hat{A}_1^\dagger) + ib(\hat{A}_2 - \hat{A}_2^\dagger)] \\
\hat{x}_2 &= \tfrac{1}{\sqrt{2}}[b(\hat{A}_2 + \hat{A}_2^\dagger) + ia(\hat{A}_1 - \hat{A}_1^\dagger)] \\
\hat{p}_1 &= \tfrac{1}{\sqrt{2}}[\rho b(\hat{A}_2 + \hat{A}_2^\dagger) - i\sigma a(\hat{A}_1 - \hat{A}_1^\dagger)] \\
\hat{p}_2 &= \tfrac{1}{\sqrt{2}}[\sigma a(\hat{A}_1 + \hat{A}_1^\dagger) - i\rho b(\hat{A}_2 - \hat{A}_2^\dagger)]
\end{aligned} \tag{5}$$

Here, $a = \sqrt{(\hbar - \mu\rho)/(\sigma+\rho)}$ and $b = \sqrt{(\hbar+\mu\sigma)/(\sigma+\rho)}$, which are introduced to make the mathematical expressions simpler.

By choosing $\sigma\rho = m^2\omega^2$, the Hamiltonian of the isotropic harmonic oscillator is diagonalized by the annihilation operators of Eqs. (2a) and (2b) and the corresponding creation operators.

$$\hat{H}_0 = \frac{\hat{p}_1^2 + \hat{p}_2^2}{2m} + \frac{1}{2}m\omega^2(\hat{x}_1^2 + \hat{x}_2^2) = \hbar\omega_{10}(\hat{A}_1^\dagger\hat{A}_1 + 1/2) + \hbar\omega_{20}(\hat{A}_2^\dagger\hat{A}_2 + 1/2) \tag{6}$$

Here, $\hbar\omega_{10} = a^2(\sigma^2 + m^2\omega^2)/m,\ \hbar\omega_{20} = b^2(\rho^2 + m^2\omega^2)/m$. Using the relation from Eq. (3) and the condition $\sigma\rho = m^2\omega^2$, we obtain the following.

$$\begin{aligned}
\sigma &= \sqrt{\left(\tfrac{\nu - \mu m^2\omega^2}{2\hbar}\right)^2 + m^2\omega^2} - \tfrac{\nu - \mu m^2\omega^2}{2\hbar} > 0 \\
\rho &= \sqrt{\left(\tfrac{\nu - \mu m^2\omega^2}{2\hbar}\right)^2 + m^2\omega^2} + \tfrac{\nu - \mu m^2\omega^2}{2\hbar} > 0
\end{aligned} \tag{7}$$

In these expressions, the noncommutative parameters can take any values.

We suppose that initially there is no electric field and the initial state $|\psi(t)\rangle$ is one of the eigenstates of the Hamiltonian (6). At some time the electric field is suddenly switched on. The electric field can be written as $E(t) = E\theta(t)$, where $\theta(t)$ is the Heaviside step function. After the electric field is applied along the x_1-axis, the Hamiltonian becomes the following.

$$\begin{aligned}\hat{H}_1 \quad = \quad & \tfrac{\hat{p}_1^2+\hat{p}_2^2}{2m} + \tfrac{1}{2}m\omega^2(\hat{x}_1^2+\hat{x}_2^2) - qE\hat{x}_1 = \hbar\omega_{10}(\hat{A}_1^\dagger\hat{A}_1 + 1/2) - (aqE/\sqrt{2})(\hat{A}_1 + \hat{A}_1^\dagger) \\ & +\hbar\omega_{20}(\hat{A}_2^\dagger\hat{A}_2 + 1/2) - i(bqE/\sqrt{2})(\hat{A}_2 - \hat{A}_2^\dagger) = \hbar\omega_{10}D_1^\dagger(\alpha_1)(\hat{A}_1^\dagger\hat{A}_1 + 1/2 - \alpha_1^2)D_1(\alpha_1) \\ & +\hbar\omega_{20}D_2^\dagger(\alpha_2)(\hat{A}_2^\dagger\hat{A}_2 + 1/2 - \alpha_2^2)D_2(\alpha_2)\end{aligned} \tag{8}$$

Here, $\alpha_1 = -[aqE/(\sqrt{2}\hbar\omega_{10})]$ and $\alpha_2 = [bqE/(\sqrt{2}\hbar\omega_{20})]$ are constants. The displacement operators are

$$D_1(\alpha_1) = \exp[\alpha_1(\hat{A}_1^\dagger - \hat{A}_1)],\ D_2(\alpha_2) = \exp[i\alpha_2(\hat{A}_2^\dagger + \hat{A}_2)], \tag{9}$$

for which there exist the relations

$$D_1^\dagger(\alpha_1)\hat{A}_1D_1(\alpha_1) = \hat{A}_1 + \alpha_1,\ D_2^\dagger(\alpha_2)\hat{A}_2D_2(\alpha_2) = \hat{A}_2 + i\alpha_2. \tag{10}$$

In the presence of the electric field, time evolution of wave functions is governed by the Schrödinger equation.

$$H_1\,|\psi(t)\rangle = i\hbar\frac{\partial\,|\psi(t)\rangle}{\partial t} \tag{11}$$

As the Hamiltonian of Eq. (8) is time-independent, the wave function at any time is as follows.

$$\begin{aligned}|\psi(t)\rangle = \quad & \exp\left(-i\tfrac{H_1t}{\hbar}\right)|\psi(t)\rangle = D_1^\dagger(\alpha_1)\exp(-i\omega_{10}\,t\hat{A}_1^\dagger\hat{A}_1)D_1(\alpha_1)\exp[i\omega_{10}\,t(\alpha_1^2 - 1/2)] \\ & D_2^\dagger(\alpha_2)\exp(-i\omega_{20}\,t\hat{A}_2^\dagger\hat{A}_2)D_2(\alpha_2)\exp[i\omega_{20}\,t(\alpha_2^2 - 1/2)]\,|\psi(t)\rangle\end{aligned} \tag{12}$$

In the case of $|\psi(t)\rangle = |\psi(t)\rangle$, the wave function of Eq. (12) finally becomes the following.

$$\begin{aligned}|\psi(t)\rangle = \quad & \exp\left(-i\tfrac{H_1t}{\hbar}\right)|\psi(t)\rangle = \exp[\beta_1(t)\hat{A}_1^\dagger - \beta_1^*(t)\hat{A}_1]\exp[i\beta_2(t)\hat{A}_2^\dagger + i\beta_2^*(t)\hat{A}_2]\,|\psi(t)\rangle \\ & \exp[i\omega_{10}\,t(\alpha_1^2 - 1/2) - i\alpha_1^2\sin(\omega_{10}\,t)]\exp[i\omega_{20}\,t(\alpha_2^2 - 1/2) - i\alpha_2^2\sin(\omega_{20}\,t)]\end{aligned} \tag{13}$$

Here, $\beta_1(t) = \alpha_1[\exp(-i\omega_{10}\,t) - 1]$, $\beta_2(t) = \alpha_2[\exp(-i\omega_{20}\,t) - 1]$. Under the action of the electric field, the state of the system is transformed from the ground state into the wave function of Eq. (13). Acting as the annihilation operators on the wave function of Eq. (13), we have

$$\hat{A}_1\,|\psi(t)\rangle = \beta_1(t)\,|\psi(t)\rangle\,,\ \hat{A}_2\,|\psi(t)\rangle = i\beta_2(t)\,|\psi(t)\rangle\,, \tag{14}$$

which means the wave function of Eq. (13) is the standard coherent state [32]. Now we calculate the geometric phase related to this standard coherent state.

While studying the interference of light, Pancharatnam came up with a brilliant idea regarding the general evolution of polarized light, which was then generalized to an arbitrary evolution in quantum mechanics [24–26]. Generally, the total phase for the evolution from an initial state to a final state is:

$$\phi_P(t) = \arg\langle 00 \mid \psi(t)\rangle = \omega_{10}\,t(\alpha_1^2 - 1/2) - \alpha_1^2\sin(\omega_{10}\,t) + \omega_{20}\,t(\alpha_2^2 - 1/2) - \alpha_2^2\sin(\omega_{10}\,t). \tag{15}$$

The dynamical phase is:

$$\varphi_d(t) = -\frac{1}{\hbar}\int_0^t \langle\psi(\tau)|H_1\,|\psi(t)\rangle\,d\tau = -\frac{\omega_{10}\,t}{2} - \frac{\omega_{20}\,t}{2}. \tag{16}$$

The difference is named the geometric phase [25,26]:

$$\phi_g(t) = \phi_P(t) - \phi_d(t) = \alpha_1^2[\omega_{10}\,t - \sin(\omega_{10}\,t)] + \alpha_2^2[\omega_{20}\,t - \sin(\omega_{20}\,t)], \tag{17}$$

which is the result corresponding to the nonadiabatic noncyclical evolution. Recalling that $\alpha_1 = -[aqE/(\sqrt{2}\hbar\omega_{10})]$ and $\alpha_2 = [bqE/(\sqrt{2}\hbar\omega_{20})]$, one clearly sees that the geometric phase is induced by the external electric field. In the commutative limit, the geometric phase of Eq. (17) reduces to

$$\varphi_{0g}(t) = \frac{q^2E^2}{2\hbar m\omega^3}[\omega\,t - \sin(\omega t)]. \tag{18}$$

The phase difference $\Delta\varphi_g(t) = \varphi_g(t) - \varphi_{0g}(t)$ is a signature of noncommutativity. Based on this geometric phase, one may test the noncommutativity of spaces. However, in the first order approximation of the noncommutative parameters, it is found that $\Delta\varphi_g(t) = \varphi_g(t) - \varphi_{0g}(t) = 0$. That is to say, noncommutative correction to the geometric phase is at least second order in terms of the noncommutative parameters. Detecting the noncommutativity through such kinds of geometric phases is difficult.

3. Geometric phase induced by magnetic field

Similar to the electric field, the magnetic field can be written as $B(t) = B\theta(t)$. We suppose that the magnetic field is perpendicular to the plane of the 2D harmonic oscillator. In symmetry gauge, the vector potential of the magnetic field is $(-B\hat{x}_2/2,\ B\hat{x}_1/2)$. After the magnetic field is switched on, the Hamiltonian of Eq. (6) becomes the following.

$$\begin{aligned}\hat{H}_2 &= \tfrac{(\hat{p}_1+qB\hat{x}_2/2)^2+(\hat{p}_2-qB\hat{x}_1/2)^2}{2m} + \tfrac{1}{2}m\omega^2(\hat{x}_1^2+\hat{x}_2^2)\\ &= \hat{H}_0 + \tfrac{1}{2}m\omega_L^2(\hat{x}_1^2+\hat{x}_2^2) - \omega_L(\hat{x}_1\hat{p}_2 - \hat{x}_2\hat{p}_1)\\ &= \hbar\omega_1(\hat{A}_1^\dagger\hat{A}_1+\tfrac{1}{2}) + \hbar\omega_2(\hat{A}_2^\dagger\hat{A}_2+\tfrac{1}{2}) + ab[m\omega_L^2 - \omega_L(\sigma-\rho)]i(\hat{A}_1\hat{A}_2 - A_1^\dagger A_2^\dagger)\end{aligned} \tag{19}$$

Here, $\omega_L = qB/(2m)$ and

$$\begin{aligned}\omega_1 &= \omega_{10} + (m\omega_L^2a^2 - 2\omega_L\sigma a^2)/\hbar\\ \omega_2 &= \omega_{20} + (m\omega_L^2b^2 + 2\omega_L\rho b^2)/\hbar\end{aligned}\ . \tag{20}$$

As is well known, the operators $\hat{K}_- = \hat{A}_1\hat{A}_2$, $\hat{K}_+ = \hat{A}_1^\dagger\hat{A}_2^\dagger$, and $\hat{K}_0 = \frac{1}{2}(\hat{A}_1^\dagger\hat{A}_1 + \hat{A}_2^\dagger\hat{A}_2 + 1)$ form the Lie algebra of the SU(1,1) group, $[\hat{K}_-,\ \hat{K}_+] = 2\hat{K}_0$ and $[\hat{K}_0,\ \hat{K}_\pm] = \pm\hat{K}_\pm$, or the Hamiltonian of Eq. (19) is composed of SU(1,1) generators. Using the 2-mode squeeze operator [33,34]

$$S(r) = \exp[ir(\hat{A}_1\hat{A}_2 + \hat{A}_1^\dagger\hat{A}_2^\dagger)], \tag{21}$$

the Hamiltonian of Eq. (19) can be written in the following form:

$$H_2 = S^\dagger(r)[\hbar\Omega(\hat{A}_1^\dagger\hat{A}_1 + \hat{A}_2^\dagger\hat{A}_2) - \hbar\Delta_B(\hat{A}_1^\dagger\hat{A}_1 - \hat{A}_2^\dagger\hat{A}_2) + E_{0r}]S(r), \tag{22}$$

where

$$\begin{array}{l} \Omega = \frac{\omega_1+\omega_2}{2\cosh(2r)},\ \tanh(2r) = \frac{2ab\omega_L(\sigma-\rho-m\omega_L)}{\hbar(\omega_1+\omega_2)} \\ \Delta_B = \frac{\omega_2-\omega_1}{2},\ E_{0r} = \frac{1}{2}\hbar(\omega_1+\omega_2) - 2\hbar\Omega\sinh^2 r \end{array}. \tag{23}$$

After the magnetic field is turned on, the wave function at any time is determined by

$$\begin{aligned} |\psi(t)\rangle &= \exp\left(-i\tfrac{H_2 t}{\hbar}\right)|\psi(t)\rangle \\ &= S^\dagger(r)\exp[-i\Omega t(\hat{A}_1^\dagger\hat{A}_1+\hat{A}_2^\dagger\hat{A}_2)+i\Delta_B t(\hat{A}_1^\dagger\hat{A}_1-\hat{A}_2^\dagger\hat{A}_2)]S(r)\,|\psi(t)\rangle\exp(-iE_{0r}t/\hbar) \\ &= U_1(r,t)\exp[-i(\Omega-\Delta_B)t\hat{A}_1^\dagger\hat{A}_1-i(\Omega+\Delta_B)t\hat{A}_2^\dagger\hat{A}_2]\,|\psi(t)\rangle\exp(-iE_{0r}t/\hbar) \end{aligned} \tag{24}$$

where

$$\begin{array}{l} U_1(r,t) = S^\dagger(r)\exp[-i(\Omega-\Delta_B)t\hat{A}_1^\dagger\hat{A}_1-i(\Omega+\Delta_B)t\hat{A}_2^\dagger\hat{A}_2]S(r) \\ \exp[i(\Omega-\Delta_B)t\hat{A}_1^\dagger\hat{A}_1+i(\Omega+\Delta_B)t\hat{A}_2^\dagger\hat{A}_2] \end{array}. \tag{25}$$

Our next task is to find the wave function for a given initial state. However, to get a simple form for the wave function is much more complicated than that in the above section. Now we must simplify the operator of Eq. (25). After some calculations, we obtain

$$U_1^{-1}(r,t)\hat{A}_1U_1(r,t) = \mu\hat{A}_1 - \nu\hat{A}_2^\dagger, \tag{26}$$

$$U_1^{-1}(r,t)\hat{A}_2U_1(r,t) = \mu\hat{A}_2 - \nu\hat{A}_1^\dagger, \tag{27}$$

where

$$\begin{array}{l} \mu = \cosh^2 r - \exp(2i\Omega t)\sinh^2 r \\ \nu = i[1-\exp(-2i\Omega t)]\sinh r\cosh r \end{array}. \tag{28}$$

Calculations show that $|\mu|^2 - |\nu|^2 = 1$, which means Eqs. (26) and (27) are Bogoliubov transformations. The parameters of Eq. (28) can be rewritten as

$$\mu = \cosh s\exp(i\phi_\mu),\ \nu = \sinh s\exp(i\phi_\nu), \tag{29}$$

where

$$\begin{array}{l} \cosh s = |\mu| = \sqrt{1+\frac{1}{2}\sinh^2(2r)[1-\cos(2\Omega t)]} \\ \varphi_\mu = \arctan\frac{-\sin(2\Omega t)\sinh^2 r}{\cosh^2 r-\cos(2\Omega t)\sinh^2 r} \\ \sinh s = |\nu| = \sqrt{\frac{1}{2}\sinh^2(2r)[1-\cos(2\Omega t)]} \\ \varphi_\nu = \arctan\frac{1-\cos(2\Omega t)}{-\sin(2\Omega t)} \end{array}. \tag{30}$$

After some investigations, it is found that the Bogoliubov transformations of Eqs. (26) and (27) can be realized by the following operator.

$$U_2(r,t) = \exp\left(-\frac{v}{\mu^*}\hat{A}_1^\dagger\hat{A}_2^\dagger\right)\exp\left[-(\hat{A}_1^\dagger\hat{A}_1+\hat{A}_2^\dagger\hat{A}_2+1)\ln\mu^*\right]\exp\left(\frac{v^*}{\mu^*}\hat{A}_1\hat{A}_2\right) \tag{31}$$

That is to say, by replacing $U_1(r,t)$ in Eqs. (26) and (27) with $U_2(r,t)$, the transformations of Eqs. (26) and (27) remain valid. In other words, there exist the following relations.

$$U_1^{-1}(r,t)\hat{A}_1U_1(r,t) = U_2^{-1}(r,t)\hat{A}_1U_2(r,t) \tag{32a}$$

$$U_1^{-1}(r,t)\hat{A}_2U_1(r,t) = U_2^{-1}(r,t)\hat{A}_2U_2(r,t) \tag{32b}$$

From Eqs. (32a) and (32b), one can get $[U_2(r,t)U_1^{-1}(r,t), \hat{A}_1] = 0 = [U_2(r,t)U_1^{-1}(r,t), \hat{A}_2]$. That is to say, the operator $U_2(r,t)U_1^{-1}(r,t)$ commutes with the annihilation operators $\hat{A}_1$ and $\hat{A}_2$. By similar arguments, we can get that $U_2(r,t)U_1^{-1}(r,t)$ commutes with the creation operators $\hat{A}_1^\dagger$ and $\hat{A}_2^\dagger$ too. Thus, $U_2(r,t)U_1^{-1}(r,t)$ commutes with any combinations of the operators $\hat{A}_1$, $\hat{A}_2$, $\hat{A}_1^\dagger$, and $\hat{A}_2^\dagger$, which means $U_2(r,t)U_1^{-1}(r,t)$ must be a constant. Choosing a special case $r=0$, we see that this constant is unit,

$$U_2(r,t)U_1^{-1}(r,t) = 1, \text{ or } U_1(r,t) = U_2(r,t). \tag{33}$$

The operator of Eq. (25) has another form, as seen in Eq. (31). Using Eq. (31), the final state can be obtained more easily. For example, when $|\psi(t)\rangle = |\psi(t)\rangle$, the final state is

$$\begin{aligned} |\psi(t)\rangle &= \exp\left(-\tfrac{v}{\mu^*}\hat{A}_1^\dagger\hat{A}_2^\dagger\right)\exp(-\ln\mu^*)\,|\psi(t)\rangle\exp(-iE_{0r}t/\hbar) \\ &= \tfrac{\exp[i(\phi_\mu - E_{0r}t/\hbar)]}{\cosh s}\sum_{n=0}^{\infty}\tanh^2 s\,|\psi(t)\rangle\exp[ni(\pi+\phi_\mu+\phi_\nu)] \end{aligned}. \tag{34}$$

In the process of time evolution, the structure of the state in Eq. (34) is preserved. At the moment $\phi_\mu + \phi_\nu = \pi$, the wave function of Eq. (34) is the 2-mode squeezed vacuum state with s the squeezing coefficient [33,34], which belongs to the SU(1,1) coherent states. Through the expressions of Eq. (30), we see that the parameters s, ϕ_μ, and ϕ_ν are all functions of time with the period $T = \pi/\Omega$. The total and dynamical phases of the state in Eq. (34) are as follows.

$$\begin{aligned} \varphi_t(t) &= \arg\langle 00 \mid \psi(t)\rangle = \phi_\mu - E_{0r}t/\hbar \\ \varphi_d(t) &= -\tfrac{1}{\hbar}\int_0^t \langle\psi(\tau)|\,H_2\,|\psi(t)\rangle\,d\tau = -\tfrac{1}{2}(\omega_1+\omega_2)t \end{aligned} \tag{35}$$

Thus, the geometric phase is $\varphi_g(t) = \varphi_t(t) - \varphi_d(t) = \phi_\mu + 2\Omega t\sinh^2 r$, which is the result of nonadiabatic evolution. At the time $T = \pi/\Omega$, $\phi_\mu = 0$, so the geometric phase for one period becomes

$$\varphi_g(T) = 2\pi\sinh^2 r, \tag{36}$$

which is the geometric phase that corresponds to the nonadiabatic cyclic evolution [24]. From Eq. (28), we have $\mu = 1$ and $\nu = 0$ at $T = \pi/\Omega$. Now the wave function of Eq. (34) returns to the initial one:

$$|\psi(t)\rangle = |\psi(t)\rangle\exp(-iE_{0r}T/\hbar). \tag{37}$$

The phase gained is the total phase in Eq. (35). In the first order approximation, Eq. (36) becomes the following.

$$\begin{aligned} \varphi_g(T) &= 2\pi\sinh^2 r \to 2\pi\left(\tfrac{2\omega^2+\omega_L^2}{2\omega\sqrt{\omega^2+\omega_L^2}} - 1\right) \\ &\quad +2\pi\tfrac{2\omega^2+\omega_L^2}{2\omega\sqrt{\omega^2+\omega_L^2}}\left[\tfrac{\omega_L(\nu+\mu m^2\omega^2)}{m\hbar(2\omega^2+\omega_L^2)} - \tfrac{\mu m\omega_L}{\hbar} - \tfrac{\nu\omega_L}{2m\hbar(\omega^2+\omega_L^2)}\right] \end{aligned} \tag{38}$$

The first order correction is nonzero. Hence, as far as the geometric phase is concerned, the noncommutative effect is more easily detected with the magnetic field than the electric field.

4. Conclusions

The geometric phases by abrupt turn-on of an electric or magnetic field for a 2D harmonic oscillator in noncommutative phase space are calculated. For the geometric phase induced by electric field, the noncommutative correction is at least second order in terms of the noncommutative parameters. If the electric field is replaced by a magnetic field, the SU(1,1) coherent states are generated, which are seldom addressed in noncommutative phase space. In this situation, the first order correction to the geometric phase is nonzero. Therefore, to detect noncommutativity through the geometric phase, it is better to use the magnetic field rather than the electric field.

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