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Delta excitation calculation studies in the ground state of the compressed finite heavy doubly-magic nucleus ^{100}Sn

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Abstract

The energies and matter densities of finite nuclei under radial compression are investigated by using a constrained Hartree-Fock method with the Δ degree of freedom included. The results are presented for the doubly-magic nucleus ^{100}Sn in an effective baryon-baryon interaction. It is found that as the nucleus is compressed to about three time of the ordinary nuclear density, the Δ component is sharply increased to about 17% of all baryons in the system. This result is consistent with the values extracted from relativistic heavy-ion collisions. The single particle energy levels calculated and their behaviors under compression examined too. A good agreement between results with effective Hamiltonian and the phenomenological shell model for the low lying single-particle spectra obtained. A considerable reduction in compressibility for the nucleus, and softening of the equation of state with the inclusion of the Δ's in the nuclear dynamics are suggested by the results.

Key Words: Nuclear structure, compressed finite nuclei, Δ-nesonance; Hartree-Fock method, single particle energy

1. Introduction

Nuclei having neutron and proton numbers both equal to one of the magic numbers are called "double-magic." Doubly-magic nuclei are the cornerstones of the nuclear structure. Properties of these nuclei are essential for understanding the evolution of the nuclear structure far from the line of stability. The ^{100}Sn region, where the $N = Z$ line crosses the proton drip line and where the astrophysical rp-process is proposed to terminate, has been an aim of numerous experimental studies [1].

A nuclei model is extended to include $\Delta(3,3)$ isobars in addition to nucleons. The $\Delta(3,3)$ isobar has spin $s = 3/2$ and isospin $\tau = 3/2$. Therefore, it has a group of four charge states: Δ^{++}, Δ^{+}, Δ^{0} and Δ^{-}. Each has a mass of 1236 MeV (neglecting effects due to its finite width). The nuclear structure calculations for

^{100}Sn nucleus are examined by this model. A nonrelativistic microscopic mean-field approach is presented for a finite ^{100}Sn nucleus. It includes nucleon and Δ degrees of freedom. The ground state properties are calculated for ^{100}Sn nucleus at zero temperature within a constrained spherical Hartree-Fock (CSHF) approximation. A central goal of the present effort is to explore the role of Δ resonances on the properties of finite ^{100}Sn nucleus under compression.

The results of the role of Δ's in finite nuclei have been investigated [2–15]. The nucleus has been considered as a collection of nucleons and Δ-resonances. The effects of including the Δ-degrees of freedom on the Hartree-Fock energy, density distribution, and Δ-orbital occupations in the ground state and under large amplitude static compression at temperature $T = 0$, and model space consisting of seven major oscillator shells have also been examined. The selected nuclei were: ^{16}O, ^{40}Ca, ^{56}Ni, ^{90}Zr, ^{100}Sn, and ^{132}Sn.

In this study of the heaviest doubly-magic nucleus ^{100}Sn, the previous effects with different adjusting parameters are considered. The emphasis is on single particle energy levels for nucleons and deltas and the large amplitude static compression with a model space consisting of six major oscillator shells with CSHF approximation. The calculation was done with the use of a realistic effective Hamiltonian with different potentials. The Bruekner G matrices used are generated from coupled channels NN, NΔ, and πNN [16–20]. This is done to give a good description of NN data up to 1 GeV. The method for calculating the effective interactions of the nuclear shell model [21] is being used in this work. It is a good tool to study the highly compressed nuclei at densities accessible to relativistic heavy ion collisions.

The detailed calculations demonstrated the effective Hamiltonian, H_{eff}, model space, the calculation procedures and strategy in references [12–15]. Based on these studies, the two-body matrix elements are scaled in the N-N sector to an optimal value of ω, the oscillator energy for ^{100}Sn nucleus in the six major oscillator shells with the six delta orbitals. The adjusting parameters and ω' for ^{100}Sn nucleus in a given model space at equilibrium with the Δ channel turned off, are obtained in table 1. This table appears the difference between our adjusting parameters and other studies [4, 9].

Table 1. Adjusting parameters λ_1, λ_2, and $\hbar\omega'$ of the effective Hamiltonian for ^{100}Sn for the model space of 6 oscillator shells for which the calculations were performed. The binding energy (point mass r_{rms}) that was fitted was -826 MeV (5.11 fm) for ^{100}Sn.

Nucleus ^{100}Sn	λ_1	λ_2	$\hbar\omega'$(MeV)
Our model	0.996	1.395	5.542
Reference [4]	0.998	1.141	5.30
Reference [9]	0.998	1.423	5.15

The remainder of this paper is organized as follows. Section 2 specifies the results and presents a discussion. Conclusions are presented in Section 3.

2. Results and discussion

In references [4, 9], some selected results for ^{100}Sn demonstrating the behavior of self-consistent single-particle spectra as a function of compression were presented. Here, more detailed results for ^{100}Sn are presented in order to examine its properties under static compression. The N-Δ and $\Delta - \Delta$ interaction were employed as they were activated in a model space consisting of six major oscillator shells for nucleons and six orbitals for Δ's making a total of 27 baryons orbitals.

The performed calculations were done for ^{100}Sn. The Hartree-Fock energies, E_{HF}, versus r_{rms} using RSC potential are displayed in Figure 1. Figure 1 clearly shows that there is virtually no difference in the results with and without Δ's at equilibrium. It is seen that, without the Δ-degree of freedom in the system, E_{HF} increases steeply towards zero binding energy under compression. As the volume (based on the root mean square radius) of nucleus decreased by about 12%, the binding energy will be about 105.15 MeV, when Δ-excitations are included at the results obtained when nucleons are considered only. That means, it shows about 131.19 MeV, and 26.04 MeV of excitation energy to achieve a 12% volume reduction in the nucleon-only results, and nucleons and Δ^+'s results, respectively.

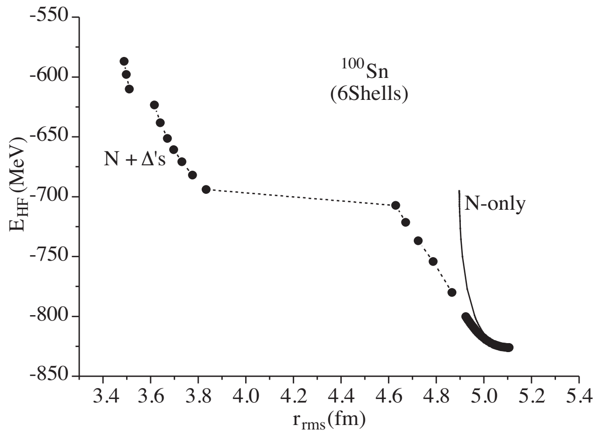

Figure 1. (CSHF) energy as a function of the point mass r_{rms} for ^{100}Sn evaluated in 6 major oscillator shells with 6 Δ-orbitals. The dashed curve corresponds to CSHF the full calculations including the Δ's while the solid curve corresponding to CSHF with nucleons only.

That it costs 26.04 MeV of excitation energy to reduce the volume by 12%, and the same amount of additional energy to reduce it by a further 3% suggests that the less dense outer part of the nucleus initially responds to the external load more readily than the inner part.

The difference between the results of the Hartree-Fock binding energy obtained here and those in references [4, 9] is the size of the nucleon model space, the number of the Δ orbitals included, different potentials, and more compression, and also it is worth mentioning that at equilibrium (no constraint) in ^{100}Sn, it was not found any mixing between nucleon states and the Δ states. As same as in references [4, 9], all curves of E_{HF} agree near equilibrium, (r_{rms} = 5.11 fm). This is implied that results for the system at equilibrium do not depend on model space. In comparison with results in previous studies [4, 9], current results are consistent with results obtained for ^{100}Sn for E_{HF}, but the curve of N-only is very steep. This is due to consider smaller model space than reference [9], so the static compression modulus is increased significantly by reduced the nucleon model space. The current results show more compression than previous. The results show that there is a significant reduction in the static compression modulus for RSC static compressions is reduced by including the Δ excitations. The consequence of this reduction is a softening of the nuclear equation of state at larger compression.

It is shown from Figure 1 that, as the static load force increases, the compression of nuclei with nucleons only is less than the compression of nuclei with both nucleons and Δ's.

To get an impression of the role of the Δ's as a function of compression, the number of Δ's against r_{rms} radius is plotted in Figure 2. The total number of Δ's, the number of Δ^+'s and Δ^0's are separately shown.

In Figure 2, the number of deltas increased rapidly as volume decreased. When the nucleus volume reaches about 68% of its volume at equilibrium, the number of deltas is increased to about 17% of all constituents of ^{100}Sn. It is interesting to note in Figure 2 that the number of Δ^0's and Δ^+'s are the same at all r.m.s. radii obtained. The creation of Δ^0's becomes more favorable as the compression continues. The last result is consistent with reference [4, 9] while differs with model size.

Although there is a rapid rise of the Δ-population as compression increases, the change in the total number of Δ's is 0.17. Note that there is a consistency of the amount of $N-\Delta$ maxing with the amount of excitation energy exhibited with compression. That is, when 0.17Δ's are presented, the excitation energy is the order of $0.17(M-m) \quad \approx 50.49$ MeV. Thus, on the scale of the unperturbed single-particle energies, a substantial fraction of the compressive energy is delivered, through the N-Δ and $\Delta-\Delta$ interactions, to create more massive baryons in the lowest energy configuration of the nucleus. By another way, the number of Δ's can be increased to about 17 at $r_{rms} = 3.49$ fm, which corresponds to about 3 times normal density.

In other words, Figure 2 shows that the number of created Δ's increase sharply, when ^{100}Sn nucleus compressed to a volume of about 0.68 of its equilibrium size. However, at this nuclear density, which is three times the normal density, the percentage of nucleons converted to Δ is only about 17% in ^{100}Sn.

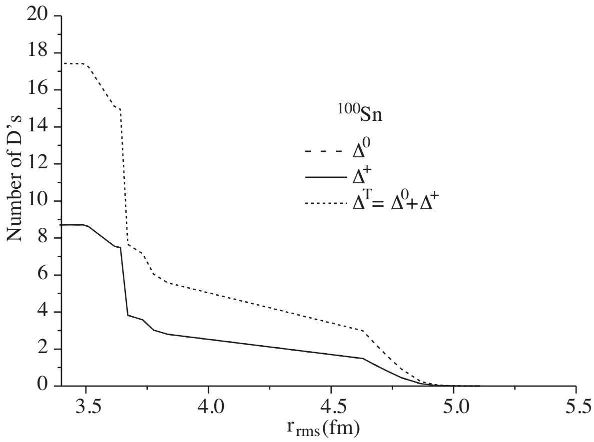

Figure 2. Number of Δ's as a function of r_{rms} for ^{100}Sn in six major shells model space. The upper curve is for the total number of Δ's. The solid curve is for the number of Δ^+, and the dashed curve is for Δ^0.

To compare results in Figure 5 of reference [4] and Figure 3 of reference [9] with the present results in Figure 2, the number of Δ's increases as the model space decreases. From Figure 6 in reference [4], the creation of Δ's becomes more favorable as the compression continues as model space decreases. The current results in this work show a major difference than in other findings [4, 9]; the number of Δ's at $r_{rms} = 3.70$ fm increase very sharply at this radius. This behavior may be artifacts of the small number of Δ-orbitals employed, and may be due to the small gap between the $n = 0$ and $n = 1$ single particle energies. As moving to larger compression, including the Δ states reduces the static compression modulus, but their role in reducing the static modulus is less dramatic than enlarging the size of the nucleon model space. The role of Δ states in reducing the static compression modulus is the largest in the smallest space.

In terms of relativistic heavy-ion collisions, the nucleus that can more easily penetrate when the Δ degree of freedom becomes explicit is implied by Figure 1. Because of the limitations of the model space, the calculations for higher densities are more speculative. Nevertheless it can give us some idea about how the Δ population can be increased as the nucleus is compressed to higher densities accessible to relativistic heavy-ion collisions. The results shown in Figures 1 and 2 are consistent with the results extracted from relativistic heavy-ions collisions [22–24].

Figure 3 displays the radial density distribution for ^{100}Sn at large compression and point mass radius r_{rms} = 4.92 fm in a model space of six major oscillator shells with Δ excitation restricted to the six orbitals: $0s_{3/2}$, $0p_{1/2}$, $0p_{3/2}$, $1s_{3/2}$, $1p_{1/2}$, $1p_{3/2}$.

Figure 3 shows the radial density distribution for neutrons ρ_n ,protons ρ_p, deltas ρ_Δ, and their sum ρ_T as a function of the radial distance from the center of the nucleus at large compression in five-oscillator model space. From this figure, the neutron radial density is higher than the proton density at all values of r. This is due to Coulomb repulsion between the protons. Even though the Δ-density appears to be zero at equilibrium, the Δ-radial density distribution, under high compression (point mass r_{rms} = 4.92 fm), reaches a peak value of about 0.10 of the proton (or neutron) radial density at r = 2.0 fm. Δ-mixing with the nucleons in the $0p_{1/2}$,$0p_{3/2}$,$1p_{1/2}$, and $1p_{3/2}$ orbitals occurs, which explains the shape of the Δ-radial distribution presented in Figure 3.

Figure 4 displays the radial density distributions of ^{100}Sn evaluated at an about 0.39 reduced volume (r_{rms} = 3.74 fm). In this case, the Δ-radial density distribution reaches a peak value of about 0.94 of the proton radial density.

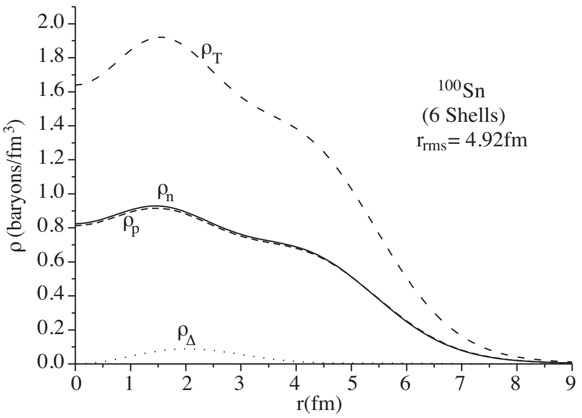

Figure 3. Total ρ_T, proton ρ_p (dashed line), neutron ρ_n (solid line), and deltaρ_Δ (dotted line) radial density distribution for ^{100}Sn at point mass r_{rms}=4.92fm in a model space of six major oscillator shells.

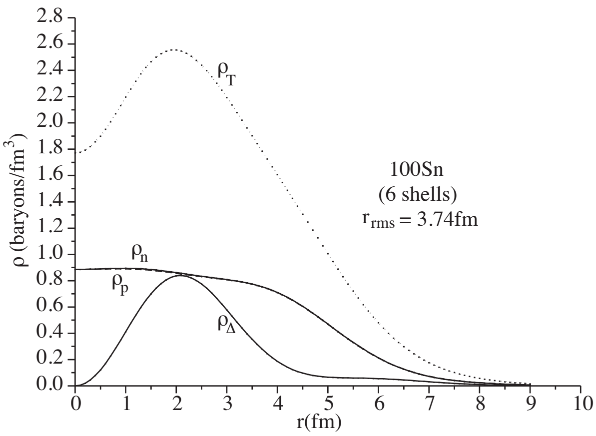

Figure 4. Total ρ_T, proton ρ_p (dashed line), neutron ρ_n (solid line), and deltaρ_Δ (solid line) radial density distribution for ^{100}Sn at point mass r_{rms}=3.74fm in a model space of six major oscillator shells.

Figure 5 shows that the Δ-radial density distribution reaches a peak value of about 2.27 of the proton radial density of ^{100}Sn evaluated at higher compression an about 0.14 reduced volume (r_{rms} = 2.68 fm).

It can be seen from Figures 3, 4 and 5, as the total radial density increases, the radial density distribution of Δ's increases sharply; yet, radial density of nucleons decreases sharply.

It can be seen from Figure 6 the total radial density for ^{100}Sn in six oscillator shells at large compression

(r_{rms} = 3.74 fm) and at equilibrium (point mass radius r_{rms} = 5.11 fm). This figure shows when the volume of the nucleus is decreased by 0.39 of the equilibrium volume; the radial density is increased by 1.53 of its value at equilibrium case.

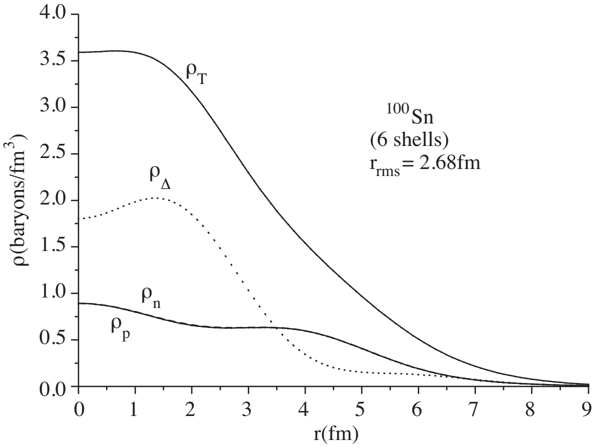

Figure 5. Total ρ_T, proton ρ_p (dashed line), neutron ρ_n (solid line), and delta ρ_Δ (dotted line) radial density distribution for ^{100}Sn at point mass r_{rms}=2.68fm in a model space of six major oscillator shells.

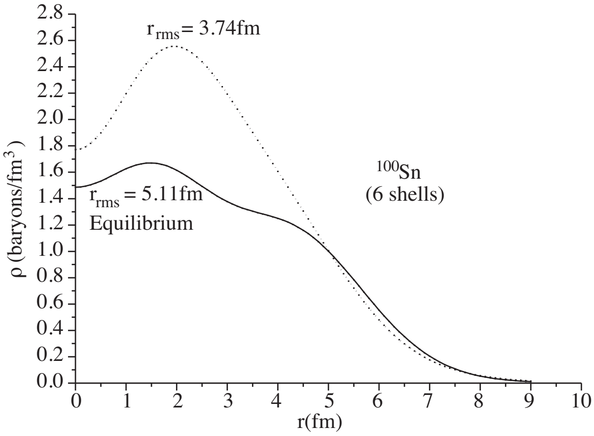

Figure 6. Total radial density distribution for ^{100}Sn at equilibrium (r_{rms}=5.11fm) (solid line) and at r_{rms} = 3.74fm (dashed line).

The results for the matter distribution of ^{100}Sn ground state are shown in Figure 6. For the nucleon distribution, they are very close to that from the calculation of figure7 in reference [4] with a different model space.

Clearly, the density in the interior rises relative to the interior density at equilibrium as one compresses the nucleus. This is in contrast to the behavior of the radial density on the outer-surface, where the radial density distribution is higher at equilibrium than the radial density when the static load is applied.

In Figure 7, the lowest self-consistent zero-change single particle energy levels as a function of I_{rms} were displayed. The orbits curved up as the load on the nucleus increased. This is because the kinetic energy of the baryons which is positive quantity becomes more influential than the attractive mean field of the baryons. The single particle energy levels of ^{100}Sn have not been studied before [4, 9].

The single particle energy spectrum also exhibits the gaps between the shells. As the nucleus is compressed, the single particle level ordering and the gaps are preserved. The general trend exhibited the single particle energies (except the deepest bound orbital, which actually drops with compression) shift to higher energies as the nucleus is compressed. The curvature increases further as the orbital becomes closer to the surface. This implies that the surface is more responsive to compression than the interior of nucleus.

Figure 8 shows the last three unoccupied zero-charge orbitals (low curves in the figure) and the six orbitals, which are dominantly Δ^0 (higher curves in the figure). Note the gap of about 271.05 MeV between the last dominant neutron orbital and the first dominant Δ^0 orbitals, due to the difference in rest mass of baryons (neutrons and Δ^0). Contrary to previous works [4, 9], the present results show the gap between nucleons and delta levels.

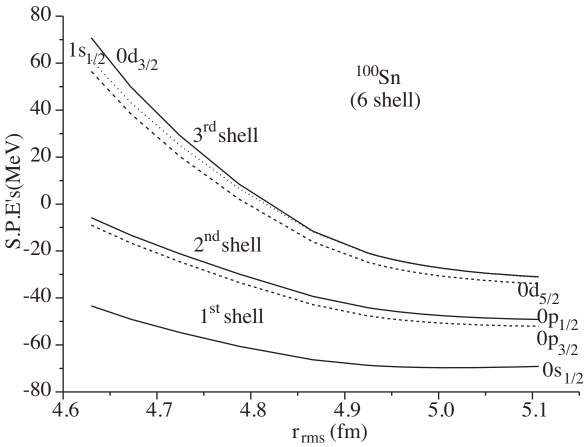

Figure 7. Single Particle Energy of Lowest Six Neutron states for ^{100}Sn in Six- Oscillator shells as a function of r_{rms}.

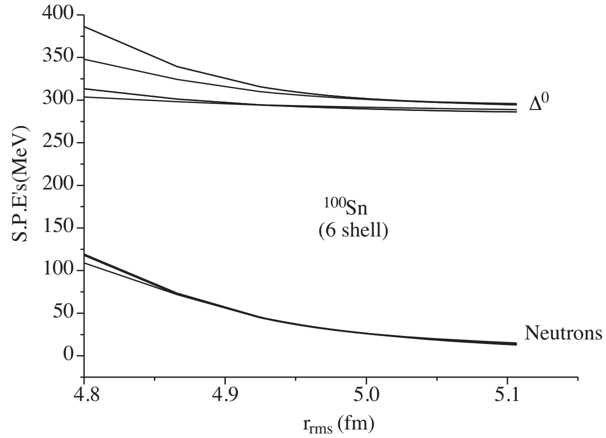

Figure 8. Single Particle Energy vs. r_{rms} of the highest three zero charge (dominantly nucleon) orbitals (low curves) and six orbitals, which are dominantly Δ^0 for ^{100}Sn in model space of six major oscillator shells.

Finally, it is important to note that the ordering of six dominantly Δ^0 proceed from lowest to highest as: $0s_{3/2}$, $0p_{1/2}$, $0p_{3/2}$, $1s_{3/2}$, $1p_{1/2}$, $1p_{3/2}$. The behavior of the positively charged baryon orbitals were not separately presented here as they exhibit properties similar to those of the zero-charge baryons.

3. Conclusions

Using a realistic effective baryon-baryon Hamiltonian, the ground state properties of spherical nucleus ^{100}Sn have been examined in the constrained Hartree-Fock approximation. It is found considerable decrease in the compression energy at fixed radius when the Δ degrees of freedom are included. The nuclear shell model is derived in this approach with single particle levels occupied by baryons which are mixture of nucleons and deltas. As shown in ^{100}Sn, the results compare favorably with those of the phenomenological successful shell model.

It can be concluded that the Hartree-Fock energy calculated with much larger decrease in compression as the size of the model space increases for either the N-only or both N and Δ cases. The nucleus becomes more compressible when delta particle resonances occur. A more modern potential and the inclusion of Δ resonances together induce a significant softening of the nuclear equation of state for large amplitude compression.

Finally, a large fraction of the excitation energy required to compress the nucleus used to create mass in the form of Δ's.

References

[1] D. Seweryniak, M. Carpenter, S. Gros, A. Hecht, N. Hoteling, R. Janssens, T. Khoo, T. Lauritsen, C. Lister, G. Lotay, D. Peterson, A. Robinson, W. Walters, X. Wang, P. Woods, S. Zhu, *Acta Physica Polonica B*, **40**, (2009), 621.

[2] M. Hasan, T. Lee and J. Vary, *Phys. Rev. C*, **61**, (1999), 014301.

[3] M. Hasan, T. Lee and J. Vary, *Phys. Rev. C*, **56**, (1997), 3063.

[4] M. Hasan, J. Vary and T. S. H. Lee, *Phys. Rev. C*, **64**, (2001), 024306.

[5] M. Hasan, S. Köhler, and J. Vary, *Phys. Rev. C*, **36**, (1987), 2180.

[6] M. Hasan, S. Köhler, and J. Vary, *Phys. Rev. C*, **36**, (1987), 2649.

[7] J. Vary and M. Hasan, *Phys. Rep.*, **242**, (1994), 139.

[8] J. Vary and M. Hasan, *Nucl. Phys. A*, **570**, (1994), 355.

[9] M. Hasan, *Dirasat J.*, **22**, (1995), 777.

[10] M. Hasan and J. Vary, *Phys. Rev. C*, **50**, (1994), 202.

[11] M. Hasan and J. Vary, *Phys. Rev. C*, **54**, (1996), 3035.

[12] M. Abu-Sei'leek, and M. Hasan, *Comm. Theort. Phys.*, **54,** (2010), 339.

[13] M. Abu-Sei'leek, *Comm. Theort. Phys.*, **55,** (2011), 115.

[14] M. Abu-Sei'leek, *Pramana- J. of Phys.*,**76,** (2011), 753.

[15] M. Abu-Sei'leek,*International J. of Pure and Applied Phys.*, **7,** (2011), 73.

[16] H. Brandow, *Rev. Mod. Phys.* ,**39**, (1967), 77.

[17] T. Lee, *Phys. Rev. Lett.*, **20**, (1983), 1571.

[18] T. Lee, *Phys. Rev. C*, **29**, (1994), 195.

[19] T. Lee, and A. Matsuyama, *Phys. Rev. C*, **32**, (1985), 516.

[20] T. Lee, and A. Matsuyama, *Phys. Rev. C*, **36**, (1987), 1459.

[21] Y. Tzeng, T. Kuo, and T. Lee, *Phys. Sc.*, **53**, (1996), 300.

[22] U. Mosol and V. Metag, *Nucl. Phys. News*, **3**, (1993), 25.

[23] L. Xiong and C. Ko, *Nucl. Phys. A*, **512**, (1990), 772.

[24] M. Hofmann, R. Mattiello, H. Sorge, H. Stöcker, and W. Greiner, *Phys. Rev. C*, **51**, (1995), 2095.