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Convective inertia effects on the squeeze film between rectangular parallel plates

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Abstract

This paper investigates the effects of convective inertia on the lubricating characteristics of the squeeze film between parallel rectangular plates. Relaxing a simplifying assumption of the conventional lubrication theory, the paper retains some terms of the convective inertia terms of the Navier-Stokes equations, leading to a derivation of a modified Reynolds equation. The introduction of the convective inertia terms induces two inversely related dimensionless parameters, m and R (the Reynolds number), into the characteristics of the flow behavior. The dimensionless normal (or squeezing) velocity increases monotonically from the lower plate to the upper plate for given value of R. At every point of the flow space, the squeezing velocity decreases with increase in R due to an increase in the fluid's resistance to the motion of the upper plate and an increase in the rate of loss of momentum per unit volume of fluid flow. The dimensionless pressure and load capacity decrease with increase in the Reynolds number R due to an increase the kinetic energy of the fluid.

Key Words: Navier-Stokes equations, Reynolds equation, convective inertia terms, velocity, pressure, load capacity

AMS subject classification: 35B40, 35K57, 80A25.

1. Introduction

This paper investigates a nonlinear system of partial differential equations describing the squeezing incompressible fluid flow between parallel impermeable rectangular plates, with the lower plate stationary and the upper plate moving with constant velocity (see Figure 1). Reynolds [1] formulated a mathematical model which has since provoked rich literature on this subject. His model consists of deriving a single autonomous equation (now referred to as Reynolds equation) in the pressure field, using the Navier-Stokes equations and the continuity equation, subject to a number of simplifying assumptions which includes the neglecting of the nonlinear convective inertia terms. The original equation has been modified by authors to address new situations. Such authors include Wu [2], Prakash and Vij [3], Sanni [4], Shah et al [5], Sattler and Wachutka [6], Deheri et al [7] and Bujurke and Kudenatti [8], to mention a few.

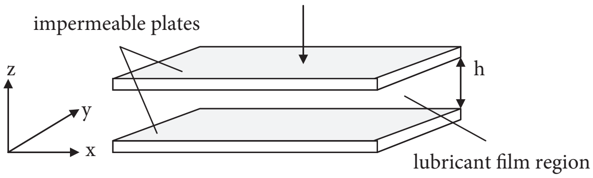

Figure 1. The geometry of the squeeze film bearing.

Authors who have presented approximate solutions to the incompressible Navier-Stokes equations describing squeezing flow, including the nonlinear convective inertia terms, include Gupta and Gupta [9], who analyzed two dimensional flow between two infinite parallel plates approaching or receding from each other symmetrically. The authors obtained approximate solution presented as regular perturbation for small values of Reynolds number. Singh and Verma [10] also obtained approximate solution for the same problem by a variational technique proposed by Gyarmati [11, 12], called the governing principle of dissipative processes. Their solution compares favorably with that of [9] for small values of the Reynolds number. The numerical solution of the combined effects of non-Newtonian rheology and fluid inertia forces on the non-linear transient behavior in circular squeeze films was presented by Lin and Hung [13]; while the same authors derived a closed-form solution for the squeeze film characteristics including the film pressure, the load capacity and the response time, in their study of the combined effects of non-Newtonian couple stresses and fluid inertia on the squeeze film characteristics between a long cylinder and an infinite plates [14]. Other authors who have investigated the squeeze film characteristics including the convective inertia terms include [15, 16, 17, 18] and [19].

The conventional lubricating theory assumes that the nonlinear convective inertia terms are negligible (see for example Wu [2]). We relax this assumption in the present analysis. We retain some terms of the nonlinear convective inertia terms on the assumption that they dominate over the other terms; and obtain an exact solution to the ensuing governing system of equations. A similar case in the lubrication theory is that in which the z-derivatives of the velocity components is assumed to dominate. If we define the operator $u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}+w\frac{\partial}{\partial z}$ as the convective inertia operator, then we refer to $u\frac{\partial U'}{\partial x}$, $v\frac{\partial U'}{\partial y}$ and $w\frac{\partial U'}{\partial z}$ as the x-, y- and z-convective inertia derivatives of U', respectively. Precisely, we assume that the the z-convective inertia derivatives of the velocity components dominate over the others. In addition to other assumptions of the conventional lubricating theory, this assumption gave rise to a modified Reynolds equation satisfied by the squeeze film. Effects of convection on the pressure distribution, load carrying capacity and squeezing velocity field are studied.

Remainder of this paper is organized as follows. Section 2 presents a general analysis, setting up the constraints of the problem. Section 3 presents results and discussion. Section 4 offers a conclusion; and section 5 gives a table of nomenclature used in this treatment.

2. General analysis

Assuming that the z-convective inertia derivatives of the velocity components dominate over the others and upholding the other assumptions of the conventional lubrication theory, the flow is governed by the following system of equations:

$$w'\frac{\partial u'}{\partial z'} = \frac{1}{\rho}\frac{\partial p'}{\partial x'} + \nu\frac{\partial^2 u'}{\partial z'^2} \tag{1}$$

$$w'\frac{\partial v'}{\partial z'} = \frac{1}{\rho}\frac{\partial p'}{\partial y'} + \nu\frac{\partial^2 v'}{\partial z'^2} \tag{2}$$

$$w'\frac{\partial w'}{\partial z'} = \nu\frac{\partial^2 w'}{\partial z'^2}, \tag{3}$$

and the continuity equation

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} + \frac{\partial w'}{\partial z'} = 0 \tag{4}$$

with the boundary conditions

$$u'(x', y', 0) = u'(x', y', h) = 0 \tag{5}$$

$$v'(x', y', 0) = v'(x', y', h) = 0 \tag{6}$$

$$w'(x', y', 0) = 0, \quad w'(x', y', h) = \dot{h} \tag{7}$$

$$p'(0, y', z') = p'(a, y', z') = p'(x', 0, z') = p'(x', b, z') = 0 \tag{8}$$

$$\frac{\partial p'}{\partial z'}(x', y', 0) = \frac{\partial p'}{\partial z'}(x', y', h) = 0, \tag{9}$$

where u', v', w' are the velocity components along the x', y', z' directions; p' is the pressure, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, μ is the dynamic viscosity and ρ is the density of the fluid.

Introducing dimensionless quantities

$$u = \frac{hu'}{a\dot{h}},\ v = \frac{hv'}{b\dot{h}},\ w = \frac{w'}{\dot{h}},\ p = \frac{h^3}{\mu a^2 \dot{h}}p',\ n = \frac{a}{b}$$

$$x = \frac{\pi x'}{a},\ y = \frac{\pi y'}{b},\ z = \frac{z'}{h},\ \frac{\dot{h}h}{\nu} = R \text{ (Reynolds number)}, \tag{10}$$

and substituting into equations (1)–(9) yields the governing equations

$$Rw\frac{\partial u}{\partial z} = -\pi\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial z^2} \tag{11}$$

$$Rw\frac{\partial v}{\partial z} = -\pi n^2\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial z^2} \tag{12}$$

$$Rw\frac{\partial w}{\partial z} = \frac{\partial^2 w}{\partial z^2} \tag{13}$$

$$\pi\frac{\partial u}{\partial x} + \pi\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{14}$$

with the boundary conditions

$$u(x,y,0) = u(x,y,1) = 0 \tag{15}$$

$$v(x,y,0) = v(x,y,1) = 0 \tag{16}$$

$$w(x,y,0) = 0,\ \ w(x,y,1) = 1 \tag{17}$$

$$p(0,y,z) = p(\pi,y,z) = p(x,0,z) = p(x,\pi,z) = 0 \tag{18}$$

$$\frac{\partial p}{\partial z}(x,y,0) = \frac{\partial p}{\partial z}(x,y,1) = 0. \tag{19}$$

Solving equations (11)–(14) with boundary conditions (15)–(17) gives

$$u = \frac{\pi}{mR}(z-1)\frac{\partial p}{\partial x}\tan(\frac{1}{2}mRz) \tag{20}$$

$$v = \frac{\pi n^2}{mR}(z-1)\frac{\partial p}{\partial y}\tan(\frac{1}{2}mRz) \tag{21}$$

$$w = \frac{\tan(\frac{1}{2}mRz)}{\tan(\frac{1}{2}mR)}, \tag{22}$$

where m is given by

$$m\tan(\frac{1}{2}mR) = 1. \tag{23}$$

We remark that (23) stems from the application of the second boundary conditions of (17) to the solution $w = m\tan(\frac{1}{2}mRz)$ of (13) which satisfies only the first boundary condition. Thus the parameters m and R depend on each other, so that neither can be arbitrarily chosen. Indeed (23) may be solved explicitly to get

$$R = \frac{2}{m}\tan^{-1}(\frac{1}{m}) \tag{24}$$

Substituting equations (20)–(22) into (14) and integrating the ensuing equation from $z = 0$ to $z = 1$ give the modified Reynolds equation

$$\frac{\partial^2 p}{\partial x^2} + n^2\frac{\partial^2 p}{\partial y^2} = \frac{-m^2R^2}{2\pi^2\int_0^1 \ln|\cos(\frac{1}{2}mRz)|dz}. \tag{25}$$

Applying the finite sine transform

$$\overline{f}(\alpha,\beta) = \int_0^\pi\int_0^\pi f(x,y)\sin(\alpha x)\sin(\beta y)dxdy \quad (\alpha,\beta = 1,2,3...) \tag{26}$$

to equation (25), we deduce that

$$\overline{p}(\alpha,\beta) = \frac{-2m^2R^2}{\pi^2\int_0^1 \ln|\cos(\frac{1}{2}mRz)|dz\alpha\beta(\alpha^2+n^2\beta^2)}, \quad (\alpha,\beta \text{ odd}). \tag{27}$$

The inversion theorem applied to equation (27) yields the dimensionless pressure distribution

$$p(x,y)=\frac{h^3}{a^2\mu\dot{h}}p'=\frac{-8m^2R^2}{\pi^4\int_0^1 \ln|\cos(\frac{1}{2}mRz)|dz}\sum_{\substack{\alpha=1\\(\alpha,\beta}}^{\infty}\sum_{\substack{\beta=1\\ \text{odd})}}^{\infty}\frac{\sin(\alpha x)\sin(\beta y)}{\alpha\beta(\alpha^2+n^2\beta^2)}. \tag{28}$$

The dimensional load capacity is obtainable by integrating the dimensional pressure over the plate, viz.

$$\int_0^a\int_0^b p'(x',y')dx'dy'. \tag{29}$$

For the current case, using (29), the dimensionless load capacity is obtained as

$$W=\frac{h^3}{a^3b\mu\dot{h}}W'=\frac{-32m^2R^2}{\pi^6\int_0^1 \ln|\cos(\frac{1}{2}mRz)|dz}\sum_{\substack{\alpha=1\\(\alpha,\beta}}^{\infty}\sum_{\substack{\beta=1\\ \text{odd})}}^{\infty}\frac{1}{\alpha^2\beta^2(\alpha^2+n^2\beta^2)}. \tag{30}$$

In the limit as $R\to 0$, we obtain from equation (30) the result of Hays [20] for the non-convective inertia squeeze film, namely

$$W=\frac{h^3}{a^3b\mu\dot{h}}W'=-\frac{768}{\pi^6}\sum_{\substack{\alpha=1\\(\alpha,\beta}}^{\infty}\sum_{\substack{\beta=1\\ \text{odd})}}^{\infty}\frac{1}{\alpha^2\beta^2(\alpha^2+n^2\beta^2)}.$$

3. Results and discussion

The flow is characterized by two dependent parameters m and R given by equation (23). The profile of the parameter m versus R is displayed in Figure 2, showing an inverse variation between the duo.

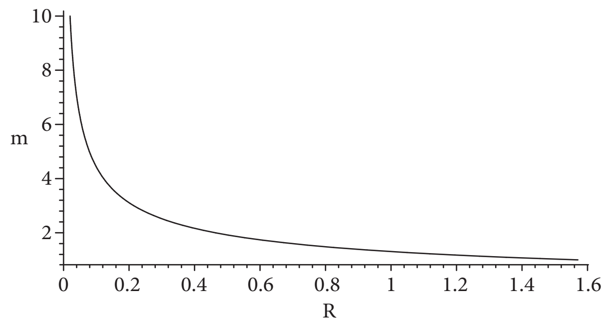

Figure 2. The curve of the parameter m versus R.

As observed in Figure 3, the normal velocity decreases with increase in R. This suggests an increase in the fluid's resistance to the motion of the upper plate as R increases. Furthermore, since the convective inertia terms represent the rate of loss of momentum per unit volume of fluid flow [21], an increase in R necessarily implies an increase in the rate of loss of momentum per unit volume of fluid flow. Consequently, the observed decrease in the dimensionless normal velocity is provoked by the increase in the loss of momentum per unit volume of fluid. For given values of R, w increases monotonically from $z=0$ to $z=1$. It exhibits a nonlinear profile for $R>0$ in contrast to the linear profile for the non-convective inertia flow ($R=0$). We remark that the profile is similar to that of Figure 2 of Reference [4]. Keeping all other parameters constant, the higher the fluid's density (a coefficient of the convective inertia terms), the lesser the squeezing velocity; so that the time of approach of the plates is reduced.

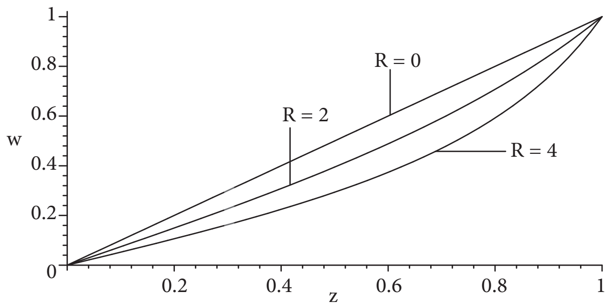

Figure 3. The dimensionless squeezing velocity w versus z for various values of R.

Figure 4 shows the profiles of the dimensionless pressure for some values of R. The dimensionless pressure decreases with increase in R. The decrease is provoked by the increase in the kinetic energy of the fluid. The latter deduction stems from the fact that an increase in R implies an increase in the kinetic energy of the fluid, so that a fluid pressure does work against the intermolecular forces in the fluids, thereby leading to a reduction in pressure. This deduction relates with the Bernoulli's principle. The dimensionless pressure exercises a profile similar to that of Figures 3 of References [2] and [4]. Keeping all other parameters constant, the lower the fluid's density (a coefficient of the convective inertia terms), the higher the pressure. Hence, to enhance pressure, a fluid of low density should be used in squeeze film bearings.

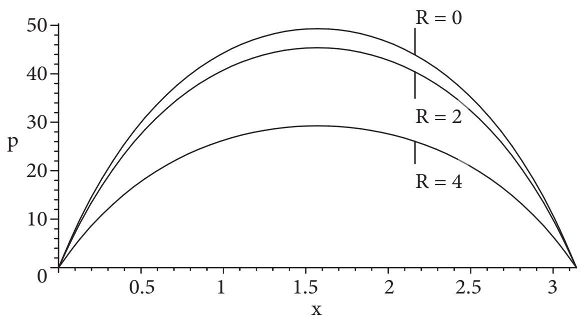

Figure 4. The dimensionless pressure p versus x for some values of R (n = 1, y = .5).

In sympathy with the decrease observed in the dimensionless pressure, the dimensionless load capacity decreases with increase in R, as shown in Table. Since the dimensionless load capacity is derived from pressure, the same physical explanations for the pressure holds for the dimensionless load capacity.

Table 1. Dimensionless load capacity for some values of R.

R	0	1	2	3	4	5
W	0.421691	0.402939	0.388036	0.375995	0.366095	0.357823

4. Conclusions

The flow is characterized by the dimensionless parameters m and R, which bear an inverse relationship with each other.

For given value of R, the dimensionless normal velocity w increases monotonically from $z = 0$ to $z = 1$; but it decreases along the film thickness as R increases, due to an increase in the rate of loss of momentum per unit volume of fluid flow. An increase in the density of the fluid reduces the time of approach of the plates. The profile of w is non-linear for $R > 0$ in contrast to the profile for the non-convective inertia case ($R = 0$).

The dimensionless pressure and the dimensionless load capacity decrease with increase in the Reynolds number R, due to an increase in the kinetic energy of the fluid, in sympathy with the Bernoulli's principle. Hence, a reduction in the density of the fluid enhances the pressure and the load-carrying capacity of the squeeze film.

In the manufacture of the squeeze film bearing, attention should be given to the density of the fluid, so as to ensure a desirable performance with respect to the approach of the plates and the load-carrying capacity.

Nomenclature

u', v', w'	velocity components
p'	pressure
h	film thickness
x', y', z'	rectangular coordinates
a	length of plate
b	breadth of plate
W'	load capacity
$\dot{h}$	relative normal velocity of the plates
u, v, w	dimensionless velocity components defined in equation (10)
x, y, z	dimensionless rectangular coordinates defined in equation (10)
R	Reynolds number
m	a dimensionless parameter given by equation (23)
n	the ratio a/b
W	dimensionless load capacity defined by equation (30).

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