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A case study on ΣΛ conversion in p-shell hypernuclei

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Abstract

The important process of $\Sigma - \Lambda$ conversion is presented and discussed in this report and a case example has been presented reviewing some cross sections and decay widths calculated for the process in a p-shell hypernucleus in recent studies.

Key Words: $\Sigma - \Lambda$ conversion, hypernucleus.

1. Introduction

Λ and Σ are two of the fundamental and most extensively studied hyperons in the physics of hypernuclei. There have been numerous studies in the last three decades citing the possibility of inter-conversion of these hyperons. Several reports of broad widths in Σ hypernuclear spectra and many experimental studies reporting complete absence of any bound states found in the ΣN system in the last two decades have been a strong underlying reason supporting the existence of $\Sigma - \Lambda$ conversion. This report presents a brief introduction to the $\Sigma - \Lambda$ conversion process, while presenting a review of some cross sections and decay widths calculated for the process in p-shell hypernuclei in some recent studies.

$\Sigma - \Lambda$ conversion is an in situ process which is in essence a $\Sigma - \Lambda$ coupling at the level of sub-baryonic (quark) degrees of freedom. At the nuclear level, it can be possible that the processes such as isospin mixing and hyperon-nucleon absorption are the underlying mechanisms responsible for this process. Figure 1 depicts a quark flow diagram for one of the modes of the process in form of a two-step conversion. This process occurs within the nuclear medium. As a Σ does not exist as a bound state in nucleus, this conversion is mediated via the strong interaction with one of the neighbouring nucleons. If a bound state existed, it would convert by decay into a Λ by means of the strong interaction, $\Sigma\mathrm{N} \leftarrow \rightarrow \Lambda\mathrm{N}$. This can be a mixing process, such as the mixing of mass or weak eigenstates as observed in fundamental particles. This conversion takes place as an intermediate state of the admixture, when both the nucleon and Σ hyperon undergo a mixing and as a consequence, a Λ and another nucleon emerge.

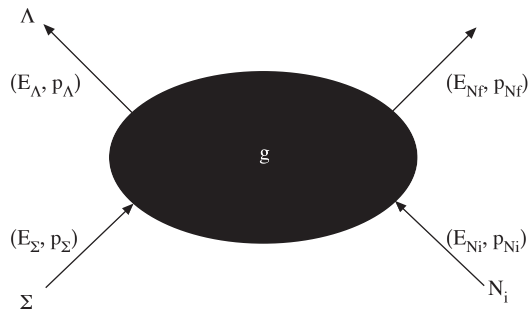

Figure 1. A Feynman diagram depicting the $\Sigma^0 \leftarrow\rightarrow \Lambda^0$ conversion via a gluon exchange.

The mixings of spin and isospin, as well as spin-isospin coupling can be represented in a form reported elsewhere [1], as given in the equation

$$V_{\Sigma N_{mix}} = V_\sigma \vec{\sigma}_N \cdot \vec{\sigma}_\Sigma + V_\tau \vec{t}_N \cdot \vec{\tau}_\Sigma + V_{\sigma\tau} \vec{\sigma}_N \cdot \vec{\sigma}_\Sigma \vec{t}_N \cdot \vec{\tau}_\Sigma. \tag{1}$$

On the production of Λ from mixing, which happens in the continuum of states, the hyperon readily decays, making it possible to be detected and measured. Its decay products and their kinematics thus bear the signature of the conversion process.

Dating back to the fifties, $\Sigma - \Lambda$ conversion was a possible way to explain the broad widths in hypernuclear spectra of various systems, for example, as pointed out by Lichtenberg and Ross [2]. In the seventies, during the early work [3] with ^{3}He and ^{4}He bubble chambers, the magnitude of the Σ-N interaction was found comparable to the ΛN interaction, which reaffirmed the possibility of a dominant role of $\Sigma - \Lambda$ conversion in this potential. Investigations by Batty et al. [4] led to the conclusion that the conversion width would be too broad to achieve any meaningful Σ-nucleus spectroscopic data. Elsewhere [5, 6], analyses of the x-ray data from the Σ^- atom studies found the real part of the potential at the centre of a nucleus to be very shallow or even repulsive, making the formation of bound states very unlikely. A study by Barakat and Hungerford [7] also reported a negative outcome in search for bound Σ states. To date, no acceptable experimental evidence has been presented, showing narrow peaks in Σ spectra or existence of their bound states, except in the case of Σ ^{4}He hypernucleus [8]. Therefore, both the possibility of the existence of Σ bound states or narrow spectra are considered very remote. An important criterion for the bound state formation is the hyperon width cannot exceed the separation between two adjacent hypernuclear levels. So far all the widths observed for Σ's in various experiments have been broader than the difference between two adjacent Σ-hypernuclear levels. Band et al., in their extremely detailed and enlightening treatise [9] on hypernuclei, forecast the conversion width to be around 10-20MeV.

A study [10] of the process $\mathrm{K^-d} \rightarrow \Lambda\mathrm{N}$ used an analysis of bubble chamber film (from the Lawrence Livermore Laboratory 25″ bubble chamber) irradiated with a K^- beam of 400 MeV/c. The momentum was chosen so that the nearby occurrence of the $\Lambda(1520)$ state in the $\mathrm{K^-p}$ channel results in highly polarized Σ^+ production. The study concluded that the Λp interaction plays an important role and suggested the existence of a resonance in the Λp system at a mass very near Σn threshold.

The conversion of a Σ into Λ involves states which are believed to possess high angular momentum values and induce the conversion at the surface of the nuclear core. The lifetimes and quantum numbers of these states have been calculated in a number of studies, such as by Batty et al. [11]. As the conversion is a strong interaction, the role of the pion-mediated Yukawa force is significant. One is led to investigate a pion-mediated $\pi\Lambda\Sigma$ state, dealing with the problem as a three-body coupling. This approach has been taken in a number of studies, most notably by Lichtenberg et al. [2] and Loiseau et al. [12].

2. A DWIA treatment-cross section and widths calculation

A useful step in quantitatively understanding the conversion process is the calculation of a general cross-section for this process. In this context, one can either use an elementary potential, as discussed earlier, or an effective potential, such as an impulse approximation. Here, an enhanced form of the *Born Approximation*, i.e d*istorted-wave born approximation* (DWBA) [13] is utilized. It is customary to use variations of the *impulse approximations* in such cases, such as DWIA (Distorted-Wave Impulse Approximation) and PWIA (Plane-Wave Impulse Approximation). Details of the customary DWBA methods, adopted in the context of the many-body hypernuclear problem, can be found elsewhere [14].

In this case, the calculations of Oset et al. [15] and Plano [16] are discussed, which used a model based on the DWBA approximation to calculate a hypernuclear cross-section for the $\Sigma - \Lambda$ interaction.

In this approach, the Σ-N interaction is taken as a strong interaction-mediated, direct (one-step) process. As the first step, the Feynman diagram for the strong interaction $\Sigma - \Lambda$ conversion can be drawn, as illustrated in Figure 2.

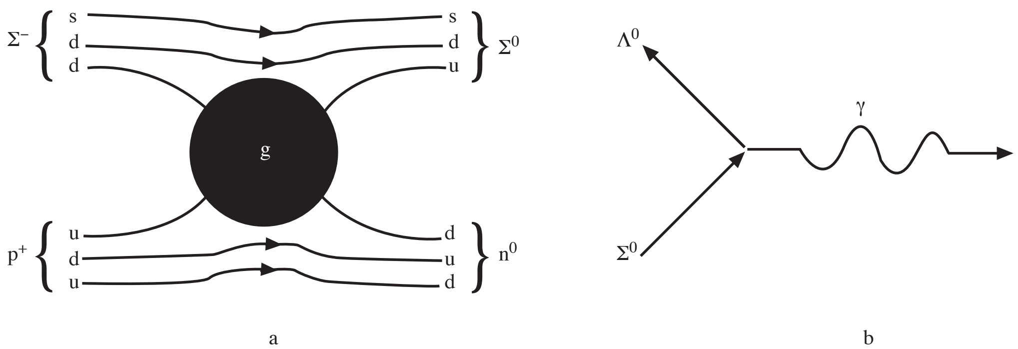

Figure 2. A quark flow diagram depicting the two-step $\Sigma - \Lambda$ conversion, via the $\Sigma^- p^+ \to \Lambda^0 n^0$ channel. Figure 2(a) illustrates the first step, i.e. $\Sigma^- \to \Sigma^0$ conversion, and 2(b) depicts the second step of $\Sigma^0 \leftarrow \to \Lambda^0$ conversion.

This shows the hyperon and nucleon vertices and the interaction between them is mediated by a sea of gluons. With this, we can readily construct a scattering matrix:

$$\langle f|S-1|i\rangle = -i\Im_{fi}\frac{1}{(2\pi)^6}\sqrt{\frac{M_N}{E_{N_i}}}\sqrt{\frac{M_N}{E_{N_f}}}\sqrt{\frac{M_\Sigma}{E_\Sigma}}\sqrt{\frac{M_\Lambda}{E_\Lambda}}(2\pi)^4\delta(p_\Sigma + p_{N_i} - p_\Lambda - p_{N_f}). \quad (2)$$

Here, M_N are the masses of the two nucleons in initial and final states (i.e. proton and neutron, respectively) and M_Σ and M_Λ are the masses of the two hyperons, respectively; and whereas $\Im_{fi}$ is the transition amplitude, or the matrix element for the S-matrix for time-development of the initial states to final states, and is related to the transition probability $\boldsymbol{W}_{fi}$. The transition probability can be expressed as

$$W_{fi} = \frac{\pi(2l+1)}{k^2} \left| S_{fi} - \delta_{fi} \right|. \tag{3}$$

From this, one can derive the cross section, an example of which is one calculated by Oset [16], expressed as

$$\sigma_{\Sigma-\Lambda} = \left(\frac{1}{v_{rel}}\right) \int \frac{d^3k'}{(2\pi)^3} \int \frac{d^3p'}{(2\pi)^3} \frac{M_\Sigma}{E_\Sigma(k)} \frac{M_\Lambda}{E_\Lambda(k')} \frac{M_N}{E_N(p)} \frac{M_N}{E_N(p')} \Im^2_{fi} (2\pi)^4 \delta(p_\Sigma + p_{N_i} - p_\Lambda - p_{N_f}). \tag{4}$$

Then, the calculation proceeds by using the relevant Feynman rules and taking the *quasi-particle approach* (i.e. assuming Σ^- in an infinite nuclear medium), and making certain approximations like *fermi sea* averaging, the usual steps in a cross section calculation, and taking the non-relativistic limit, i.e.

$$E_\Sigma - E_\Lambda \approx M_\Sigma - M_\Lambda, \tag{5}$$

This way, the average cross section was obtained by Plano [15] as

$$\langle \sigma \nu_{rel} \rangle_{ave} = \frac{1}{\pi} \mu \bar{q} \bar{\Sigma} \Sigma \left| T \right|^2 \xi(\bar{q}), \tag{6}$$

where, ν_{rel} is the relative velocity of the ΣN system (in lab), $\bar{\Sigma}\Sigma$ denotes the summed-over isospin and spin sums, μ is the reduced nucleon and lambda mass, $\xi(q)$ is the Pauli Blocking term

$$\xi(\bar{q}) = 1 - \theta(2 - \left|\vec{q}\right| / k_F),$$

k_F is the Fermi momentum, and q is the momentum transfer,

$$\bar{q} = \sqrt{2\mu(M_\Sigma - M_\Lambda)}.$$

This is a general cross section for a $\Sigma - \Lambda$ coupling in the non-relativistic limit. The transition amplitude can be calculated from the relevant Feynman diagrams. The detailed procedure of this calculation, within the paradigm of quantum field theoretical treatment, can be found in a detailed treatise on quantum field theory [17].

The next step after determination of cross section is the resolution of the Σ widths involved in this process. Although the widths for finite nuclei are usually calculated from *local density approximations*, this cross section can be utilized to calculate them in a simpler manner.

In a study of proton and neutron spectra following Λ decay, Ramos et al. [18], calculate the self-energy and decay widths for Λ hypernucleus. Expressions for Σ self-energy and decay width can be written in a similar fashion.

In order to calculate the width of a Σ^- in nuclear matter (during this process) and account for the medium corrections, one writes the width in terms of the imaginary component of the Σ^- self energy, following the *quasi-particle* approach. This self energy is denoted by Σ (a generic symbol, not limited to the 'Σ' hyperon only) and is proportional to the optical potential

$$\Gamma_\Sigma = -2Im(\Sigma). \tag{7}$$

A many-body graph (lowest order) of the Σ^- self energy Σ, while undergoing the conversion process $\Sigma^- p^+ \rightarrow \Lambda^0 n^0$, is shown in the Figure 3. The dashed line depicts the transition with the participating nucleon and hyperon initial and final states in the Fermi sea of the nuclear medium.

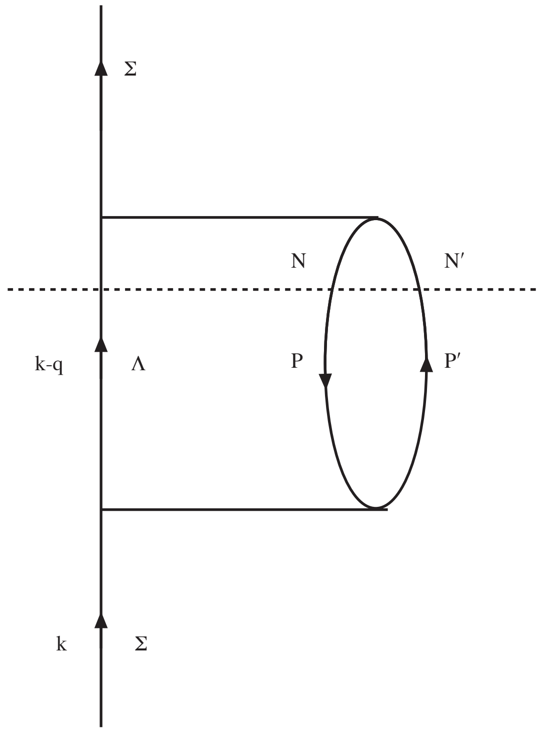

Figure 3. A many-body graph (lowest order) of the Σ^- self energy Σ, while in the conversion process $\Sigma^-\,\mathrm{p}^+ \longleftrightarrow \Lambda^0\,\mathrm{n}^0$.

This *self energy* is related to the cross section of the process by the relationship

$$Im(\Sigma) = -\frac{1}{2}\rho \left\langle \sigma v_{rel} \right\rangle_{f.a}, \tag{8}$$

where ρ is the density of the nucleon (in this case, a proton) which is undergoing an interaction with the Σ^-, and the quantity $\left\langle \sigma v_{rel} \right\rangle_{f.a}$ is the Fermi-averaged cross section for the process.

Thus the decay width is expressed in terms of the cross section

$$\Gamma_\Sigma = \rho \left\langle \sigma v_{rel} \right\rangle_{f.a}. \tag{9}$$

Oset et al. evaluate this width in a *local density approximation* in a manner adopted by Ramos et al. [18] for calculation of Λ widths. Ramos calculated widths for Λ by using the form

$$\Gamma(\vec{k}) = \int d^3r \left|\Psi_\Lambda(r)\right|^2 \Gamma\left(\vec{k}, \rho(r)\right). \tag{10}$$

Here, $\Psi(r)$ is the Λ eigenfunction obtained from a solution of the harmonic oscillator with the energy eigenvalue parameter $\hbar\omega$, and substituting values for it from hypernuclear binding energies data. The width, $\Gamma\left(\vec{k}, \rho(r)\right)$, is calculated from a method involving averages over the momentum distribution of $\Psi(r)$ eigenfunction and using the dressed pion propagators.

The width expressed here had been calculated for the case of $^{16}\mathrm{O}_{\Sigma}\Sigma^{-}$ hypernucleus (1s and 1p states), with and without the Pauli blocking factor. They found widths to be on the order of:

$$\Gamma^{1s}_{\Sigma^-} = 15.1\,MeV, \Gamma^{1s}_{\Sigma^-} = 19.4\,MeV$$

$$\Gamma^{\Sigma^-}_{1p} = 11.1\,MeV, \Gamma^{\Sigma^-}_{1p} = 13.6\,MeV$$

The calculated widths seem quite reasonable and expected (although the result was obtained from the studies carried out with p-shell systems only).

3. Conclusion and discussion

There is very little known about the mechanism and manifestations of the conversion of a Σ particle into a Λ. Some decay widths and cross section calculations carried out elsewhere have been presented in this report to illustrate the magnitude of this process. In addition to that, we also saw similar results in the case of an s-shell hypernuclear system the work was reported as part of our study, as presented in a doctoral dissertation [19] to partially address the vacuum of available knowledge on the $\Sigma\Lambda$ conversion. Our analysis aimed to study the three-body system ΣNN and the $\Sigma - \Lambda$ conversion mechanics and rates and rates and we found results hinting towards a strong Σ to Λ conversion, as reported in [19]. Since early work with He bubble chambers [3], the magnitude of the Σ-N interaction was found comparable to the $\Lambda - N$ interaction. It is believed that a dominant role is played by the $\Sigma - \Lambda$ conversion. Interconversions [20] between the two hyperons have been cited as a strong possibility. Recently, it has been suggested [21] that $\Sigma - \Lambda$ inter-conversion is a crucial prerequisite for the binding of the hypertriton. These and other factors make the investigation of this conversion a strong candidate to understand the empirical structural differences between the two important hyperons and their interactions.

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