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Effect of a wiggler magnetic field and the ponderomotive force on the second harmonic generation in laser-plasma interactions

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Abstract

In this paper, we consider second harmonic generation and its phase-matching in an underdense plasma in the presence of a wiggler magnetic field. Wiggler magnetic field plays both a dynamic role in producing the traverse harmonic current and a kinematical role in ensuring phase-matching. The inertial ponderomotive force $\rho(\vec{u}\cdot\vec{\nabla})\vec{u}$ is a source of harmonic generation. $\vec{u}$ beats with itself to produce a different harmonic. The inertial ponderomotive force can also affect the efficiency of second harmonic generation; and its effect on second harmonic generation is also considered.

Key Words: Second harmonic generation, wiggler magnetic field, nonlinear optics, phase-matching, plasma.

1. Introduction

The interaction of intense short pulse lasers with plasma leading to harmonic generation has been an active area of research for more than four decades, especially in recent years [1–6]. This interaction indicates that, because of the mismatch between the phase velocities of the laser pulse and the generated harmonic, and because of the collective response of the plasma, the conversion efficiency should be low without a means for phase-matching [4].

Phase matching is a group of techniques for achieving efficient nonlinear interactions so as to ensure that a proper phase relationship between the interacting waves is maintained along the propagation direction for optimum nonlinear frequency conversion. Nonlinear process of second harmonic generation requires *phase matching* to be efficient. In second harmonic generation, two photons of energy $\hbar\omega$ and momentum $\hbar\vec{k}_1$ combine to produce a photon of second harmonic radiation of energy $\hbar\omega_2$ and momentum $\hbar\vec{k}_2$, where $(\omega, \vec{k}_1)$

and $(2\omega, \vec{k}_2)$ satisfy the linear dispersion condition for electromagnetic waves [7].

The energy conservation in a second harmonic process is subject to the condition $\hbar\omega_2 = 2\hbar\omega$ and the momentum conservation $\hbar\vec{k}_2 = 2\hbar\vec{k}$ can not be satisfied because of the dispersion relation in plasma.

Agrawal et al. [8] have studied resonant second harmonic generation of a millimeter wave in a plasma filled waveguide in the presence of a helical magnetic wiggler. Weissman et al. [9] have studied second harmonic generation in Bragg-resonant quasi-phase-matched periodically segmented waveguides. Ding et al. [10] have developed a theory for quasi-phase-matched backward second and third harmonic generation in a periodically doped semiconductor.

Singh et al. [11] show a density ripple in a plasma could be properly employed for resonant second harmonic generation and the efficiency of the process is sensitive to the angle between the density ripple and the incident laser and beam energy. The ripple density provides the additional momentum required by the second-harmonic for phase matching. Singh et al. [12] show when a p-polarized high power laser, obliquely incident on underdense plasma, undergoes second and third harmonic generation and intensity of the second and third-harmonic is proportional to the square of the current density. Singh et al. [13] show that, if a high intensity laser is obliquely incident on a vacuum-plasma, the interface produces second-harmonic radiation in the reflected component and the efficiency of second-harmonic generation increases with the angle of incidence, up to critical angle of incidence, with the efficiency also a function of electron density. Nitikant and Sharma [14, 15] found that wiggler magnetic field plays both a dynamic role in producing the traverse harmonic current and a kinematical one in ensuring phase-matching.

In this paper, (A) second harmonic generation in an underdense plasma in the presence of wiggler magnetic field is considered by two causes. First, interaction of an ultra-short laser pulse with nonmagnetic isotropic plasma produces many nonlinear phenomena such as generation of odd harmonics. In the presence of a magnetic field, an isotropic plasma is converted to a non-isotropic plasma and leads to production of even harmonics. Second, the wave vector $\vec{k}_0$ of wiggler magnetic field acts as a virtual photon of quantum energy 0 and momentum $\hbar\vec{k}_0$ and the Gaussian phase matching condition, which is an important character to obtain a large output, is given by the conservation of the momentum [9, 10, 16]. (B) The ponderomotive force acting on electron consists of two terms: the inertial term $\rho(\vec{u}\cdot\vec{\nabla})\vec{u}$, and Lorentz term $\rho(\vec{u}\times\vec{B})$ [17, 18]. In [14, 15], the inertial term had been neglected, but in this paper, the effect of inertial ponderomotive force $\rho(\vec{u}\cdot\vec{\nabla})\vec{u}$ on efficiency of second harmonic generation is considered. It is shown the effect of inertial ponderomotive force in second harmonic generation is considerable at definite range of frequency and negligible at the rest.

2. Solutions of nonlinear equations of plasma by using perturbation theory

The continuity equations for electron number density, the hydrodynamic equation for the average velocity of electron and the electromagnetic wave in a weakly collisional cold plasma are given by following relations [18, 19]:

$$\frac{\partial n}{\partial t} + \vec{\nabla}\cdot n\vec{u} = 0 \tag{1}$$

$$\frac{\partial\vec{u}}{\partial t}+\left(\vec{u}\cdot\vec{\nabla}\right)\vec{u}+\frac{e}{m}\vec{E}+\frac{e}{m}\vec{u}\times\vec{B}=\vec{0} \tag{2}$$

$$\nabla^2\vec{E}-\vec{\nabla}(\vec{\nabla}\cdot\vec{E})-\frac{\partial^2\vec{E}}{\partial t^2}=\frac{4\pi}{c^2}\frac{\partial\vec{j}}{\partial t}, \tag{3}$$

where$\vec{j}=-ne\vec{u}$, $\vec{B}$,$\vec{E}$, c, $\vec{u}$, n, m and e are current density, magnetic and electric fields, the velocity of light in vacuum, the average velocity, number density, mass and charge of electron, respectively. Term $(\vec{u}\cdot\vec{\nabla})\vec{u}$ in relation (2) is called inertial term [18]. Assume a plasma with uniform density n_0 in the presence of the wiggler magnetic field

$$\vec{B}_w(z)=B_{ow}e^{ik_0z}\hat{e}_y, \tag{4}$$

where B_{ow} and k_0 are amplitude and wave number of the background wiggler magnetic field. Consider an intense, short laser pulse with frequency ω, the wave number $\vec{k}_1$ and polarization in the x-direction propagating through the plasma along the positive z-axis. The electric field of the laser induces on plasma electrons an oscillatory velocity $\hat{\vec{u}}_1^0$ at $(\omega,\,\vec{k}_1)$. The velocity $\hat{\vec{u}}_1^0$ and $\vec{B}_w(z)$ beat to exert a ponderomotive force $-(e/2mc)\,(\hat{\vec{u}}_1^0\times\hat{\vec{B}}_w)$ and imparts an oscillatory velocity $\hat{\vec{u}}_1^1$ at $(\omega,\,\vec{k}_1+\vec{k}_o)$; and $\hat{\vec{u}}_1^1$ and $\vec{B}_w(z)$ also exert a ponderomotive force $-(e/2mc)\,(\hat{\vec{u}}_1^1\times\hat{\vec{B}}_w)$ and gives the oscillatory velocity $\hat{\vec{u}}_1^2$ on electron at $(\omega,\,\vec{k}_1+2\vec{k}_0)$ and so on. Under this circumstance, it is better to solve the above equations by means of perturbation expansion. For this purpose, physical quantities are shown as

$$\hat{Q}_n=\hat{Q}_n^0+\hat{Q}_n^1+\hat{Q}_n^2+\hat{Q}_n^3+\cdots \tag{5}$$

so that lower and upper indexes show harmonic and order of perturbation, respectively. The envelope of the wave, $\hat{E}_n$ varies when propagating through the medium. If we assume that this variation is slow, both in amplitude and in phase, over distances of the order of the wavelength, then

$$\left|\frac{\partial^2E_n}{\partial z^2}\right|<<\left|k_n\frac{\partial E_n}{\partial z}\right|. \tag{6}$$

This is known as the slowly varying envelope approximation (SVEA) and is almost always valid in nonlinear processes. By invoking this approximation, equation (2) can be written for fundamental harmonic ω as

$$\frac{\partial(\hat{\vec{u}}_1^0+\hat{\vec{u}}_1^1+\hat{\vec{u}}_1^2+\cdots)}{\partial t}+\frac{e}{m}(\hat{\vec{E}}_1^0+\hat{\vec{E}}_1^1+\hat{\vec{E}}_1^2+\cdots)+\frac{e}{mc}(\hat{\vec{u}}_1^0+\hat{\vec{u}}_1^1+\hat{\vec{u}}_1^2+\cdots)\times\hat{\vec{B}}_w=\vec{0}, \tag{7}$$

so that its solutions to second order perturbation $\hat{\vec{B}}_w(z)$are obtained by the relations

$$\hat{\vec{u}}_1^0=\frac{e}{im\omega}\hat{\vec{E}}_1^0, \tag{8}$$

$$\hat{\vec{u}}_1^1=\frac{e}{im\omega}\hat{\vec{E}}_1^1+\frac{1}{2c}\left(\frac{e}{im\omega}\right)^2\hat{\vec{E}}_1^0\times\hat{\vec{B}}_w, \tag{9}$$

$$\hat{\vec{u}}_1^2=\frac{e}{im\omega}\hat{\vec{E}}_1^2+\frac{1}{2c}\left(\frac{e}{im\omega}\right)^2\hat{\vec{E}}_1^1\times\hat{\vec{B}}_w+\frac{1}{4c^2}\left(\frac{e}{im\omega}\right)^3\left(\hat{\vec{E}}_1^0\times\hat{\vec{B}}_w\right)\times\hat{\vec{B}}_w, \tag{10}$$

where the symbol "∧" over quantities denotes their complex representation. By writing equation (1) for the fundamental harmonic as

$$\frac{\partial \widehat{n}_1}{\partial t} + n_0 \vec{\nabla} \cdot \hat{\vec{u}}_1 = 0, \tag{11}$$

the plasma density and the linear current density are obtained via

$$\hat{n}_1^0 = \hat{n}_1^2 = 0, \quad \hat{n}_1^1 = \frac{n_0 k_1 \hat{u}_{1z}^1}{\omega} \tag{12}$$

$$\hat{\vec{j}}_1^0 = -n_0 e \hat{\vec{u}}_1^0, \quad \hat{\vec{j}}_1^1 = -n_0 e \hat{\vec{u}}_1^1, \quad \hat{\vec{j}}_1^2 = -n_0 e \hat{\vec{u}}_1^2. \tag{13}$$

By substituting the linear current density in the electromagnetic wave equation, equation (3), the fundamental harmonic wave equation is given as

$$\nabla^2 \hat{\vec{E}}_1 - \vec{\nabla}(\vec{\nabla} \cdot \hat{\vec{E}}_1) + \frac{\omega^2}{c^2} \hat{\vec{E}}_1 = -\frac{4\pi e n_0}{c^2} \frac{\partial \hat{\vec{u}}_1}{\partial t}. \tag{14}$$

By solving the zero order perturbation of wave equation, the zero order electric field becomes

$$\hat{\vec{E}}_1^0(z,t) = A_1(z - v_{g1} t) e^{i(k_1 z - \omega t)} \hat{e}_x, \tag{15}$$

where k_1 is the wave number of wave with frequency ω in plasma that is given by

$$k_1(\omega, \omega_p) = \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2}. \tag{16}$$

In the above, $\omega_p^2 = 4\pi e^2 n_0 / m$ is plasma frequency and $A_1(z - v_{g1} t)$ is the amplitude of fundamental electric field that is characterized by the temporal profile of the laser pulse; and the group velocity v_{g1} of fundamental wave is given by

$$v_{g1} = c \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2} \quad . \tag{17}$$

By solving the first and second order perturbation of the wave equation, the electric fields are obtained as

$$\hat{\vec{E}}_1^1(z,t) = \frac{\Omega_p^2 \Omega_c}{2i(1 - \Omega_p^2)} A_1(z - v_{g1} t) e^{i((k_1 + k_0\} z - \omega t)} \hat{e}_z, \tag{18}$$

$$\hat{\vec{E}}_1^2(z,t) = -\frac{\omega^2 \Omega_p^2 \Omega_c^2}{16 c^2 k_0 (k_1 + k_0)} \left(\frac{\Omega_p^2}{1 - \Omega_p^2} + 1 \right) A_1(z - v_{g1} t) e^{i((k_1 + 2k_0\} z - \omega t)} \hat{e}_x, \tag{19}$$

where $\Omega_p = \omega_p / \omega$, $\Omega_c = \omega_c / \omega$ and $\omega_c = e B_{0w} / mc$ is the cyclotron frequency. These solutions show that the zero, first and second order perturbations of fundamental waves are transversal for $(\omega, \vec{k}_1)$, longitudinal for $(\omega, \vec{k}_1 + \vec{k}_o)$ and transversal for $(\omega, \vec{k}_1 + 2\vec{k}_o)$, respectively.

In a similar way, the electron second harmonic equation of motion is written

$$\frac{\partial \hat{\vec{u}}_2}{\partial t} + \frac{1}{2} \left(\hat{\vec{u}}_1 \cdot \vec{\nabla} \right) \hat{\vec{u}}_1 + \frac{e}{m} \hat{\vec{E}}_2 + \frac{e}{2mc} \hat{\vec{u}}_2 \times \hat{\vec{B}}_w + \frac{e}{2mc} \hat{\vec{u}}_1 \times \hat{\vec{B}}_1 = \vec{0} \tag{20}$$

where

$$\hat{\vec{B}}_1 = \frac{\vec{k}_1 \times \hat{\vec{E}}_1}{\omega}, \tag{21}$$

and its solutions are obtained as

$$\hat{\vec{u}}_2^0 = \frac{e}{2im\omega}\hat{\vec{E}}_2^0 + \frac{1}{4c}\left(\frac{e}{im\omega}\right)^2 \hat{\vec{E}}_1^0 \times \hat{\vec{B}}_1^0 \tag{22}$$

$$\begin{aligned} \hat{\vec{u}}_2^1 = \quad & \tfrac{e}{2im\omega}\hat{\vec{E}}_2^1 + \tfrac{1}{4c}\left(\tfrac{e}{im\omega}\right)^2 \hat{\vec{E}}_1^1 \times \hat{\vec{B}}_1^0 + \tfrac{1}{8c}\left(\tfrac{e}{im\omega}\right)^2 \hat{\vec{E}}_2^0 \times \hat{\vec{B}}_w + \tfrac{1}{16c^2}\left(\tfrac{e}{im\omega}\right)^3 \left(\hat{\vec{E}}_1^0 \times \hat{\vec{B}}_1^0\right) \times \hat{\vec{B}}_w \\ & + \tfrac{1}{8c^2}\left(\tfrac{e}{im\omega}\right)^3 \left(\hat{\vec{E}}_1^0 \times \hat{\vec{B}}_w\right) \times \hat{\vec{B}}_1^0 + \tfrac{k_1}{4\omega}\left(\tfrac{e}{im\omega}\right)^2 \hat{E}_1^1 \hat{\vec{E}}_1^{\,0} + \tfrac{k_1}{4\omega}\left(\tfrac{e}{im\omega}\right)^3 \hat{E}_1^0 \hat{B}_w \hat{\vec{E}}_1^{\,0}. \end{aligned} \tag{23}$$

The plasma density related to second harmonic is satisfied with the equation

$$\frac{\partial \widehat{n}_2}{\partial t} + n_0 \vec{\nabla} \cdot \hat{\vec{u}}_2 + \frac{1}{2}\vec{\nabla} \cdot \hat{n}_1 \hat{\vec{u}}_1 = 0. \tag{24}$$

Current density related to the second harmonic is obtained by

$$\hat{\vec{j}}_2 = -n_0 e \hat{\vec{u}}_2^0 - n_0 e \hat{\vec{u}}_2^1 - \frac{1}{2}\hat{n}_1^1 e \hat{\vec{u}}_1^0. \tag{25}$$

This nonlinear current density produces a second-harmonic wave. Then by substituting $\hat{\vec{j}}_2$ in the second-harmonic wave, we have

$$\nabla^2 \hat{\vec{E}}_2 - \vec{\nabla}(\vec{\nabla} \cdot \hat{\vec{E}}_2) + \frac{4\omega^2}{c^2}\hat{\vec{E}}_2 = \frac{4\pi}{c^2}\frac{\partial \hat{\vec{j}}_2}{\partial t}, \tag{26}$$

and introducing the solution

$$\hat{\vec{E}}_2(z,t) = \hat{\vec{E}}_2^0(z,t) + \hat{\vec{E}}_2^1(z,t) + \cdots, \tag{27}$$

the zero-order wave equation is obtained as

$$\nabla^2 \hat{\vec{E}}_2^0 - \vec{\nabla}(\vec{\nabla} \cdot \hat{\vec{E}}_2^0) + \frac{4\omega^2}{c^2}\hat{\vec{E}}_2^0 = \frac{\omega_p^2}{\omega\, c^2} k_1 \left(A_1(z - v_{g1}t)\right)^2 e^{2i(k_1 z - \omega t)} \hat{e}_z. \tag{28}$$

Its solution is a longitudinal wave at $(2\omega, 2\vec{k}_1)$ as

$$\hat{\vec{E}}_2^0(z,t) = \frac{i\Omega_p^2\, e\sqrt{1-\Omega_p^2}}{2\, m\omega\, c(1-\Omega_p^2)} \left(A_1(z - v_{g1}t)\right)^2 e^{2i(k_1 z - \omega t)} \hat{e}_z, \tag{29}$$

where k_2 is the wave number associated with the 2ω component in the plasma, and the dispersion relation for the second harmonic wave in plasma is given by

$$k_2(\omega, \omega_p) = \frac{2\omega}{c}\sqrt{1 - \left(\frac{\omega_p}{2\omega}\right)^2}. \tag{30}$$

It is obvious the wave number depends on characteristics of plasma, ω_p and laser frequency ω. The first order wave equation is obtained as

$$\nabla^2 \hat{\vec{E}}_2^0 - \vec{\nabla}(\vec{\nabla} \cdot \hat{\vec{E}}_2^0) + \frac{4\omega^2}{c^2} \hat{\vec{E}}_2^0 = -\frac{4\pi e n_0}{c^2} \frac{\partial \hat{\vec{u}}_2^1}{\partial t} - \frac{2\pi e \hat{n}_1^1}{c^2} \frac{\partial \hat{\vec{u}}_1^0}{\partial t} - \frac{2\pi e \hat{\vec{u}}_1^0}{c^2} \frac{\partial \hat{n}_1^1}{\partial t}. \tag{31}$$

By using equations (12), (22) and (23), and substituting into the above equation, we find the inhomogeneous term is a vector along the x-axis with coefficient $\exp[-i(2\omega t - (2k_1 + k_0)z]$. It is obvious that its solution is a transversal wave for $(2\omega,\ 2\vec{k}_1 + \vec{k}_0)$. Therefore, in order to satisfy the phase matching, it is necessary $k_2 = 2k_1 + k_0$. By substituting k_1 (equation (16)) and k_2 (equation (30)) in the condition of phase matching, $k_2 = 2k_1 + k_0$, where k_0 is determined as

$$k_0(\omega, \omega_p) = \frac{2\omega}{c} \left(\sqrt{1 - \left(\frac{\omega_p}{2\omega}\right)^2} - \sqrt{1 - \left(\frac{\omega_p}{\omega}\right)^2} \right).$$

The above equation shows that the wave number of wiggler magnetic field, k_0, depends on the characteristic plasma frequency ω_p and laser frequency ω. By introducing the solution as

$$\hat{\vec{E}}_2^1(z,t) = A_2^1(z,t) e^{i(k_{21}z - 2\omega t)} \hat{e}_x, \tag{32}$$

And using new variables $\xi = z - v_{1g}t$ and $\eta = z$ so that

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial \xi} - \frac{\partial}{\partial \eta}, \quad \frac{\partial}{\partial t} = -v_{1g} \frac{\partial}{\partial \xi}, \tag{33}$$

and the condition of phase matching, the first order wave equation reduces to

$$\frac{\partial A_{2x}^1}{\partial \xi} + \beta \frac{\partial A_{2x}^1}{\partial \eta} = \alpha_i \left(A_1(z - v_{1g}t)\right)^2, \tag{34}$$

where

$$\beta = 1 - \frac{v_{1g}}{v_{2g}} \tag{35}$$

and the group velocity of second harmonic v_{g2} is given by

$$v_{2g} = c\sqrt{1 - \left(\frac{\omega_p}{2\omega}\right)^2} \tag{36}$$

and

$$\alpha_i = \frac{i\Omega_p^2\, e\, k_1 \Omega_c}{4\, m\, c^2 k_2} \left(\frac{3}{4} + \frac{\Omega_p^2}{4\,(4 - \Omega_p^2)} + \frac{\Omega_p^2}{(1 - \Omega_p^2)} \right) + \frac{i\Omega_p^2\, e\, k_0 \Omega_c}{4\, m\, c^2 k_2} \left(1 + \frac{\Omega_p^2}{(1 - \Omega_p^2)} \right). \tag{37}$$

Consider the temporal profile of the laser pulse to be Gaussian with the form

$$A_1(z,t) = A_0 \exp\left(-\left(\frac{z - v_{1g}t}{\xi_0} \right)^2 \right) \hat{e}_x, \tag{38}$$

where $\xi_0 = \tau\, v_{1g}$ and τ is the pulse duration. Hence, the solution of equation (34) can be obtained as

$$A_{2x}^{i1}(z,t) = \frac{\alpha_i A_0^2 \xi_0 \sqrt{\pi}}{2\beta}\left[\operatorname{erf}\left(\frac{z}{\xi_0} - \frac{v_{1g}t}{\xi_0}\right) - \operatorname{erf}\left((1-\beta)\frac{z}{\xi_0} - \frac{v_{1g}t}{\xi_0}\right)\right], \tag{39}$$

where $\operatorname{erf}[\phi]$ is the error function of argument ϕ. Superscript i on $A_{2x}^{i1}(z,t)$ denotes that the inertial ponderomotive force is also considered. Ratio of the second harmonic amplitude $A_{2x}^{i1}(z,t)$ to that of the laser, A_0, called the normalized amplitude, is

$$\left|\frac{A_{2x}^{i1}}{A_0}\right| = \frac{\alpha_i A_0 \xi_0 \sqrt{\pi}}{2\beta}\left[\operatorname{erf}\left(\frac{z}{\xi_0} - \frac{v_{1g}t}{\xi_0}\right) - \operatorname{erf}\left((1-\beta)\frac{z}{\xi_0} - \frac{v_{1g}t}{\xi_0}\right)\right]. \tag{40}$$

By using the definition of Poynting vector,

$$\vec{P}_i = \frac{c}{4\pi}\hat{\vec{E}} \times \hat{\vec{H}}_i^*, \tag{41}$$

the ratio of Poynting vectors associated with the second harmonic and fundamental waves is

$$\left|\frac{P_2^{i1}}{A_0}\right| = \left|\frac{\alpha_i A_0 \xi_0 \sqrt{\pi}}{2\beta}\left[\operatorname{erf}\left(\frac{z}{\xi_0} - \frac{v_{1g}t}{\xi_0}\right) - \operatorname{erf}\left((1-\beta)\frac{z}{\xi_0} - \frac{v_{1g}t}{\xi_0}\right)\right]\exp\left(-\left(\frac{(z-v_{1g}t)}{\xi_0}\right)^2\right)\right|^2. \tag{42}$$

When the inertial ponderomotive force $\rho(\vec{u}\cdot\vec{\nabla})\vec{u}$ is neglected, equation (34) is also valid, but the value α_i is given by

$$\alpha_w = \frac{i\Omega_p^2 e\Omega_c}{4c^2 m k_2}\left(\frac{7}{4}k_1 + k_0\right), \tag{43}$$

and the normalized amplitude of the second harmonic(without inertial force) A_{2x}^{w1}/A_0 is given by

$$\left|\frac{A_{2x}^{w1}}{A_0}\right| = \frac{\alpha_{wi} A_0 \xi_0 \sqrt{\pi}}{2\beta}\left[\operatorname{erf}\left(\frac{z}{\xi_0} - \frac{v_{1g}t}{\xi_0}\right) - \operatorname{erf}\left((1-\beta)\frac{z}{\xi_0} - \frac{v_{1g}t}{\xi_0}\right)\right]. \tag{44}$$

3. Discussions and conclusions

In this paper, we consider the effect of an inertial ponderomotive force $\rho(\vec{u}\cdot\vec{\nabla})\vec{u}$ on the efficiency of second harmonic generation. The sources of second harmonic generation are linear and nonlinear current densities. Self-consistent electric field $\hat{\vec{E}}_2^{\,1}$ produces velocity $\hat{\vec{u}}_2^1$ and a linear current density at $(2\omega, 2\vec{k}_1 + \vec{k}_o)$. $\hat{\vec{u}}_2^0$ and $\vec{B}_w(z)$; $\hat{\vec{u}}_1^0$ and $\vec{B}_1^1$, and $\hat{\vec{u}}_1^1$ and $\vec{B}_1^0$ beat to exert a ponderomotive force on electrons and imparts an oscillatory velocity $\hat{\vec{u}}_2^1$ at $(2\omega, 2\vec{k}_1 + \vec{k}_o)$. This velocity produces nonlinear current density $-n_0 e\hat{\vec{u}}_2^1$ at $(2\omega, 2\vec{k}_1 + \vec{k}_o)$. On the other hand, terms $\hat{\vec{E}}_1^{\,1}$ and $\hat{\vec{u}}_1^0 \times \hat{\vec{B}}_w$ produce a velocity $\hat{\vec{u}}_1^1$ and consequently $\hat{n}_1^1$, and a nonlinear current density $-n_0 e\hat{\vec{u}}_2^0\big/2$ at $(2\omega,\ 2\vec{k}_1 + \vec{k}_0)$. The normalized amplitude of second harmonic generation due to above sources is denoted by $A_{2x}^{w1}\big/A_0$.

In inertial term $\rho(\vec{u}\cdot\vec{\nabla})\vec{u}$, the velocities $\hat{\vec{u}}_1^0$ and $\hat{\vec{u}}_1^1$ beat to exert an acceleration $\frac{1}{2}\Big(\rho(\hat{\vec{u}}_1^0\cdot\vec{\nabla})\hat{\vec{u}}_1^1+\rho(\hat{\vec{u}}_1^1\cdot\vec{\nabla})\hat{\vec{u}}_1^0\Big)$ on electrons and imparts an oscillatory velocity $\hat{\vec{u}}_2^1$ at $(2\omega, 2\vec{k}_1+\vec{k}_o)$. This velocity produce a nonlinear current density at $(2\omega, 2\vec{k}_1+\vec{k}_o)$. The normalized amplitude of second harmonic generation due to total sources is denoted by A_{2x}^{i1}/A_0.

Assuming $n_0 = 10^{17}$ cm^3 gives $\omega_p = 1.8\times10^{13}$ Hz, $B_{ow} = 1.1\times10^5$ Guass, which gives $\omega_c = 1.95\times10^{12}$ Hz, $A_0 = 9.76\times10^6$ statvolt/m and temporal length short laser wave $\tau = 40$fs. From the foregoing, amplitudes A_{2x}^{i1}/A_0 and A_{2x}^{w1}/A_0 can be determined and are shown in Figure 1, denoted by solid and dashed curves, respectively. The amplitudes are plotted in terms of ω_p/ω for $z = 30$ μm and $t = 120$ fs. As indicated in Figure 1, the effect of inertial ponderomotive force is negligible to$\omega_p/\omega < 0.7$ but is considerable for $\omega_p/\omega > 0.7$. A_{2x}^{w1}/A_0 increases up to $\omega_p/\omega = 0.792$, then falls; while A_{2x}^{i1}/A_0 increases throughout the range. This means that, at laser pulse frequencies near the plasma frequency, the effect of inertial ponderomotive force is large (see Figure 2).

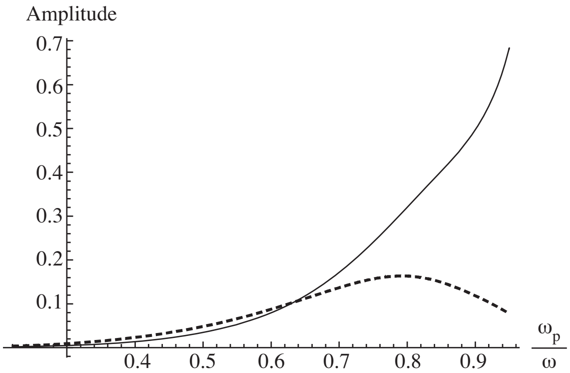

Figure 1. Diagram of normalized amplitudes A_{2x}^{i1}/A_0 and A_{2x}^{w1}/A_0, denoted by solid and dashed curves, respectively, in terms of $\Omega_p = \omega_p/\omega$ at distance $z = 30$ μm and time $t = 120$ fs.

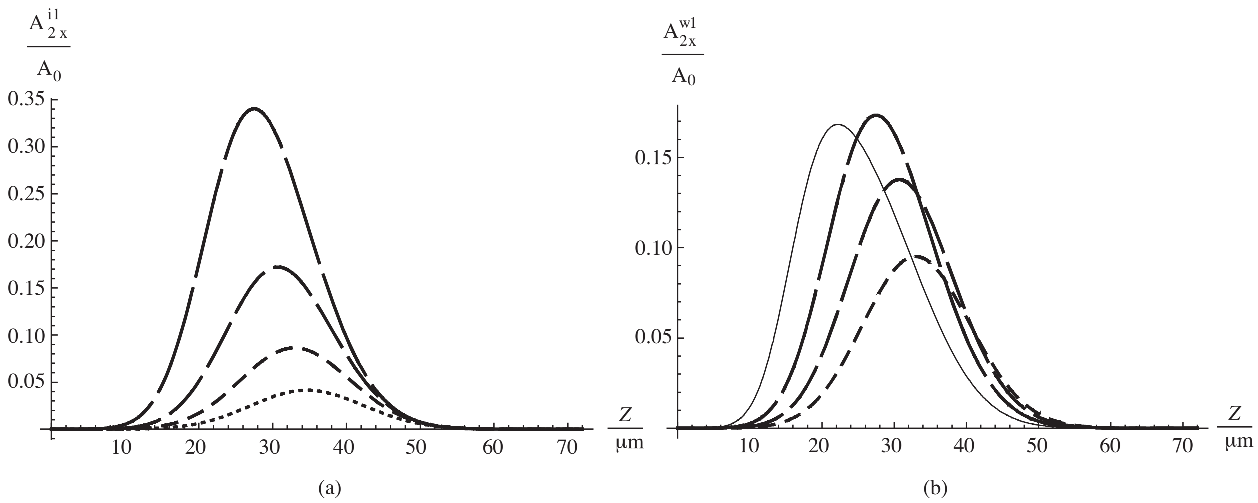

Figure 2. Diagrams of normalized amplitudes (a) A_{2x}^{i1}/A_0 and (b) A_{2x}^{w1}/A_0, in terms of z at time $t = 120$ fs for $\Omega_p = \omega_p/\omega = 0.5$, 0.6, 0.7, 0.8 and 0.792 (denoted, respectively, by increasingly dashed curves).

Electromagnetic wave propagation follows the linear dispersion relation $k=\omega\sqrt{\varepsilon}/c$, where ε is a dielectric coefficient. For an isotropic plasma, dielectric coefficient $\varepsilon=1-\omega_p^2/\omega^2$, and causes vector wave $kk$to increase more than linearly with frequency ω; hence $k_2>2k_1$ and the momentum conservation cannot be satisfied. The difference between of momentums $\hbar k_2$ and $2\hbar k_1$ can be provided to the second harmonic photon by virtual photon from the wiggler magnetic field with quantum energy 0 and momentum $\hbar\vec{k}_0$, so that $k_0=k_2-2k_1$. For

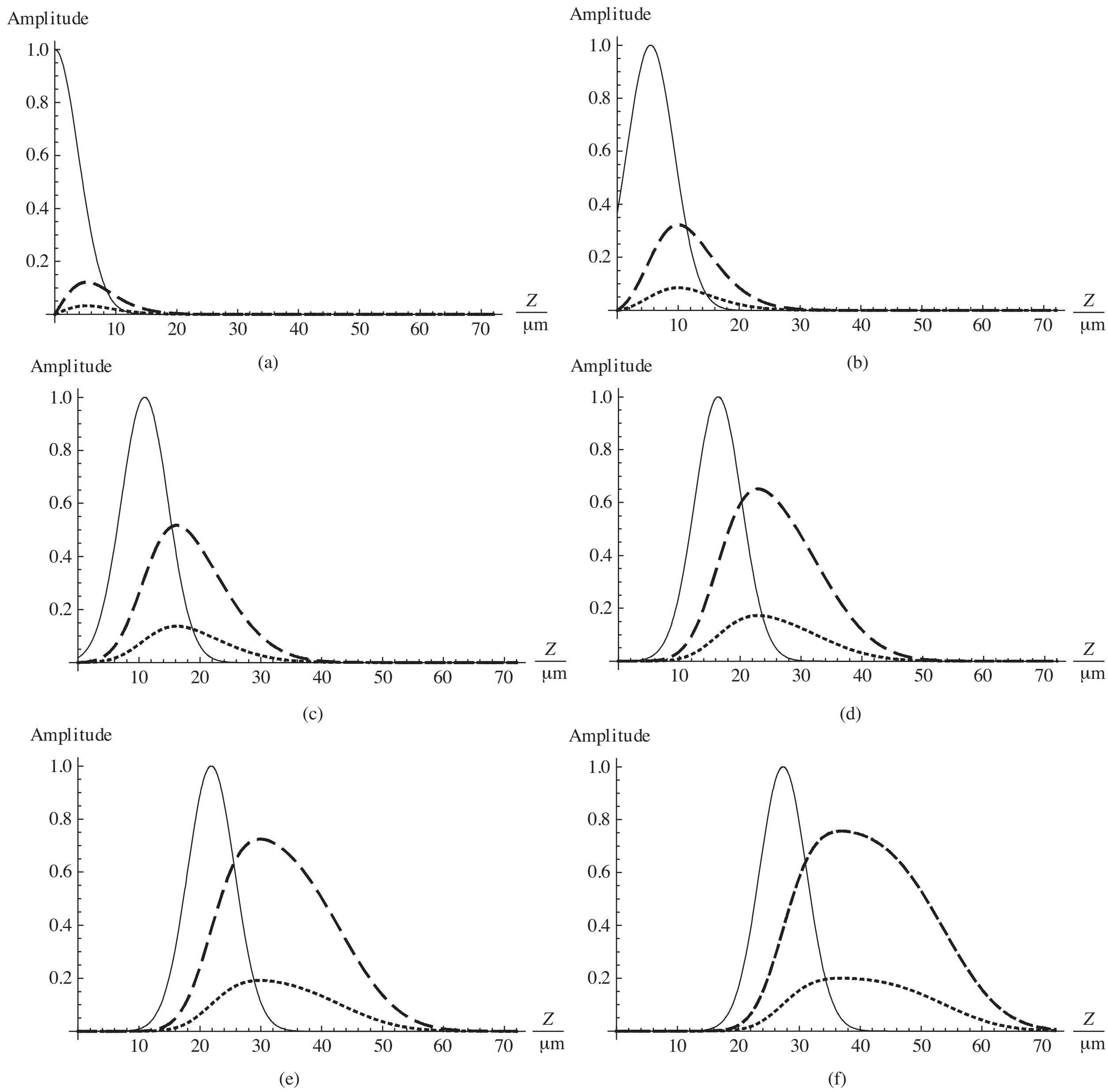

Figure 3. Diagrams of normalized amplitudes A_1/A_0, A_{2x}^{i1}/A_0 and A_{2x}^{w1}/A_0, denoted by solid, dotted and dashed curves, respectively, in terms of distance z for $\Omega_p=\omega_p/\omega=0.89$ and different values of (a) $t=0$, (b) $t=40$ fs, (c) $t=80$ fs, (d) $t=130$ fs, (e) $t=170$ fs, (f) $t=200$ fs.

example, k_0 has the value 594.925 cm^{-1} for $\omega \approx 2 \times 10^{13}$ Hz from, for example, a methanol laser. A wiggler magnetic field can be produced by propagating two identical plane waves in opposite directions.

The group velocity mismatch between the fundamental laser and the second harmonic is significant in high-density plasma. According to equations (16) and (35), the second harmonic's group velocity exceeds that of the fundamental ($v_{2g} = c\sqrt{1-\omega_p^2/4\omega^2} > v_{1g} = c\sqrt{1-\omega_p^2/\omega^2}$), which leads to the escape of the second harmonic from the fundamental laser pulse, limiting the yield of harmonic generation and its amplitude saturates. The effect becomes more important as plasma density increases. Normalized amplitudes A_1^0/A_0, A_{2x}^{i1}/A_0 and A_{2x}^{w1}/A_0, denoted by solid, dashed and dotted curves, respectively, are plotted in term of distance z for different values of time t in Figures 3(a–f). In Figure 3(a), at $t = 0$, the second harmonic pulse, which has a small amplitude, is generated in the domain of the fundamental laser pulse. In Figure 3(b), one can see the second harmonic's amplitude has increased by $t = 40$fs. Figure 3(c) shows amplitude of the second harmonic increased further at $t = 80$ fs, with the pulse now slipping beyond the fundamental laser pulse. For $t > 130$

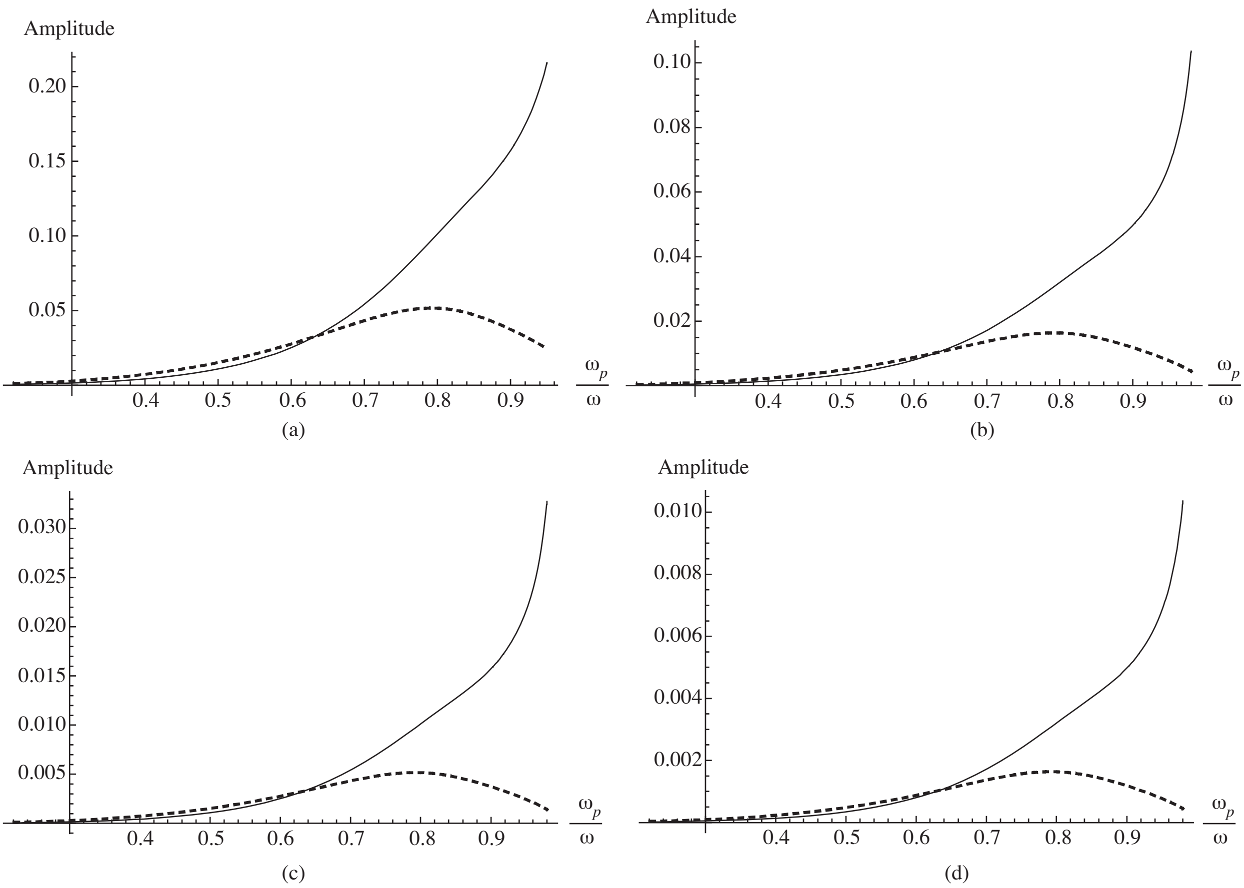

Figure 4. Diagram of normalized amplitudes A_{2x}^{i1}/A_0 and A_{2x}^{w1}/A_0, denoted by solid and dashed curves, respectively, in terms of $\Omega_p = \omega_p/\omega$ at distance $z = 30$ μm and time t=120 fs for (a) $n_0 = 10^{18}\,\mathrm{cm}^{-3}$ and $\omega_w^{\max} \approx 7 \times 10^{13}$ Hz; (b) $n_0 = 10^{19}\,\mathrm{cm}^{-3}$ and $\omega_w^{\max} \approx 2 \times 10^{14}$ Hz; (c) $n_0 = 10^{20}\,\mathrm{cm}^{-3}$ and $\omega_w^{\max} \approx 7 \times 10^{14}$ Hz; (d) $n_0 = 10^{21}\,\mathrm{cm}^{-3}$ and $\omega_w^{\max} \approx 7 \times 10^{15}$ Hz.

fs, as shown in Figures 3(d–f), the generated pulse slips out of the main laser pulse with unchanged amplitude, but with increased pulse duration.

Efficiency of second harmonic generation depends on laser and plasma frequencies. Increasing plasma frequency (electron number density) reduces efficiency. In Figure 4 normalized amplitudes A_{2x}^{i1}/A_0 and A_{2x}^{w1}/A_0 are plotted in terms of ω_p/ω at $z = 30$ μm and $t = 120$ fs for four values of number density, $\mathrm{n}_0 = 10^{18}$, 10^{19}, 10^{20} and 10^{21} cm^{-3}. As such, effect of the inertial ponderomotive force is negligible to $\omega_p/\omega < 0.7$ but is considerable for $\omega_p/\omega > 0.7$. While A_{2x}^{w1}/A_0 increases up to $\omega_p/\omega = 0.792$, and then reducing beyond, A_{2x}^{i1}/A_0 increases for any range.

For $\omega_p/\omega = 0.792$ and $\mathrm{n}_0 = 10^{18}$, 10^{19}, 10^{20} and 10^{21} cm^{-3}, respectively, the following laser frequencies are obtained: $\omega_w^{\max} \approx 7 \times 10^{13}$Hz, $\omega_w^{\max} \approx 2 \times 10^{14}$Hz, $\omega_w^{\max} \approx 7 \times 10^{14}$Hz, and $\omega_w^{\max} \approx 7 \times 10^{15}$Hz, for such devices such as InGaAsP (808 nm), Nd:Yag (1.03 μm) and Er:Yag (2.94 μm) lasers. The superscript “max” denotes the frequency at which A_{2x}^{w1}/A_0 is maximum.

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References

[1] P. A. Franken, A. E. Mill, C. W. Peters, and G. Weinreich, *Phys. Rev. Lett.*, **7**, (1961), 118.

[2] T. Ishizawa, T. Kanai, T. Ozaki, and H. Kuroda, *IEEE J Quant. Electro*, **37**, (2001), 384.

[3] Q. H. Park, J. E. Boyd, J. E. Sipe and A. L. Gaeta, *IEEE J. Selected topics in Quant. Electro*, **8**, (2002), 413.

[4] S. C. Wilks, W. L. Kruer, W. B. and Mori, *IEEE trans. Plasma sci.*, **21**, (1993), 120.

[5] J. M. Rax and N. J. Fisch, *IEEE Trans. Plasma Sci.*, **21**, (1993), 105.

[6] S. Shibu and V. K. Tripathi, *Phys. Lett. A*, **239**, (1998), 99.

[7] R. W. Boyd, 2003 *Nonlinear Optics*, (Elsevier Science, 2007), p. 79.

[8] R. N. Agrawal, B. K. Pandey and A. K. Sharma, *Phys. Scr.*, **63**, (2001), 243.

[9] Z. Weissman, A. Hardy, M. Katz, M. Oron and D. Eger, *Opt. Lett.,* **20**, (1995), 674.

[10] Y. J. Ding, J. U. Kang and J. B. Khurgin, *IEEE J. Quant.*, **34**, (1998), 966.

[11] M. Singh, A. P. Jain and J. Parashar, *J. Indian. Inst. of Sci.*, **82**, (2002), 183.

[12] K. P. Singh, V. L. Gupta and V. K. Tripathi, *Optic Commnications*, **226**, (2003), 377.

[13] K. P. Singh, D. N. Gupta, S. Yadav and V. K. Tripathi, *Phys. Of plasma*, **12**, (2005), 013101.

[14] Nitikant and A. K. Sharma, *J. Phys. D: Appl. Phys.*, **37**, (2004), 998.

[15] Nitikant and A. K. Sharma, *J. Phys. D: Appl. Phys.,* **37**, (2004), 2395.

[16] B. Ersfeld, R. Bischof1 and D. A. Jaroszynski1, 28 June-2 July 2004 31^{st} *conference on plasma phys. London* **28G** (2001), 5-15.

[17] Ya. N. Istomin, *Physics letters A,* **299**, (2002), 248.

[18] J. A. Bittencourt, *Fundamental of Plasma Physics,* (Pergamom Press, 1986), p. 227.

[19] J. D. Jakson, *Classical Electrodynamics*, (Reprint of the 3^{rd} John Wiley & Sons 1999), p. 320.