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Investigation of The Multipolarity of Electromagnetic Transitions in ^{88,90}Kr Nuclei

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Abstract

We have determined the most appropriate Hamiltonian that is needed for present calculations of nuclei about the A≅80 region by the view of Interacting Boson Model-2 (IBM-2). After obtaining the best Hamiltonian parameters, level energies and B(E2) probabilities of some transitions in ^{88,90}Kr nuclei were estimated. Results are compared with previous experimental and theoretical data and it is observed that they are in good agreement. Finally, $R_1 = \frac{B(E2;4_1 \to 2_1)}{B(E2;2_1 \to 0_1)}$, $R_2 = \frac{B(E2;2_2 \to 2_1)}{B(E2;2_1 \to 0_1)}$, $R_3 = \frac{B(E2;0_2 \to 2_1)}{B(E2;2_1 \to 0_1)}$, $R_4 = \frac{B(E2;2_2 \to 0_1)}{B(E2;2_2 \to 2_1)}$, $R_5 = \frac{B(E2;3_1 \to 2_1)}{B(E2;3_1 \to 4_1)}$, $R_6 = \frac{B(E2;4_2 \to 4_1)}{B(E2;4_2 \to 2_2)}$ and $R_7 = \frac{B(E2;4_1 \to 2_1)}{B(E2;2_2 \to 2_1)}$ ratios are compared with the values of dynamic symmetry limits. (SO(6), SU(5), SU(3)).

Key Words: Electromagnetic transition, multipolarity, Interacting Boson Model-2 (IBM2), deformation parameters.

1. Introduction

The mass region of A ≅ 80 is a new region of neutron-excess nuclei. In view of the growth of this kind of theoretical interest, the Interacting Boson Model-2 (IBM-2) is one of those attemps that has been successful in describing the low-lying nuclear collective motion in medium and heavy mass nuclei [1,2].

The aim of this study is to carry out some doubly even Kr nuclei, which are around the mass region A ≅ 80, and to provide a clear description of their structure in the dynamic symmetry limits of IBM. Therefore, we have carried out a microscopic study of the energy levels and some selected transition probabilities of B(E2) for the ^{88,90}Kr nuclei.

In Section 2, we present the calculational framework. Section 3 contains the comparison of the estimated B(E2) transition probabilities of some transitions in ^{88,90}Kr, with available experimental and theoretical results. In this last section, the investigated isotopes of even-even Kr nuclei are set up in the dynamic symmetry limits of IBM and some concluding remarks of the study are given.

2. The Model

In nuclear structure, structural changes have been proposed to be related to exceptionally strong neutron-proton interaction. It is also suggested that the neutron-proton effective interactions have a deformation-producing tendency, while the neutron-neutron and proton-proton interactions are of a spheriphying nature [3,4].

Within the region of medium-heavy and heavy nuclei, a large number of nuclei exhibit properties that are neither close to anharmonic quadrupole vibrational spectra nor to deformed rotors [5]. The standard description of these phenomena has been given in terms of nuclear triaxiality [6], going from rigid triaxial shapes to more soft potential energy surfaces, when describing such nuclei in a geometric description [7]. Within the Interacting Boson Model, when no distinction is made between proton and neutron variables (IBM-1) [8], triaxiality can be described explicitly, through the introduction of cubic terms in the boson operators [9,10]. This is in contrast to the recent work of Dieperink and Bijker [11,12] who showed that triaxiality also occurs in particular dynamic symmetries of the IBM-2, which does distinguish between protons and neutrons.

According to A.Arima et. al. [13], the IBM Hamiltonian takes different forms depending on the regions (SU(5), SU(3), SO(6)) of the traditional IBM triangle. The Hamiltonian that we consider is in the form of [9],

$$H = H_{sd} + \Sigma\theta_L [d^+ d^+ d^+]^{(L)} [d^\sim d^\sim d^\sim]^{(L)} \tag{1}$$

where H_{sd} is the standard Hamiltonian of the IBM [14,15],

$$H_{sd} = \in_d \eta_d + \kappa Q \cdot Q + \kappa' L \cdot L + \kappa'' P^+ \cdot P + q_3 T_3 \cdot T_3 + q_4 T_4 \cdot T_4 \tag{2}$$

It is indicated that the Hamiltonian is not diagonal in any of the IBM chains, but is a mixture of the SU(5), SU(3) and SO(6) chains. Note that L^2 does not commute with all the generators of SO(5). In the IBM-2 model, the neutron's and proton's degrees of freedom are explicitly taken into account. Thus the Hamiltonian [16] can be written as,

$$H = \varepsilon_v n_{dv} + \varepsilon_\pi n_{d\pi} + \kappa Q_\pi . Q_v + V_{\pi\pi} + V_{vv} + M_{\pi v} \tag{3}$$

$$n_{d\rho} = d^+ d^\sim, \rho = \pi, \nu \tag{4}$$

where $n_{d\rho}$ is the neutron (proton) d-boson number operator. The rest of the operators in equation (3) are defined as

$$Q_\rho = (s_\rho^+ d \sim_\rho + d_\rho^+ s_\rho) + \chi_\rho (d_\rho^+ d_\rho^\sim)$$

$$V_{\rho\rho} = \sum_{L=0,2,4} C_{L\rho} ((d_\rho^+ d_\rho^+)^{(L)} (d_\rho^+ d_\rho^\sim)^{(L)})^{(0)}; \quad \rho = \pi, \nu \tag{5}$$

where $s_\rho^+, d_{\rho m}^+$, and s_ρ, $d_{\rho m}$ represent the s- and d-boson creation and annihilation operators, and

$$M_{\pi v} = \sum_{L=1,3} \xi_L (d_v^+ d_\pi^+)^{(L)} (d_v d_\pi)^{(L)} + \xi_2 (s_v d_\pi^\sim - s_\pi d_v^\sim)^{(2)} . (s_v^+ d_\pi^+ - s_\pi^+ d_v^+)^{(2)} \tag{6}$$

where $d^{\sim}_{\rho m}$= (-1)$^m d_{\rho-m}$. In this case, $M_{\pi v}$ affects only the position of the non-fully symmetric states relative to the symmetric ones. For this reason, $M_{\pi v}$ is often referred to as the Majorana force [16]. The rule of choice for the total angular momentum is given as follows:

$$|J_i - J_f| \leq L\gamma \leq |J_i + J_f| \tag{7}$$

$T(E2; J_i \rightarrow J_f)$ is the number of E2 transitions per second, from $J_i \rightarrow J_f$. The electric quadropole (E2) transition operator is an important factor within the collective nuclear structure. In IBM, the general linear E2 transition operator, with L=2 for one body, is given by,

$$\begin{aligned} T(E2) &= e_B Q & \text{in} \quad IBM-1 \\ T(E2) &= e_\pi Q_\pi + e_\nu Q_\nu & \text{in} \quad IBM-2 \end{aligned} \tag{8}$$

where e_B, e_π, and e_ν are effective boson charges. Below we show how the $B(E2;\ J_i \rightarrow J_f)$ prescription is implemented.

$$\begin{aligned} B(E2; J_i \rightarrow J_f) &= \sum_{mM'} |< J_f M'|T(E2)m|J_i M >|^2 \\ B(E2; J_i \rightarrow J_f) &= \frac{1}{2J+1}|< J_f||T(E2)||J_i >|^2 \end{aligned} \tag{9}$$

3. Results and Discussion

In this paper, we have presented a phenomenological analysis of B(E2) of some selected transitions in ^{88}Kr and ^{90}Kr nuclei in terms of the neutron-proton IBM model. The estimated energy levels are generally in good agreement with the experiment. Although the energy spectrums of ^{88}Kr and ^{90}Kr display vibrational-like structures, the use of the complete IBM-2 Hamiltonian shows some SO(6) behaviors. The wave functions obtained by diagonalization of the IBM-2 Hamiltonian have been used by the program PHINT[17] to estimate the reduced transition probabilities of E2 transitions.

The isotopes ^{88,90}Kr have $N_\pi = 4$, and N_ν varies from 1 to 2. In addition, the parameters κ, χ_ρ and ε, as well as $C_{L\rho}$, with L=0,2,4, were treated as free parameters and their values were estimated by fitting them to the measured level energies. This procedure was made by selecting the “traditional” values of parameters and then allowing one parameter to vary while keeping the others held constant until a best fit was obtained. This was carried out iteratively until an overall fit was achieved. The best fit values for the Hamiltonian parameters are given in Table 1 and the estimated energy levels are compared with the experimental data, which are shown in Table 2 (^{88}Kr) and in Table 3 (^{90}Kr).

Table 1. The best fit values of the Hamiltonian parameters for ^{88,90}Kr.

^A_ZX	N_π	N_ν	N	ε	κ	χ_ν	χ_π	$C_{L\nu}$	$C_{L\pi}$
$^{88}_{36}Kr_{52}$	4	1	5	0.930	-0.100	0.60	-1.20	0.18	0.18
$^{90}_{36}Kr_{54}$	4	2	6	0.860	-0.990	0.58	-1.20	0.17	0.17

The estimated energy levels in Table 2 and in Table 3 are generally in good agreement with the experiment. The estimated values of $E(4_1^+)/E(2_1^+)$ ratio for both isotopes in the Tables are equal to 2.12 and 3.00. The value of $R_{4/2}$ ratio has the limiting value 2 for a quadrupole vibrator, 2.5 for a non-axial gama-soft rotor, and 3.33 for an ideally symmetric rotor.

Table 2. The comparison of estimated energy levels with the experiment for ^{88}Kr.

Isotope	Spin Parity (I^{π})	IBA energies (MeV)	experiment energies (MeV) [18,21,22]
$^{88}_{36}Kr_{52}$	2_1^+	0.779	0.775
	4_1^+	1.650	1.644
	6_1^+	2.608	-
	8_1^+	3.651	-
	10_1^+	4.777	-
	2_3^+	2.807	2.216
	3_1^+	2.581	2.342
	4_3^+	3.602	2.550
	2_2^+	1.640	1.577
	4_2^+	2.591	2.420
	6_2^+	3.625	-
	8_2^+	4.741	-
	0_2^+	1.808	2.370
	0_3^+	2.576	2.776

Table 3. The comparison of estimated energy levels with the experiment for ^{90}Kr.

Isotope	Spin Parity (I^{π})	IBA energies (MeV)	experiment energies (MeV) [18,21,22]
$^{90}_{36}Kr_{54}$	2_1^+	0.711	0.707
	4_1^+	2.163	2.148
	6_1^+	4.309	-
	8_1^+	7.141	-
	10_1^+	10.676	-
	2_3^+	6.559	-
	3_1^+	4.513	-
	4_3^+	7.127	-
	5_1^+	6.954	-
	6_3^+	10.232	-
	7_1^+	10.133	-
	9_1^+	14.075	-
	2_2^+	3.124	-
	4_2^+	5.007	-
	6_2^+	7.479	-
	8_2^+	10.650	-
	10_2^+	14.578	-

To have B(E2) probabilities of some selected transitions in $^{88}_{36}Kr_{52}$ and $^{90}_{36}Kr_{54}$ nuclei, the best fitted values of e_{π} and e_{ν} are obtained. The values of e_{π} are 0.0890 and 0.0850 for $^{88}_{36}Kr_{52}$ and $^{90}_{36}Kr_{54}$, respectively. Moreover, the e_{ν} values are equal to 0.910 for $^{88}_{36}Kr_{52}$ and 0.0830 for $^{90}_{36}Kr_{54}$. The estimated B(E2) values for the nuclei ^{88}Kr and ^{90}Kr can't be compared with the experimental and theoretical results in Table 4, because no theoretical and experimental B(E2) values exist for ^{88}Kr and ^{90}Kr. Therefore, only estimated B(E2) values of the present study are shown in the Table. During all of the transitions from 2_1^+,0_2^+, 2_2^+ and 4_1^+ states, B(E2) values indicate some collectivity, but not an overwhelming contribution.

Table 4. The estimated (B(E2)) probabilities of the present study for ^{88,90}Kr. Experimental and theoretical results do not exist.

Isotope	$J_i^+ \rightarrow J_s^+$	B(E2) (e^2b^2)
$^{88}_{36}Kr_{52}$	$2_1^+ \rightarrow 0_1^+$	0.0466
	$0_2^+ \rightarrow 2_1^+$	0.0514
	$2_2^+ \rightarrow 0_1^+$	0.0002
	$2_2^+ \rightarrow 2_1^+$	0.0715
	$3_1^+ \rightarrow 2_1^+$	0.0002
	$3_1^+ \rightarrow 4_1^+$	0.0225
	$4_1^+ \rightarrow 2_1^+$	0.0699
	$4_2^+ \rightarrow 2_2^+$	0.0400
	$4_2^+ \rightarrow 4_1^+$	0.0368
	$6_1^+ \rightarrow 4_1^+$	0.0748
$^{90}_{36}Kr_{54}$	$2_1^+ \rightarrow 0_1^+$	0.0643
	$0_2^+ \rightarrow 2_1^+$	0.0000
	$2_2^+ \rightarrow 0_1^+$	0.0142
	$2_2^+ \rightarrow 2_1^+$	0.0469
	$3_1^+ \rightarrow 2_1^+$	0.0191
	$3_1^+ \rightarrow 4_1^+$	0.0261
	$4_1^+ \rightarrow 2_1^+$	0.0873
	$4_2^+ \rightarrow 4_1^+$	0.0336
	$4_2^+ \rightarrow 2_2^+$	0.0654

Some B(E2) transition ratios of ^{88}Kr and ^{90}Kr isotopes are taken as $R_1 = \frac{B(E2; 4_1 \rightarrow 2_1)}{B(E2; 2_1 \rightarrow 0_1)}$, $R_2 = \frac{B(E2; 2_2 \rightarrow 2_1)}{B(E2; 2_1 \rightarrow 0_1)}$, $R_3 = \frac{B(E2; 0_2 \rightarrow 2_1)}{B(E2; 2_1 \rightarrow 0_1)}$, $R_4 = \frac{B(E2; 2_2 \rightarrow 0_1)}{B(E2; 2_2 \rightarrow 2_1)}$, $R_5 = \frac{B(E2; 3_1 \rightarrow 2_1)}{B(E2; 3_1 \rightarrow 4_1)}$, $R_6 = \frac{B(E2; 4_2 \rightarrow 4_1)}{B(E2; 4_2 \rightarrow 2_2)}$ and $R_7 = \frac{B(E2; 4_1 \rightarrow 2_1)}{B(E2; 2_2 \rightarrow 2_1)}$ and then the estimated ratios are compared with the SU(5),SO(6),SU(3) dynamical symmetry limits in Table 5.

Table 5. Comparison of R_1, R_2, R_3, R_4, R_5, R_6, and R_7 ratios of ^{88}Kr and ^{90}Kr isotopes to the IBM. symmetry ratio.

Limit and Nucleus	R_1	R_2	R_3	R_4	R_5	R_6	R_7
SU(5)[a]	2.00	2	2.00	0.011	0.060	0.72	1.0
SU(3)[a]	1.60	0.02	0.00	0.700	2.500	0.03	6.93
SO(6)[a]	1.60	0.79	0.00	0.070	0.120	0.75	1.84
$^{88}_{36}Kr_{52}$	1.50	1.53	1.10	0.003	0.009	0.92	0.98
$^{90}_{36}Kr_{54}$	1.36	0.73	0.00	0.300	0.732	0.51	1.86

[a][23]

The shape transition predicted by this study is consistent with the spectroscopic data for these nuclei. ^{88}Kr and ^{90}Kr are typical examples of isotopes that exhibit a smooth phase transition from vibrational nuclei to soft triaxial rotors. The comparison of some B(E2) ratios of ^{88}Kr and ^{90}Kr nuclei with that of SU(3),SU(5) and SO(6) limits show that these nuclei exist along the SU(5)-SO(6) side of the IBM triangle. That is, they exist around the closed shell and lie along the SU(5)-SO(6) proportion of the IBM triangle with a tendency to SO(6) symmetry. As a result, the use of the complete Hamiltonian shows that vibrational features are dominant in ^{88}Kr and ^{90}Kr, but with the presence of some SO(6) characteristics.

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