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Contribution to the Relationship Between Solar Radiation and Sunshine Duration in the Tropics: A Case Study of Experimental Data at Ilorin, Nigeria

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Abstract

From monthly average daily values of clearness index and relative sunshine spanning a period of two years, two equations (linear and quadratic) of the Angstrom-Prescott model have been developed for the estimation of global solar irradiation at Ilorin (Lat. $8°32'$N, Long. $4°34'$E), a tropical location. Comparison of the regression coefficients of the linear type model showed that the values are quite consistent with those quoted in the literature for the Tropics.

The predictive efficiencies of these two models are also compared with those which are believed to be applicable globally, and those developed for the Nigerian environment. Although some of the models seem fairly adequate for this location, their predictive efficiencies in terms of Mean bias error (MBE), mean absolute bias error (MABE) and root mean square error (RMSE) are poorer than those models developed in this study. Everything, therefore, seem to point to the fact that the Angstrom-Prescott model, whether in a linear or quadratic form, is locality dependent.

1. Introduction

Development of a solar energy research program must always start with a study of solar radiation data at a site or region of interest. Long-term measurements of solar radiation on a horizontal surface exists for only relatively few meteorological stations. For places where it is not directly measured, solar radiation can be estimated by using models and empirical correlations. However, the computational complexity and associated time and input data requirements discourage many researchers and users from basing their calculations of solar energy irradiation on models which have strong links to the fundamental radiative equation. Rather, they are encouraged by simplicity and expediency of calculations using empirically based methods.

While it is appreciated that a number of commonly measurable atmospheric and meteorological parameters such as turbidity, relative humidity, degree of cloudiness, temperature and sunshine duration taken severally or jointly, affect the magnitude of the global irradiation incident on a given location, the preponderance of data now clearly, perhaps incontestably, point to the fact that the greatest influence is exerted by sunshine hours.

Several investigators [1-13] have demonstrated the predictive ability of the Angstrom-type one-parameter equation correlating the global solar radiation to the percentage of bright sunshine hours in a simple linear regression form. The Angstrom-type equation, for history sake, according to Martinez-Lazano et al [14] and Gueymard et al [15], should be called the Angstrom-Prescott formula. This name will be retained here. A maximum-likelihood quadratic fit was later employed by Ogelman et al [6] to estimate monthly global solar radiation. This quadratic fit method has been utilised by a number of authors like Akinoglu and Ecevit [16] and Fagbenle [17] to estimate global solar radiation. The maximum-likelihood quadratic fit method according to Fagbenle appears to widen the range of applicability of the one-parameter correlation to cover climatologically different zones of the same geographical region. All these empirical models have been shown severally to work reasonably well on a daily or longer-term basis.

In this work, the Angstrom-Prescott one-parameter model in linear and quadratic forms are developed and compared in terms of their predictive abilities for the Ilorin location. The predictive efficiencies of these two equations will be compared with other sunshine-based models of linear [3,5] and quadratic [6,16] forms. The results from this study will also be compared with results obtained at other locations in Nigeria (in particular) and the Tropics (generally).

2. Equations Used and Data Base

Clearness index K_T(=H/H_0) gives the percentage deflection by the sky of the incoming global radiation and therefore indicates both the level of availability of solar radiation and changes in atmospheric conditions in a given locality while relative sunshine S(=n/N) is a measure of the cloud cover. Here, H represents the daily global radiation, H_0 the daily extraterrestrial radiation, n the daily sunshine duration and N the maximum sunshine duration or day-length. H_0 and N were evaluated according to equations reported in lqbal [18].

The analysis is based on data collected at Ilorin (latitude: 8°32′N; longitude: 4°34′E; altitude: 375m) for a two-year period (September 1992 – August 1994).

It has to be pointed out that, although a two-year data seems a bit inadequate for major climatological conclusions to be drawn, however, extensive comparison of some meteorological parameters in the West African sub-region as reported in Udo [19] has shown that, in this case, it is quite representative of Ilorin climate. Measurement of global solar irradiation H are made according to International Scale of Pyrheliometry, 1976, using Eppley Precision Spectral Pyranometer (PSP) complete with a data acquisition system, a CR10 Campbel data logger, and a storage module. The calibration factor for the PSP was provided by the manufacturer (Eppley Laboratory) at the beginning and World Radiation Centre, Davos at the end of data acquisition, respectively. Other details concerning this and other radiometers installed at this station and methods of data acquisition are described in Udo and Aro [20] and Udo and Aro [21]. The daily sunshine duration n was obtained from the Nigerian Meteorological Services, Oshodi, Lagos and is for Ilorin International Airport (a distance of about 12 km NW from the project site). This distance is within the distance which radiation-sunshine data can be reliably extrapolated [22-23], n was measured by Campbell-Stokes sunshine recorder with heat sensitive paper.

3. Angstrom – Prescott Equations

3.1 *Linear*: The linear form of the Angstrom – Prescott-one- parameter formula is written as:

$$\begin{aligned} H/H_0 &= a + b(n/N) \\ \text{i.e.}\quad K_T &= a + bS, \end{aligned} \tag{1}$$

for which Prescott originally proposed $a = 0.22$ and $b = 0.54$.

Although the Angstrom-Prescott coefficients a and b, are considered to be pure constants by some researchers, they are considered to depend on S, κ (latitude) or seasonal variation by others.

Similar one-parameter linear equations to be considered for comparison are:

(i) Rietveld.

Rietveld [5] examined several published values of a and b and noted that a is related linearly and b hyperbolically to the mean value of S such that

$$\begin{aligned} a &= 0.1 + 0.24S \\ b &= 0.37 + 0.08/S. \end{aligned}$$

Substituting a and b in Eq. (1), one gets

$$K_T = 0.18 + 0.62S. \tag{2}$$

Equation (2) is believed to be applicable anywhere in the world and yields superior results for cloudy conditions, for $S < 0.4$ [18]

(ii) Glover and McCulloch

Clover and McCulloch [3] attempted to introduce the latitude dependency to one of the Angstrom – Prescott coefficients and presented the following

$$K_T = 0.29 \cos \kappa + 0.52S, \kappa < 60° \tag{3}$$

3.2 *Maximum-likelihood quadratic*: The maximum likelihood quadratic equations to be considered are of the form:

$$K_T = a + bS + cS^2 \tag{4}$$

where a, b and c are the constant coefficients.

The models to be considered for comparison under this form are:

(i) Ogelman et al.

Ogelman et al [6] proposed the use of a correlation which relates the global solar radiation to S in a quadratic form as

$$K_T = 0.195 + 0.675S - 0.142S^2. \tag{5}$$

This fit was for daily data spread over a three – year period at Ankara and Adana, Turkey.

(ii) Akinoglu and Ecevit

Akinoglu and Ecevit [16] suggested a quadratic correlation between K_T and S to estimate the values of global solar radiation for 58 locations displaced in several countries. This equation, whose coefficients have the same values, respectively for all tested locations is:

$$K_T = 0.145 + 0.845S - 0.280S^2 \tag{6}$$

4. Results And Discussions

4.1. Equations developed in this study

The relevant meteorological and solar radiation data like H, H_0, n and N used in this study are presented in Table 1. The values of H and n are the two-year averages. Details of daily and annual (including seasonal cycles) clearness index K_T and relative sunshine S are reported in Udo [19] while seasonal variation of H for the two year period is reported in Udo and Aro [20].

Table 1. Relevant meteorological and solar radiation data H, H_0, n, N) for Ilorin.

MONTH	H	H_0	n	N
JAN.	16.0	32.7	6.3	11.3
FEB.	19.0	35.1	6.7	11.7
MAR.	20.2	37.8	7.7	12.0
APR.	19.5	37.8	7.0	12.2
MAY.	18.7	37.2	7.0	12.4
JUN.	17.1	36.5	6.3	12.5
JUL.	13.9	36.7	4.1	12.5
AUG.	13.7	37.2	3.9	12.3
SEPT.	15.7	37.1	4.4	12.1
OCT.	18.1	35.6	6.6	11.8
NOV.	17.8	33.3	7.7	11.6
DEC.	16.3	31.9	7.4	11.5

It is worth pointing out at this stage that regression models are used for prediction or estimation, data description, parameter estimation, and control. Model validation provides a measure of protection for both model developer and user. There are several methods in use for validation of models which include (i) fresh data collection, (ii) data splitting or cross-validation and (iii) data from planned experiments. In this study the second method was used mainly due to its simplicity as regard availability of data for validation and due to the obvious constraints imposed by the other two.

Accordingly a regression equation was established using 1993 data set (monthly mean of daily value of K_T and S) as the estimation data set and 1994 data set as the prediction data set. A second regression was established with the roles of the data sets reversed. This is the double cross-validation method [24].

The regression models obtained with their corresponding correlation coefficients R were:

$$K_T = 0.24 + 0.46S, R = 93.50\% \tag{7}$$

and

$$K_T = 0.22 + 0.48S, R = 94.16\% \tag{8}$$

for 1993 and 1994 data sets, respectively.

Using Eq. (7) to predict 1994 data and Eq. (8) to predict 1993 showed that the agreement with measurements is better than 8%. However, there were slight tendencies of 1993 and 1994 models to overestimate and underestimate respectively. Specifically, mean bias error (MBE) mean absolute bias error (MABE) and root mean square error (RMSE, meanings are as given by Akinoglu and Ecevit, 1990 [25]) values for 1993 and 1994 models were: 0.3583, 0.7417, 0.8732 and –0,6750, 0.8417, 0.9674, respectively. Although the 1993 model seem to perform slightly better than the 1994 model considering the above error values, the differences were small and hence it can be argued that the method of data splitting and its accompanying double cross-validation technique introduced no difficulties since the estimation data set and the prediction data set did not differ much in predictive performance and coefficient of estimates.

Since the models developed from the estimation data sets were satisfactory predictors, one way to improve the precision of estimation was to re-estimate the coefficients using the entire data: in this case, the two-year monthly average. The regression model with its correlation coefficient was:

$$K_T = 0.23 + 0.48S, R = 95.89\%. \tag{9}$$

With the high coefficients of determination R^2 of 0.874, 0.887 and 0.920 for Eq. (7), (8) and (9) it means that there is not much difference between the estimated and actual K_T values. Also, with the corresponding high R, it means that the degree of linear statistical relation in the sample observations are quite satisfactory. Further analysis showed that at 95% confidence level, the regression models are highly significant. Hence the regression models, especially Eq.(9), are useful in predicting monthly mean daily global solar radiation at Ilorin and other locations with the same agro-climatic conditions, if values of monthly mean relative sunshine S are known.

On comparing Eqs. (1) and (9), it means that a which in this case has a value of 0.23, is a measure of global solar radiation received at the ground through an overcast sky as a fraction of the extraterrestrial radiation – the transmissivity of an overcast atmosphere. The coefficient b, which has a value of 0.48, expresses the rate of increase of K_T with increase of S. For a clear sky, S becomes unity and the parameter $(a + b)$, which in this case is 0.71, is a fraction of H that reaches the earth's surface; that is, transmissivity for H under perfectly clear sky conditions [26,27].

Carrying out the analysis based on seasons, dry and rainy, rather led to a poor correlation coefficient for the dry season period (November – April) probably due to the large differences in the characteristic of the sky conditions during this period. The corresponding equations with their respective correlation coefficients were:

$$K_T = 0.38 + 0.24S, R = 52.22\% \tag{10}$$

and

$$K_T = 0.22 + 0.51S, R = 98.27\% \tag{11}$$

for dry and rainy season periods. Dry season at Ilorin consists of mainly two distinct periods: (i) Harmattan period (December and January) when cold and dust laden north – easterly trade winds from the Sahara desert keep the atmosphere over Ilorin and its environs heavily overcast with dust for many days, with characteristic hazy and cloud free weather conditions (ii) the cloud and dust – free period (November, February to April) of mainly high irradiation and clear weather conditions. The rainy season model was quite satisfactory. Other details concerning the geography of the project site are reported elsewhere [20,21]. The two equations are highly informative: as expected the coefficient a for the dry season is higher than that for rainy season. However, for estimation purpose, the non-seasonal model (Eq.9) is recommended.

The corresponding quadratic model for the two-year period was:

$$K_T = 0.053 + 1.28S - 0.83S^2, R = 96.97\% \tag{12}$$

4.2. Comparison of Coefficients

It is important to examine the regression equations developed here and compare them with others in terms of a, b, and $(a + b)$. It is observed that the values for the coefficients are quite consistent with values reported in literature for tropical locations.

For example, Massaquoi [9] reported that in general the clear-sky transmissivity of most tropical regions seems to lie between 0.68 and 0.75. Turton [8] after considering long-term averages from 25 stations in the humid tropics reported the values of a and b to be 0.30 and 0.40 for the whole data set, (0.30, 0.40) and (0.30, 0.38) for the dry and wet seasons respectively.

Bashahu and Nkundabakura [13] reported the values of 0.80, and 0.74 for Dakar, Senegal. The values of (0.37, 0.34) and (0.31, 0.48)) were reported by Agyeman and Yehoah-Amankwah [28] for Waigani and Motugore Island in Papua New Guinea. Veeran and Kumar [12] reported the value of (0.34, 0.32) and (0.270, 0.650) for two tropical locations in India, Mandras and Kodaikanal respectively.

Similar results, (0.26, 0.43), have also been reported by Sambo [29] for Northern Nigeria. After analysing data for three locations (Benin, Ibadan and Samaru) in Nigeria, Fagbenle [17] developed a linear equation of the form

$$K_T = 0.212 + 0.556S \tag{13}$$

and quadratic equation of the form

$$K_T = 0.375 - 0.128S + 0.660S^2 \tag{14}$$

for Nigeria. Although the data sets used were of short duration (for example, Benin city data was only for the year1977), as also observed by Fagbenle, the predictive abilities of these two equations will be tested. Eqs. (13) and (14) will be subsequently referred to here as Fagbenle 1 and Fagbenle 2, respectively.

4.3. Comparison of the developed models (Eqs. (9) and (12)) with other sunshine-based models

It is pertinent to compare the predictive efficiencies of models developed in this study (Eqs. (9) and (12)) with other sunshine – based models, especially those believed to be applicable universally (Eqs. (2), (3), (5) and (6)). The models will also be compared with those models developed for the Nigerian environment (Eqs. (13) and (14)). This is with a view of determining the applicability of these models to this study location, Ilorin. The results of these comparisons are in Fig. 1 and Table 2. From the values of MBE, MABE and RMSE, it can be seen that there is no significant difference between the two models, Eqs. (9) and (12), developed here. However, the quadratic model is seen to perform very slightly better than the linear model. Therefore, either of the models is adequate for use in estimating global solar radiation at Ilorin. The predictive efficiencies of the two equations (Eqs, (13) and (14)) developed for the Nigerian environment by Fagbenle [17] although slightly poorer than those developed seem quite adequate for Ilorin environment; here again the performance of the quadratic model (Eq (14) is slightly better than that of the linear model (Eq. (13)).

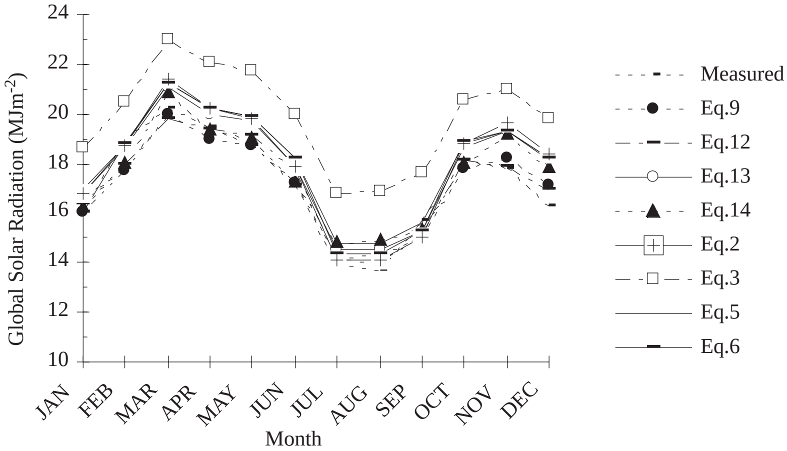

Figure 1. Comparison of measured and estimated (from different models) of monthly average daily global solar radiation.

Table 2. Comparison of error values (MJm^{-2}) for the estimated monthly average daily global solar radiation from different models.

Error Terms	Models							
	Eq.9	Eq.12	Eq.13	Eq.14	Eq.2	Eq.3	Eq.5	Eq.6
MBE	0.0667	0.1500	0.7000	0.5333	0.7333	2.6333	0.8917	0.8167
MABE	0.4333	0.2750	0.8333	0.6500	0.9000	2.6333	0.9250	0.8667
RMSE	0.5642	0.4865	0.9310	0.8114	1.0544	2.7052	1.0649	1.0239

On comparing the linear models, it is realised that the Rietveld model (Eq. (2)), although its performance is slightly poorer than that of Eq. (9), estimates reasonably well and yields good results for cloudy months, as earlier asserted. Glover and McCulloch model (Eq. (3)) has higher values of MBE, MABE and RMSE and therefore looks inadequate in this circumstance.

On comparing the quadratic models, it is realised that the performance of Ogelman et al model, (Eq. (5)) and Akinoglu and Ecevit (Eq. (6)) are poorer than that of the model (Eq. (12)) developed here. However, the performance of Akinoglu and Ecevit model is slightly better than that of Ogelman et al; this of course, is expected since the Akinoglu and Ecevit model contained data from 58 locations spread over several countries and hence is more representative while Ogelman et al model contained data for only two Turkey locations. It has to be noted that the linear model developed here still performs better than any of the other two quadratic models (Eqs, (5) and (6)): The predictive efficiency of Rietveld model, surprisingly, performs better than the quadratic models except the quadratic model developed for Nigerian environment (Eq. (13)).

From the above comparisons, everything points to the fact that Angstrom-Prescott-one parameter-model whether in a linear or quadratic form is, to a large extent, locality dependent; The performance of any general equation will always be poorer than that of the model developed for that locality.

5. Conclusion

From the monthly average daily values of clearness index and relative sunshine, two equations, Eqs. (9) and (12) of the linear and quadratic types, respectively, have been developed for use in estimating global solar irradiation at Ilorin, a tropical location. The values of the coefficients in the linear type compared favourably with values reported in the literature for the Tropics. Comparison of the predictive efficiency of these two models showed that the quadratic model performed only slightly better than the linear model.

These two models were also compared with other sunshine-based models of linear (Fagbenle, 1, Rietveld, and Glover and McCulloch) and quadratic (Fagbenle 2, Ogelman et al, and Akinoglu and Ecevit) types.

Comparative analysis showed that, although all the models except Glover and McCulloch seem to be adequate for estimating global solar irradiation at Ilorin, depending on the degree of accuracy required, the two equations developed here are more preferred.

Results also show that, in practice, the simple linear regression is sufficient and the use of quadratic models is only slightly justified. Moreover, the results show that the relationship between clearness index and relative sunshine, whether in a linear or quadratic form is to some extent locality dependent. However, more data in terms of duration and spread are needed to prove the suggestion by Fagbenle (17) that the maximum-likelihood quadratic fit method appears to widen the range of applicability of the one-parameter correlation to cover climatologically different zones of the same geographical region.

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References

[1] A. Angstrom, *Q. J. R. Meteorol. Soc.,* **50**, (1924) 121.

[2] J. N. Black, C. W. Borytton and J. A. Prescott, *Q. J. R. Meteorol. Soc.,* **80,** (1954) 231.

[3] J. Glover and J. S. F. McCulloh, *Q. J. R. Meteorol. Soc.,* **84**, (1958) 56.

[4] R. K. Swartman and O. Ogunlade, *Solar Energy,* **11,** (1967) 170.

[5] M. R. Rietveld, *Agric. Meteorol.,* **19,** (1978) 243.

[6] H. Ogelman, A. Ecevit and E. Tasdemiroglo, *Solar Energy*, **33,** (1981) 619.

[7] M. F. Hutchinson, J. H. Booth, J. P. McMahon and A. Nix, *Solar Energy,* **32,** (184) 277.

[8] S. M. Turton, *Solar Energy,* **38**, (1987) 353

[9] J. D. M. Massaquoi, *Solar Wind Technol.,* **5**, (1988) 281.

[10] A. Soler, *Solar Wind Technol.,* **7**, (1990) 305.

[11] A. Soler, *Solar Energy,* **45**, (1990) 365.

[12] P. K. Veeran and S. Kumar, *Renewable Energy,* **3**, (1993) 935.

[13] M. Bashahu and P. Nkundabakura, *Renewable Energy,* **4**, (1994) 425.

[14] J. A. Martinez – Lazano, F. Tena, J. E. Onrubia, and J. D. I. Rubia, *Agric. For. Meteorol.,* **33,** (1984) 109.

[15] C. Gueymard, P. Jindra and V. Estradalajiga, *Solar Energy,* **54**, (1995) 37.

[16] B. G. Akinoglu and A. Ecevit., *Turk. J. Physics,* **17**, (1993) 269.

[17] R. L. Fagbenle, *Renewable Energy,* **3**, (1993) 813.

[18] M. Igbal. An introduction to solar radiation (Academic Press, New York, 1983) p. 215

[19] S. O. Udo, *Solar Energy,* **69**, (2000) 45.

[20] S. O. Udo and T. O. Aro, *Renewable Energy,* **17**, (1999) 113.

[21] S. O. Udo and T. O. Aro, *Agric. For Meteorol,* **97**, (1999) 21.

[22] D. G. Baker and R. H. Skaggs, *J. Climatol,* **4**, (1984) 123.

[23] WMO. Meteorological aspects of utilization of solar radiation as an energy source. Tech. Note No. 172, WMO No. 557, 1981

[24] D. C. Montgomery and E. A. Peck, Introduction to linear regression analysis (John Wiley, New York, 1982)

[25] B. G. Akinoglu and A. Ecevit, *Solar Energy,* **45**, (1990) 85.

[26] A. Mani and S. Rangarajan, *Solar Energy,* **31**, (1983) 577.

[27] K. J. A. Revfeim, *Solar Energy,* **31**, (1983) 415.

[28] K. Agyeman and D. Yebouh-Amankwah, *Sc. in New Guinea,* **16**, (1990) 61.

[29] A. S. Sambo, *Solar Wind Technol*, **3**, (1986) 89.