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Is There an Age of the Universe Problem after the *Hipparcos* Data?

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Abstract

We have reanalyzed the age of the universe problem under the assumption that the lower limit on the age of the globular clusters is ∼11 Gyr, as predicted by the recent *Hipparcos* data. We find that the globular cluster and the expansion ages in a standard $\lambda = 0$ universe are consistent only if the present value H_0 of the Hubble constant is $\leq 60\text{kms}^{-1}Mpc^{-1}$. If $H_0 > 60\text{kms}^{-1}Mpc^{-1}$ some kind of modification of the standard $\lambda = 0$ model is required. Invoking a (time-independent) cosmological term λ in the Einstein field equations, as has been done frequently before, we have found that due to the gravitational lensing restrictions a flat universe with the present matter density parameter $\Omega_M < 0.5$ is not problem-free. A nonflat universe with $\Omega_M \leq 1$ does not suffer from the age problem if $H_0 \leq 75\text{kms}^{-1}Mpc^{-1}$.

1. Introduction

A lower limit on the present age t_0 of the universe is determined by estimating the age of the oldest objects in our galaxy, the globular clusters (hereafter GC).[1] These are stellar systems that contain about 10^5 stars in the halo surrounding the galactic disk. The key element in estimating the age of a typical GC is the determination of its distance from us. To this end, the primary observational technique is main-sequence fitting against subdwarfs with well known parallaxes. The distance obtained this way or otherwise is used to convert the measured apparent magnitude of a GC to the absolute magnitude. The

[1] The age of the universe is actually the GC age plus the time it took for the formation of GCs. The formation time is estimated to be between 0.1-2 Gyr [1]. The lower value of 0.1 Gyr is chosen for the lower limit on the age of the universe [1]. Since this difference of 0.1 Gyr in the lower limits on the ages is not significant, we will use the 'age of the universe' and the 'GC age' interchangeably, as is usually done in the literature.

age is then estimated by applying a stellar evolution model. The estimates obtained by different astronomers agree rather well. For example, Bolte and Hogan [2] find 15.8 ± 2.1 Gyr, Chaboyer *et al.* [3] find 14.6 ± 1.7 Gyr, and Sandquist *et al.* [4] find 13.5 ± 1 Gyr. These time scales are to be compared with the expansion age of the universe predicted by the standard model of cosmology (hereafter SM)[2] which requires the knowledge of the present value H_0 of the Hubble constant. Even though estimates of t_0 from GCs are based on the stellar evolution models, which are essentially the same, the situation is not the same for the H_0 estimates. There are a number of different techniques (see the review by Trimble [8]) which give values that differ substantially from each other. We present the most quoted estimates: $H_0 = 50 - 55 \text{kms}^{-1} Mpc^{-1}$ [9] and $H_0 = 73 \pm 10 \text{kms}^{-1} Mpc^{-1}$ [10] [3]. In a SM flat universe t_0 would be 13 Gyr and 8.2 Gyr if H_0 were $50 \text{kms}^{-1} Mpc^{-1}$ and $80 \text{kms}^{-1} Mpc^{-1}$, respectively, whereas in a SM open universe with $\Omega_M = 0.1$, Ω_M being the present nonrelativistic matter density parameter, the ages would be 17.6 Gyr and 11 Gyr for the same H_0 values as above. Thus researchers were rightfully led to think that, if H_0 has as large a value as determined by Freedman *et al.* [10] then, the expansion age and the GC age of the universe are in conflict with each other.

An immediate solution to this so called age of the universe problem was suggested by including a time-independent cosmological constant λ in the Einstein field equations [11-13]. The gravitational lensing studies, however, have shown that the cosmological constant cannot be as large as one desires to increase the expansion age to the level of GC age lest too many lensing events are predicted [14-16]. Recently, the supernova magnitude-redshift approach of Perlmutter *et al.* [17] has given $\Omega_\Lambda < 0.51$ (95% confidence level) for a flat universe which is significantly lower than the gravitational upper limit $\Omega_\Lambda < 0.66$ of Kochanek [15]. Thus it had been concluded that the apparent contradiction between the GC age and the axpansion age could not be reconciled in a flat universe by invoking a time-independent cosmological constant. This was the status of the age of the universe problem before *Hipparcos*. The lower limit on the age of the oldest GCs implied by the *Hipparcos* data is ~ 11 Gyr [18, 19]. The purpose of this paper is to reexamine the age problem in the light of this lower limit of 11 Gyr put by the *Hipparcos* data [18, 19].

2. The Age of the Universe Problem

The relation between the present value H_0 of the Hubble constant $H = \dot{a}/a$, where a is the scale factor of the universe and $\dot{a} = da/dt$, and the present age t_0 is given by [20] [4]

[2]Felten and Isaacman[5] call the models with $\lambda = 0$ 'standard models'. However, we follow the general trend in the literature and call the totality of them the 'standard model'and refer to each case by its k value (see, for example, Misner, Thorne and Wheeler [6]; Weinberg [7].) The SM with $k = 0$ is called the Einstein-de Sitter model [5]).

[3]Reid [18] has argued that this Freedman *et al.* [10] value of H_0 is reduces to $H_0 = 68 \pm 9 \text{kms}^{-1} Mpc^{-1}$ because the *Hipparcos* data reveal a 7% increase in the distances inferred from previous ground-based data.

[4]Equations (1a) and (1c) agree numerically with those given in Weinberg [7] where a different but equivalent functional form is used.

$$H_0 t_0 = \frac{1}{(1-\Omega_M)}\left[1 - \frac{\Omega_M}{(1-\Omega_M)^{1/2}}\sinh^{-1}(\Omega_M^{-1}-1)^{1/2}\right] \;, \quad k=-1 \tag{1a}$$

$$= 2/3 \;, \qquad k = 0 \tag{1b}$$

$$= \frac{1}{(\Omega_M - 1)}\left[\frac{\Omega_M}{(\Omega_M-1)^{1/2}}\sin^{-1}(1-\Omega_M^{-1})^{1/2} - 1\right] \;, \quad k = 1. \tag{1c}$$

Here, Ω_M is the present value of the nonrelativistic matter density parameter defined as the ratio of the present nonrelativistic matter density to the present critical energy density

$$\Omega_M = \frac{\rho_M}{\rho_c} = \frac{\rho_M}{3H_0^2/8\pi G}. \tag{2}$$

Expressing the Hubble constant as $H_0 = 100h\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$, the age in billion years is given by $t_0(\mathrm{Gyr}) = 9.78(H_0 t_0)/h$, where $(H_0 t_0)$ is given in Eq.(1) and h is a parameter assumed to be between 0.5 and 1. In Figure 1, we depict t_0 against Ω_M and h in the SM. It is seen that t_0 is below the *Hipparcos* lower limit of 11 Gyr for large values of h. Thus it can be stated safely that the age of the universe problem still survives if h is large.

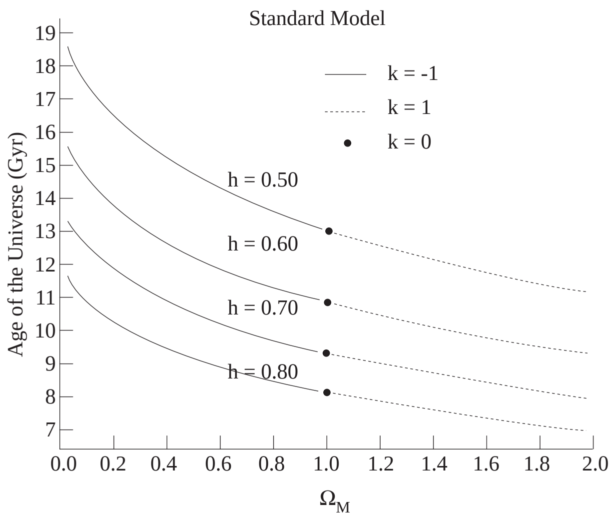

Figure 1. The age of the universe in the SM for $k = -1$ (solid lines), $k = 0$ (dots) and $k = 1$ (dashed lines) versus the present value of the matter density parameter Ω_M.

In Table 1, we display the maximum values of h for which $t_0 = 11\mathrm{Gyr}$ against Ω_M. Note that the maximum h values in Table 1 almost fall in the lower and upper limits of Freedman *et al.* [9]. Thus for each Ω_M, if h is greater than those given in Table 1, there is an age problem. For example, if $\Omega_M = 0.5$ and $h > 0.67$ or $\Omega_M = 1$ and $h > 0.593$ the age problem survives. Now the problem is, however, milder in the sense that before *Hipparcos* the age problem was thought to exist even for moderate values of h whereas it now exists for large values of h. Emphatically, the SM has no age problem if $h < 0.593 \approx 0.6$.

Table 1. Maximum values of h in the SM for which $t_0 = 11$ Gyr[a]

Ω_M	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$h_{\max}$	0.799	0.753	0.719	0.692	0.67	0.651	0.634	0.619	0.605	0.593

[a]Note that if $h = h_{\max}$ then $t_0 = 11$ Gyr, if $h > h_{\max}$ then $t_0 < 11$ Gyr and if $h < h_{\max}$ then $t_0 > 11$ Gyr.

Supposing that there is an age problem, one line of attack, as in the pre *Hipparcos* era, is to invoke a (time-independent) cosmological constant λ in the Einstein field equations [11-13]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}, \tag{3}$$

where $R = R^{\alpha}_{\alpha}$ and $T_{\mu\nu}$ is the energy-momentum tensor. For a homogeneous and isotropic universe described by the Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \tag{4}$$

the energy-momentum tensor is assumed to have the perfect fluid form

$$T_{\mu\nu} = \mathrm{diag}(\rho, p, p, p), \tag{5}$$

where p is the pressue of the matter described by ρ. Equations (3) and (4) give (with c, the speed of light, set to 1)

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho(t) + \frac{\lambda}{3} - \frac{k}{a^2}, \tag{6}$$

where $k = -1, 0, 1$ for a spatially open, flat and closed universe, respectively. At present, the universe is believed to be dominated by nonrelativistic massive matter rather than relativistic matter (radiation). It proves to be very usefull to define the current cosmological constant density parameter

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{\lambda / 8\pi G}{\rho_c} = \frac{\lambda}{3 H_0^2}, \tag{7}$$

and the current curvature density parameter

$$\Omega_k = -\frac{\rho_k}{\rho_c} = -\frac{k}{H_0^2 a_0^2}, \tag{8}$$

where a_0 is the current value of the scale factor a of the universe. When written in terms of the present values, Equation (6) gives the constraint

$$\Omega_M + \Omega_\Lambda + \Omega_k = 1, \tag{9}$$

Equations (3) and (5) under (4) give the energy conservation equation in the matter dominated era as

$$d[\rho_M(t)a^3 + \frac{\lambda}{8\pi G}a^3] + [p_M(t) - \frac{\lambda}{8\pi G}]da^3 = 0, \tag{10}$$

where the pressure p_M of nonrelativistic matter is negligible. Thus it follows from Eq.(10) that $\rho_M(t) = \rho_M a_0^3/a^3$ and the relation between H_0 and t_0 is

$$H_0 t_0 = \int_0^1 y^{1/2}[\Omega_M(1-y) + \Omega_\Lambda(y^3 - y) + y]^{-1/2}dy, \tag{11}$$

where Ω_k has been eliminated by using Eq.(9) and $y = a/a_0$. Now, a flat universe with $\Omega_M < 1$ is rendered possible by postulating the existence of the cosmological term λ such that $\Omega_M + \Omega_\Lambda = 1$. With t value of k not fixed *a priori*, a numerical investigation of Eq.(11) reveals that it is always possible to find a set of three parameters $(\Omega_M, \Omega_\Lambda, h_{\max})$ for which $t_0 = 11$Gyr.

However, the achievement of a cosmological constant to solve the age problem and to have a flat universe with $\Omega_M < 1$ may be illusory. The magnitude of Ω_Λ required to solve the age problem may turn out to be too large to predict plausible number of gravitational lensing events. Therefore, each such set of parameters $(\Omega_M, \Omega_\Lambda, h_{\max})$ need to be confronted with the gravitational lensing statistics, which we address ourselves next.

3. The Gravitational Lensing Statistics

The integrated probability, the so-called optical depth, for lensing by a population of singular isothermal spheres of constant comoving density relative to the Einstein-de Sitter model, is [21]

$$P_{\rm lens} = \frac{15}{4}\left[1 - \frac{1}{(1+z_s)^{1/2}}\right]^{-3} \int_0^{z_s} \frac{(1+z)^2}{E(z)}\left[\frac{d(0,z)d(z,z_s)}{d(0,z_s)}\right]^2 dz, \tag{12}$$

where

$$E(z)^2 = (1+z)^2(1+z\Omega_M) - z(z+2)\Omega_\Lambda \tag{13}$$

and is defined by [22]

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 E(z)^2. \tag{14}$$

Note that $P_{\rm lens} = 1$ for the Einstein-de-Sitter model (in which $\Omega_k = 0$, $\Omega_M = 1$ and $\Omega_\Lambda = 0$). $z = (a_0/a) - 1$ is the redshift and z_s is the redshift of the source (quasar). The angular diameter distance from redshift z_1 to redshift z_2 is

$$d(z_1, z_2) = \frac{1}{(1+z_2) \mid \Omega_k \mid^{1/2}} \sin n \left[\mid \Omega_k \mid^{1/2} \int_{z_1}^{z_2} \frac{dz}{E(z)} \right] \tag{15}$$

where

$$\begin{aligned} \sin n &= \sinh, \quad if \ \ \Omega_k > 0, \\ &= 1, \quad if \ \ \Omega_k = 0, \\ &= \sin, \quad if \ \ \Omega_k < 0. \end{aligned} \tag{16}$$

To determine how much of $P_{\rm lens}$ is permissible, we refer to the work of the Supernova Cosmology Project [17]. Using the initial seven of more than 28 supernovae discovered, Perlmutter *et al.* [17] have recently measured Ω_M and Ω_Λ. For $\Omega_M < 1$, they find $\Omega_\Lambda < 0.51$ at the 95% confidence level for a flat universe, and $\Omega_\Lambda < 1.1$ for the more general case $\Omega_M + \Omega_\Lambda$ unconstrained[5]. In Table 2 we present $P_{\rm lens}$ against Ω_M and Ω_Λ for a typical source redshift of z_s=2. Table 2 helps us to determine the maximum allowed value of $P_{\rm lens}$. It is seen that for $\Omega_\Lambda = 0.5$, which is the maximum allowed value according to Perlmutter *et al.* [17], the corresponding $P_{\rm lens}$ is 1.92. Thus we shall assume that $P_{\rm lens}$ cannot be much larger than 2. Having determined the upper limit on $P_{\rm lens}$, we depict in Table 3 the three parameters Ω_M, Ω_Λ and $h_{\rm max}$ in a flat universe and the corresponding gravitational lensing prediction for t_0 = 11Gyr. In preaparing Table 3, we have first fixed Ω_M and calculated $H_0 t_0$ from Eq.(11) with $\Omega_\Lambda = 1 - \Omega_M$, and finally obtained the maximum value of h from $h_{\rm max} = 9.78(H_0 t_0)/11$.

Table 2. Normalized optical depths.

Ω_M	Ω_Λ	$P_{\rm lens}$
0	1.0	13.25
0.1	0.9	5.98
0.2	0.8	3.94
0.3	0.7	2.93
0.4	0.6	2.33
0.5	0.5	1.92
0.6	0.4	1.63
0.7	0.3	1.42
0.8	0.2	1.25
0.9	0.1	1.11
1.0	0	1.00
1.0	1.1	1.61
0.8	1.1	1.99
0.6	1.1	2.57
0.4	1.1	3.61
0.2	1.1	6.05

[5]But of course $\Omega_M + \Omega_\Lambda + \Omega_k = 1$

Discarding those set of parameters which yield $P_{\rm lens} > 2$ or have $\Omega_\Lambda > 0.5$, first we confirm, from Table 3, the previous conclusions that a cosmological constant cannot solve the age problem in a flat universe with $\Omega_M < 0.5$ due to too many lensing predictions. Next, we see that the maximum allowed value of h in a flat universe is about 0.74-0.75. This is to be compared with the pre *Hipparcos* lower limits for the age. For $t_0 = 13$ and 14 Gyr the $h_{\rm max}$ values are 0.64 and 0.60 in a flat universe, respectively.

Table 3. Maximum values of h in a flat universe for which $t_0 = 11$ Gyr.

Ω_M	Ω_Λ	$h_{\rm max}$	$P^a_{\rm lens}$
0.1	0.9	1.14	5.98
0.2	0.8	0.96	3.94
0.3	0.7	0.86	2.93
0.4	0.6	0.79	2.33
0.45	0.55	0.76	2.11
0.5	0.5	0.74	1.92
0.6	0.4	0.70	1.63
0.7	0.3	0.67	1.42
0.8	0.2	0.64	1.25
0.9	0.1	0.61	1.11
1.0	0	0.59	1.00

[a] Recall that $P_{\rm lens}$ is independent of h (see equations (12)-(15)).

As for a nonflat universe, one may either fix Ω_M and Ω_Λ first and then calculate $h_{\rm max}$ to give $t_0 = 11$ Gyr, or one may fix Ω_M and $h_{\rm max}$ first and then calculate the Ω_Λ value from Eq.(11) by trial and error to give again $t_0 = 11$ Gyr. We have chosen the second option and constructed Figures 2 and 3, which are contour diagrams of $h_{\rm max}$ (for $t_0 = 11$ Gyr) in the $(\Omega_M, \Omega_\Lambda)$ and $(\Omega_M, P_{\rm lens})$ planes.

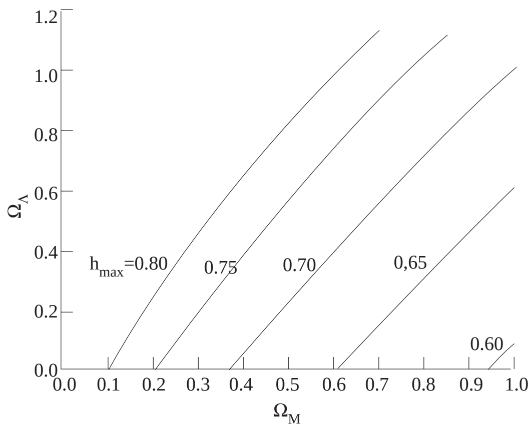

Figure 2. Contours of $h_{\rm max}$ for which $t_0 = 11$ Gyr in the $(\Omega_M, \Omega_\Lambda)$ plane.

It is seen that for each contour there is a minimum value of Ω_M before which the age is greater or equal to 11 Gyr for $\Omega_\Lambda = 0$. In drawing Figures 2 and 3, we have assumed that the maximum allowed value of Ω_Λ is about 1.1, in accordance with the findings of Perlmutter *et al.* [17]. The age problem is seen to survive for $\Omega_M \geq 0.3$ only if h is as large as 0.8 for which lensing predictions are larger than 2. There is no age problem in a non-flat universe provided $h \leq 0.75$ for all $\Omega_M \leq 1$.

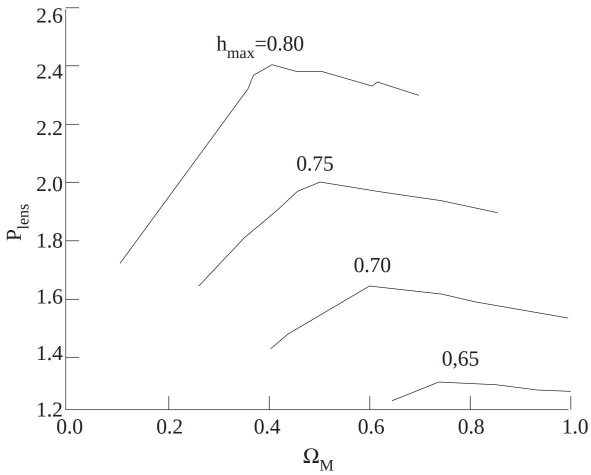

Figure 3. Contours of $h_{\rm max}$ for which $t_0 = 11$ Gyr in the $(\Omega_M, P_{\rm lens})$ plane.

Conclusions

That the *Hipparcos* data [18, 19] implies that GCs may be as young as ~11 Gyr has raised hopes to reconcile the age of GCs and the expansion age of the universe. We have studied this matter in this work. As is well known, and as born out by our results, the realization of this hope depends solely on the value of H_0. If H_0 is as large as the upper limit of the Freedman *et al.* [10] value, the age of the universe problem continues to exist in the SM. The problem, however, is now milder than it had been before *Hipparcos*. Previously, it was thought to exist even for moderate values of h, whereas it seems to exist for large values of h now. If, however, H_0 is as low as favored by Tammann and Sandage [9] then the GC and the expansion ages of the universe are consistent with each other in the SM.

Assuming that H_0 is high and hence modifying the SM by invoking a (time-independent) cosmological term in the Einstein field equations, as has been done before [11-13], we have confirmed the conclusion of previous workers that, due to lensing restrictions, the age problem still survives in a flat universe for $\Omega_M < 0.5$, and at the same time conclude that h cannot be larger than about 0.75. As for a nonflat universe, we have shown that the age problem does not exist for all $\Omega_M \leq 1$ provided $h \leq 0.75$.

The above mentioned hope is realized in the SM only if $h \leq 0.6$ (see Table 1). Otherwise, some kind of modification of the SM is called for. One such– and the most-studied– attempt is the inclusion of a cosmological term in the field equations. With such a term, the age problem has a better standing in a nonflat (open or closed) universe with $\Omega_M \leq 1$. It should be noted, in the light of recent works, that such a cosmological term need not be a pure time-independent constant. Scalar fields, cosmic strings or some kind of stable textures with an energy density varying as a^{-2} lead to viable cosmological models that stand as alternatives to the SM [23-25].

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