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Some soft topological properties and fixed soft element results in soft complex valued metric spaces

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Abstract: In this paper, first, by using the idea of soft complex numbers as in Das and Samanta [8], we introduce the notion of soft complex valued metric spaces and investigate some of their topological aspects. Next, we establish some fixed soft element theorems for various soft mappings on soft complex valued metric spaces, which are the main results of our paper.

Key words: Soft set, soft complex number, soft complex valued metric space, fixed soft element

1. Introduction

In 1999, Molodtsov [14] introduced the notion of a soft set in order to deal with ambiguities and gave the basic results of this theory. Since the new theory has a rich potential, the structures and the applications on this concept have been quickly examined [3, 10, 13, 18].

Das and Samanta [7] defined soft metric spaces using soft elements offered in [6] and presented some of their basic features. Shabir and Naz [20] gave the notion of a soft topological space. Recently, more papers have been published on soft set theory [5, 9, 12, 16, 17].

Fixed point theory has an important part in applied sciences and mathematics, especially as it shows the existence and uniqueness of the solutions of differential equations and integral equations. After Banach contraction principle, many authors have generalized and improved this theory in different ways.

In 2011, Azam et al. [4] defined the concept of a complex valued metric space, which is more general than the well-known metric space. Following the establishment of this new idea, Rouzkard et al. [19] studied some common fixed point theorems satisfying certain rational expressions in this space to generalize the result of [4]. After that, numerous researches on these spaces have been performed [1, 15, 21].

Extensions of fixed point theorems to the soft sets have been studied by some authors. Wardowski [22] defined the problem in a different way than in the literature; he used a soft mapping and proved fixed point results using soft sets. Das and Samanta [7] investigated Banach fixed point theorem in the soft setting. Abbas et al. [2] established soft metric versions of several important fixed point theorems for metric spaces. Then, Guler et al. [11] introduced soft G-metric spaces with the help of soft elements and proved fixed point theorems on these spaces.

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In this work, by using soft complex numbers, we introduce the concept of a soft complex valued metric space and investigate some of its properties. Moreover, we define a soft order relation on soft complex numbers and use this relation to obtain some fixed soft element theorems in soft complex valued metric spaces. Finally, we furnish specific examples to show the validity of our fixed point results.

2. Preliminaries

In this section, we review some fundamental definitions and operations about soft sets. Throughout this work, let Θ be an initial universe, Ξ be a set of parameters for Θ, and $P(\Theta)$ be the power set of Θ.

Definition 2.1 [14] *A soft set F on the universe Θ with the set Ξ of parameters is defined by the set of ordered pairs*

$$F = \{(e, F(e)) : e \in \Xi, F(e) \in P(\Theta)\}$$

where F is a mapping given by $F : \Xi \to P(\Theta)$.

Throughout this paper, the family of all soft sets over Θ is denoted by $S(\Theta, \Xi)$.

Definition 2.2 [3, 13, 18] Let $F, G \in S(\Theta, \Xi)$. Then,

(i) If $F(e) = \emptyset$ for every $e \in \Xi$,then F is called null soft set denoted by $\widetilde{\emptyset}$.

(ii) If $F(e) = \Theta$ for every $e \in \Xi$, then F is called absolute soft set denoted by $\widetilde{\Theta}$.

(iii) F is a soft subset of G if $F(e) \subseteq G(e)$ for every $e \in E$. It is denoted by $F \sqsubseteq G$.

(iv) The complement of F is denoted by F^c, where $F^c : \Xi \to P(\Theta)$ is a mapping defined by $F^c(e) = \Theta \setminus F(e)$ for every $e \in \Xi$. Clearly, $(F^c)^c =$ F.

(v) The union of F and G is a soft set H defined by $H(e) = F(e) \cup G(e)$ for every $e \in \Xi$. H is denoted by $F \sqcup G$.

(vi) The intersection of F and G is a soft set H defined by $H(e) = F(e) \cap G(e)$ for every $e \in \Xi$. H is denoted by $F \sqcap G$.

Definition 2.3 [6] Let Θ be a nonempty set and Ξ be a nonempty parameter set. Then, a function $\tilde{\varrho} : \Xi \to \Theta$ is said to be a soft element of Θ. A soft element $\tilde{\varrho}$ of Θ is said to belong to a soft set F of Θ, denoted by $\tilde{\varrho} \,\widetilde{\in}\, F$, if $\tilde{\varrho}(e) \in F(e)$ for every $e \in \Xi$.

Throughout this paper, the family of all soft elements in Θ with the set Ξ of parameters is denoted by Θ^{Ξ}.

Definition 2.4 [6] *Let $\tilde{\varrho} \in \Theta^{\Xi}$ and let $F_i \in S(\Theta, \Xi)$ for all $i \in J$ where J is an index set. Then,*

(i) $\tilde{\varrho} \,\widetilde{\in} \bigsqcup_{i \in J} F_i \Leftrightarrow \tilde{\varrho} \,\widetilde{\in}\, F_{i_0}, \exists i_0 \in J$.

(ii) $\tilde{\varrho} \,\widetilde{\in} \bigsqcap_{i \in J} F_i \Leftrightarrow \tilde{\varrho} \,\widetilde{\in}\, F_i, \forall i \in J$.

Definition 2.5 [6] *Let $\mathfrak{B}(\mathbb{R})$ be the family of all nonempty bounded subsets of the set of real numbers. Then,*

$$F = \{(e, F(e)) : e \in \Xi, F(e) \in \mathfrak{B}(\mathbb{R})\}$$

is said to be a soft real set.

Specifically, if F is a singleton soft set, then after identifying F with the corresponding soft element, it will be called a soft real number.

Notation 2.6 [6] *The notations $\tilde{r}$, $\tilde{s}$ is used to indicate soft real numbers while $\bar{r}$, $\bar{s}$ will indicate a special sort of soft real numbers satisfying $\bar{r}(e) = r$ for every $e \in \Xi$. For instance, $\bar{0}$ is the soft real number, where $\bar{0}(e) = 0$ for every $e \in \Xi$.*

For further information such as the sum, product, division, and modulus of soft real numbers and a sequence of soft real numbers, please see ([6]).

Theorem 2.7 [5] *Every Cauchy sequence of soft real numbers with a finite set of parameters converges to a soft real number.*

Definition 2.8 [8] *Let $\mathfrak{B}(\mathbb{C})$ be the family of all non-empty bounded subsets of the set of complex numbers. Then,*

$$F = \{(e, F(e)) : e \in \Xi, F(e) \in \mathfrak{B}(\mathbb{C})\}$$

is called a soft complex set.

Specifically, if F is a singleton soft set, then after identifying F with the corresponding soft element, it will be called a soft complex number.

Definition 2.9 [8] *Let F be a soft complex set (number). Then, the real and imaginary parts of F are denoted by ReF, and ImF and are defined by*

$ReF(e) = \{Re(z) : z \in F(e)\} \quad \big(ReF(e) = Re(F(e))\big)$

$ImF(e) = \{Im(z) : z \in F(e)\} \quad \big(ImF(e) = Im(F(e))\big)$ *for every $e \in \Xi$. It is clear that ReF and ImF are soft real sets (numbers).*

Definition 2.10 [8] *Let F, G be two soft complex numbers. For each $e \in \Xi$,*

(i) The sum is $(F + G)(e) = F(e) + G(e)$,

(ii) The difference is $(F - G)(e) = F(e) - G(e)$,

(iii) The product is $(F.G)(e) = F(e).G(e)$,

(iv) The division is $\left(\frac{F}{G}\right)(e) = \frac{F(e)}{G(e)}$, provided $G(e) \neq 0$,

(v) The scalar multiplication of F by k is $(k.F)(e) = k.F(e)$.

It is obvious that $F + G, F - G$, $F.G$, $\frac{F}{G}$ and $k.F$ are soft complex numbers.

Notation 2.11 *We use notations as $\hat{z}, \hat{u}$ to denote soft complex numbers where $\hat{z} = \widetilde{z_1} + i\widetilde{z_2}$ and $\hat{u} = \widetilde{u_1} + i\widetilde{u_2}$ with $Re\hat{z} = \widetilde{z_1}$, $Im\hat{z} = \widetilde{z_2}$ and $Re\hat{u} = \widetilde{u_1}$, $Im\hat{u} = \widetilde{u_2}$. On the other hand, $\overset{\triangle}{v} = \overline{v} + i\overline{v}$ and $\overset{\triangle}{w} = \overline{w} + i\overline{w}$ will indicate a special sort of soft complex numbers where $Re\overset{\triangle}{v} = Im\overset{\triangle}{v} = \overline{v}$ and $Re\overset{\triangle}{w} = Im\overset{\triangle}{w} = \overline{w}$. For instance, $\overset{\triangle}{0} = \overline{0} + i\overline{0}$ is the soft complex number, where $\overset{\triangle}{0}(e) = \overline{0}(e) + i\overline{0}(e) = 0 + i0$ for every $e \in \Xi$.*

Definition 2.12 [8] *Let* $\hat{z} = \widetilde{z_1} + i\widetilde{z_2}$ *be a soft complex number. Then, the complex conjugate of* $\hat{z}$ *is denoted by* $\overline{\hat{z}}$ *and is defined by* $\overline{\hat{z}} = \widetilde{z_1} - i\widetilde{z_2}$*.*

$\overline{\hat{z}}$ *is a soft complex number because of the definition of soft complex numbers.*

Definition 2.13 [8] *Let* $\hat{z} = \widetilde{z_1} + i\widetilde{z_2}$ *be a soft complex number. Then, the modulus of* $\hat{z}$ *is denoted by* $|\hat{z}|$ *and is defined by* $|\hat{z}| = \sqrt{(\widetilde{z_1})^2 + (\widetilde{z_2})^2}$*. By definition of soft real numbers it follows that* $|\hat{z}|$ *is a nonnegative soft real number (that is,* $0 \leq |\hat{z}|(e) = \sqrt{(\widetilde{z_1}(e))^2 + (\widetilde{z_2}(e))^2}$ *for every* $e \in \Xi$*).*

Theorem 2.14 [8] *Let* $\hat{z}, \hat{u} \in \mathbb{C}^{\Xi}$ *where* $\hat{z} = \widetilde{z_1} + i\widetilde{z_2}$ *and* $\hat{u} = \widetilde{u_1} + i\widetilde{u_2}$*. Then, the following properties are satisfied.*

(i) $\overline{\hat{z} + \hat{u}} = \overline{\hat{z}} + \overline{\hat{u}}$.

(ii) $|\hat{z}| = |\overline{\hat{z}}|$.

(iii) $|\hat{z}|^2 = \hat{z}.\overline{\hat{z}}$.

(iv) $|\widetilde{z_1}| \,\widetilde{\leq}\, |\hat{z}|$ *and* $|\widetilde{z_2}| \,\widetilde{\leq}\, |\hat{z}|$.

(v) $|\hat{z}.\hat{u}| = |\hat{z}|.|\hat{u}|$.

(vi) $\left|\frac{\hat{z}}{\hat{u}}\right| = \frac{|\hat{z}|}{|\hat{u}|}$.

(vii) $|\hat{z} + \hat{u}| \,\widetilde{\leq}\, |\hat{z}| + |\hat{u}|$.

Definition 2.15 [16] *Let* $S(\Theta, \Xi)$ *and* $S(\Upsilon, \Xi)$ *be the families of all soft sets over* Θ *and* Υ*, respectively. Let* $f : \Theta \to \Upsilon$ *be a mapping. Then, the mapping* f_{Ξ} *is called a soft mapping from* Θ *to* Υ*, written by* $f_{\Xi} : S(\Theta, \Xi) \to S(\Upsilon, \Xi)$.

(i) *Let* $F \in S(\Theta, \Xi)$*. Then,* $f_{\Xi}(F)$ *is the soft set over* Υ *defined as*

$$f_{\Xi}(F)(e) = f(F(e))$$

for all $e \in \Xi$*.*

(ii) *Let* $G \in S(\Upsilon, \Xi)$*. Then,* $f_{\Xi}^{-1}(G)$ *is the soft set over* Θ *defined as*

$$f_{\Xi}^{-1}(G)(e) = f^{-1}(G(e))$$

for all $e \in \Xi$*.*

In particular, if we take $S(\Theta, \Xi) = \Theta^{\Xi}$ and $S(\Upsilon, \Xi) = \Upsilon^{\Xi}$, respectively, we have

(i) If $\tilde{\varrho} \in \Theta^{\Xi}$, then $f_{\Xi}(\tilde{\varrho})$ is the soft element over Υ defined as

$$f_{\Xi}(\tilde{\varrho})(e) = f(\tilde{\varrho}(e))$$

for all $e \in \Xi$.

(ii) If f is bijective and $\tilde{\zeta} \in \Upsilon^{\Xi}$, then $f_{\Xi}^{-1}(\tilde{\zeta})$ is the soft element over Θ defined as

$$f_{\Xi}^{-1}(\tilde{\zeta})(e) = f^{-1}(\tilde{\zeta}(e))$$

for all $e \in \Xi$.

Definition 2.16 [20] *A soft topology on* Θ *is a collection* τ *of soft sets over* Θ, *which is called the soft open sets, satisfying:*

(st_1) *Both* $\widetilde{\emptyset}$ *and* $\widetilde{\Theta}$ *are elements of* τ,

(st_2) *Any union of elements of* τ *is an element of* τ,

(st_3) *Any intersection of finitely many elements of* τ *is an element of* τ.

We say the triplet (Θ, τ, Ξ) *is a soft topological space.*

Definition 2.17 [16] *A family* $\mathcal{B} \subset \tau$ *is called a soft base for a soft topological space* (Θ, τ, Ξ) *if every element of* τ *can be represented as the union of subfamily of* $\mathcal{B}$.

3. On soft complex valued metric spaces

In this part, we examine some fundamental features of soft complex numbers. Then, we present the notion of a soft complex valued metric space and investigate some of its topological aspects, which strengthen this concept.

Definition 3.1 *A sequence* $\{\widehat{z_n}\}$ *of soft complex numbers is called converges to* $\hat{z}$, *and we write* $lim_{n\to\infty}\widehat{z_n} = \hat{z}$. *if for every* $\tilde{\epsilon} \widetilde{>} \overline{0}$, *there exists an* $n_0 \in \mathbb{N}$ *such that* $|\widehat{z_n} - \hat{z}| \widetilde{<} \tilde{\epsilon}$ *for all* $n \geq n_0$.

Example 3.2 *Consider* $\widehat{z_n} = \widetilde{z_{n_1}} + i\widetilde{z_{n_2}} \in \mathbb{C}^{\Xi}$ *with* Ξ *is a finite set of parameters where* $\widetilde{z_{n_1}} = \overline{0}$ *and* $\widetilde{z_{n_2}} = \overline{\left(\frac{1}{n}\right)}$, $n \in \{1, 2, 3, ...\}$. *Then, we claim that* $lim_{n\to\infty}\widehat{z_n} = \overset{\triangle}{0}$. *Indeed, given* $\tilde{\epsilon} \widetilde{>} \overline{0}$, *we get* $k = min\{\tilde{\epsilon}(e) : e \in \Xi\} > 0$, *because* Ξ *is a finite set of parameters. By Archimedean property of real numbers, there exists an* $n_0 \in \mathbb{N}$ *such that* $\frac{1}{n_0} < k$. *Thus, from the fact that*

$$|\widehat{z_n} - \overset{\triangle}{0}|(e) = \left|\overline{\left(\tfrac{1}{n}\right)} - \overline{0}\right|(e) = \left|\overline{\left(\tfrac{1}{n}\right)}(e) - \overline{0}(e)\right| = \left|\tfrac{1}{n} - 0\right| = \tfrac{1}{n} \leq \tfrac{1}{n_0} < k \leq \tilde{\epsilon}(e) \text{ for every } e \in \Xi \text{ and } n \geq n_0,$$

it follows that $|\widehat{z_n} - \overset{\triangle}{0}| \widetilde{<} \tilde{\epsilon}$ *for all* $n \geq n_0$.

Theorem 3.3 *Let* $\{\widehat{z_n}\}$ *be a sequence of soft complex numbers where* $\widehat{z_n} = \widetilde{z_{n_1}} + i\widetilde{z_{n_2}}$ *with* $\widetilde{z_{n_1}}, \widetilde{z_{n_2}} \in \mathbb{R}^{\Xi}$. *Let* $\hat{z} = \widetilde{z_1} + i\widetilde{z_2}$ *such that* $\widetilde{z_1}, \widetilde{z_2} \in \mathbb{R}^{\Xi}$. *Then,*

$$lim_{n\to\infty}\widehat{z_n} = \hat{z} \text{ if and only if } lim_{n\to\infty}\widetilde{z_{n_1}} = \widetilde{z_1} \text{ and } lim_{n\to\infty}\widetilde{z_{n_2}} = \widetilde{z_2}.$$

Proof Necessity follows immediately from the condition (iv) of Theorem 2.14. For sufficiency, let $lim_{n\to\infty}\widetilde{z_{n_1}} = \widetilde{z_1}$ and $lim_{n\to\infty}\widetilde{z_{n_2}} = \widetilde{z_2}$. Given $\widetilde{\epsilon} \widetilde{>} \overline{0}$ there exist $m_1, m_2 \in \mathbb{N}$ such that

$$|\widetilde{z_{n_1}} - \widetilde{z_1}| \widetilde{<} \frac{\widetilde{\epsilon}}{2} \text{ for all } n \geq m_1 \quad \text{and} \quad |\widetilde{z_{n_2}} - \widetilde{z_2}| \widetilde{<} \frac{\widetilde{\epsilon}}{2} \text{ for all } n \geq m_2.$$

If we get $m = max\{m_1, m_2\}$, then for all $n \geq m$, we have

$$|\widehat{z_n} - \hat{z}| \widetilde{\leq} |\widetilde{z_{n_1}} - \widetilde{z_1}| + |\widetilde{z_{n_2}} - \widetilde{z_2}| \widetilde{<} \frac{\widetilde{\epsilon}}{2} + \frac{\widetilde{\epsilon}}{2} = \widetilde{\epsilon}.$$

Thus, we infer that $lim_{n\to\infty}\widehat{z_n} = \hat{z}$. □

Let $\hat{z},\, \hat{u} \in \mathbb{C}^{\Xi}$ where $\hat{z} = \widetilde{z_1} + i\widetilde{z_2}$ and $\hat{u} = \widetilde{u_1} + i\widetilde{u_2}$. We define a soft order $\tilde{\precsim}$ on $\mathbb{C}^{\Xi}$ for comparing two soft complex numbers as follows:

$$\hat{z} \,\tilde{\precsim}\, \hat{u} \;\Leftrightarrow\; \widetilde{z_1} \,\tilde{\leq}\, \widetilde{u_1} \;\; and \;\; \widetilde{z_2} \,\tilde{\leq}\, \widetilde{u_2}.$$

Thus, $\hat{z} \,\tilde{\precsim}\, \hat{u}$ if one of the followings holds:

(1) $\widetilde{z_1} = \widetilde{u_1}$ and $\widetilde{z_2} = \widetilde{u_2}$,

(2) $\widetilde{z_1} \,\tilde{<}\, \widetilde{u_1}$ and $\widetilde{z_2} = \widetilde{u_2}$,

(3) $\widetilde{z_1} = \widetilde{u_1}$ and $\widetilde{z_2} \,\tilde{<}\, \widetilde{u_2}$,

(4) $\widetilde{z_1} \,\tilde{<}\, \widetilde{u_1}$ and $\widetilde{z_2} \,\tilde{<}\, \widetilde{u_2}$.

Especially, we indicate $\hat{z} \,\tilde{\precnsim}\, \hat{u}$ if $\hat{z} \precsim \hat{u}$ and $\hat{z} \neq \hat{u}$ and we indicate $\hat{z} \,\tilde{\prec}\, \hat{u}$ if only (4) holds.

Lemma 3.4 *Let $m, n \in \mathbb{R}$ and $\hat{z}, \hat{u}, \hat{v}, \hat{w} \in \mathbb{C}^{\Xi}$. Then, the following properties hold.*

(i) If $m \leq n$, then $m.\hat{z} \,\tilde{\precsim}\, n.\hat{z}$.

(ii) If $\hat{z} \,\tilde{\precsim}\, \hat{u}$ and $\hat{u} \,\tilde{\prec}\, \hat{v}$, then $\hat{z} \,\tilde{\prec}\, \hat{v}$.

(iii) If $\overset{\triangle}{0} \,\tilde{\precsim}\, \hat{z} \,\tilde{\precnsim}\, \hat{u}$, then $|\hat{z}| \,\tilde{<}\, |\hat{u}|$.

(iv) If $\hat{z} \,\tilde{\precsim}\, \hat{u}$, $\hat{v} \,\tilde{\precsim}\, \hat{w}$, then $\hat{z} + \hat{v} \,\tilde{\precsim}\, \hat{u} + \hat{w}$.

Proof It can be easily checked from the definitions given in the previous section. □

Definition 3.5 *A soft complex valued metric space is a triplet (Θ, Ψ, Ξ) consisting of a set Θ and a set of parameters Ξ together with a mapping $\Psi : \Theta^{\Xi} \times \Theta^{\Xi} \to \mathbb{C}^{\Xi}$ satisfying, for all $\tilde{\varrho}, \tilde{\zeta}, \tilde{\varsigma} \in \Theta^{\Xi}$:*

(scm_1) $\overset{\triangle}{0} \,\tilde{\precsim}\, \Psi(\tilde{\varrho}, \tilde{\zeta})$,

(scm_2) $\Psi(\tilde{\varrho}, \tilde{\zeta}) = \overset{\triangle}{0}$ *if and only if* $\tilde{\varrho} = \tilde{\zeta}$,

(scm_3) $\Psi(\tilde{\varrho}, \tilde{\zeta}) = \Psi(\tilde{\zeta}, \tilde{\varrho})$,

(scm_4) $\Psi(\tilde{\varrho}, \tilde{\zeta}) \,\tilde{\precsim}\, \Psi(\tilde{\varrho}, \tilde{\varsigma}) + \Psi(\tilde{\varsigma}, \tilde{\zeta})$.

The mapping Ψ is called a soft complex valued metric on Θ^{Ξ}.

Definition 3.6 *Let (Θ, Ψ, Ξ) be a soft complex valued metric space.*

(i) A sequence $\{\widetilde{\varrho_n}\}$ of soft elements in Θ is called converges to $\tilde{\varrho} \in \Theta^{\Xi}$ if for every $\hat{c} \in \mathbb{C}^{\Xi}$ with $\overset{\triangle}{0} \,\tilde{\prec}\, \hat{c}$, there exists an $n_0 \in \mathbb{N}$ such that, for all $n \geq n_0$, $\Psi(\widetilde{\varrho_n}, \tilde{\varrho}) \,\tilde{\prec}\, \hat{c}$. We indicate this by $lim_{n\to\infty}\widetilde{\varrho_n} = \tilde{\varrho}$.

(ii) A sequence $\{\widetilde{\varrho_n}\}$ of soft elements in Θ is called a Cauchy sequence in (Θ, Ψ, Ξ) if for every $\hat{c} \in \mathbb{C}^{\Xi}$ with $\overset{\triangle}{0} \,\tilde{\prec}\, \hat{c}$, there exists an $n_0 \in \mathbb{N}$ such that, for all $n \geq n_0$, $\Psi(\widetilde{\varrho_n}, \widetilde{\varrho_{n+m}}) \,\tilde{\prec}\, \hat{c}$, where $m \in \mathbb{N}$.

(iii) (Θ, Ψ, Ξ) is called a complete soft complex valued metric space if for each Cauchy sequence $\{\widetilde{\varrho_n}\}$ in (Θ, Ψ, Ξ) there exists a $\tilde{\varrho} \in \Theta^{\Xi}$ satisfying $lim_{n\to\infty}\widetilde{\varrho_n} = \tilde{\varrho}$.

Example 3.7 *Let $X = \mathbb{C}$ and Ξ be a finite set of parameters. Consider a mapping $d : \mathbb{C}^{\Xi} \times \mathbb{C}^{\Xi} \to \mathbb{C}^{\Xi}$ by*

$$\Psi(\hat{z}, \hat{u}) = |\widetilde{z_1} - \widetilde{u_1}| + i|\widetilde{z_2} - \widetilde{u_2}| \quad for\ every\ \hat{z}, \hat{u} \in \mathbb{C}^{\Xi}.$$

Then, (Θ, Ψ, Ξ) *is a complete soft complex valued metric space.*

Definition 3.8 *Let* (Θ, Ψ, Ξ) *be a soft complex valued metric space,* $\tilde{\varrho} \in \Theta^{\Xi}$ *and* $\overset{\triangle}{0} \precsim \hat{z}$*.*

(i) *A soft open ball with centre* $\tilde{\varrho}$ *and radius* $\hat{z}$ *is denoted by* $B(\tilde{\varrho}, \hat{z})$ *and defined by*

$$B(\tilde{\varrho}, \hat{z})(e) = \bigcup \{\tilde{\zeta}(e) \in \Theta : \Psi(\tilde{\varrho}, \tilde{\zeta}) \precsim \hat{z}\}$$

for every $e \in \Xi$*.*

(ii) *A soft closed ball with centre* $\tilde{\varrho}$ *and radius* $\hat{z}$ *is denoted by* $B[\tilde{\varrho}, \hat{z}]$ *and defined by*

$$B[\tilde{\varrho}, \hat{z}](e) = \bigcup \{\tilde{\zeta}(e) \in \Theta : \Psi(\tilde{\varrho}, \tilde{\zeta}) \precsim \hat{z}\}$$

for every $e \in \Xi$*.*

Theorem 3.9 *Let* (Θ, Ψ, Ξ) *be a soft complex valued metric space. Then,*

$$\tau = \{\widetilde{\emptyset}\} \cup \{F \in S(\Theta, \Xi) : \forall \tilde{\varrho} \,\widetilde{\in}\, F, \exists \hat{z} \in \mathbb{C}^{\Xi},\ \overset{\triangle}{0} \precsim \hat{z} \textit{ such that } B(\tilde{\varrho}, \hat{z}) \sqsubseteq F\}$$

is a soft topology on Θ*.*

Proof It is easy to show that $\widetilde{\emptyset}, \widetilde{\Theta} \in \tau$.

Let $\{F_i : i \in I\} \subset \tau$, where I is any index set. Take $F = \bigsqcup_{i \in I} F_i$. If $\tilde{\varrho} \,\widetilde{\in}\, \bigsqcup_{i \in I} F_i$, then there is an $i_0 \in I$ such that $\tilde{\varrho} \,\widetilde{\in}\, F_{i_0}$. Therefore, we obtain $B(\tilde{\varrho}, \hat{z}) \sqsubseteq F_{i_0} \sqsubseteq F$ for some $\hat{z} \in \mathbb{C}^{\Xi}$ with $\hat{z} \succsim \overset{\triangle}{0}$ and so that $F \in \tau$.

Let $F_1, F_2 \in \tau$ and $\tilde{\varrho} \,\widetilde{\in}\, F_1 \sqcap F_2$. Then, we have $\tilde{\varrho} \,\widetilde{\in}\, F_1$ and $\tilde{\varrho} \,\widetilde{\in}\, F_2$. Therefore, there exist $\hat{z}, \hat{u} \in \mathbb{C}^{\Xi}$, $\hat{z} \succsim \overset{\triangle}{0}$, $\hat{u} \succsim \overset{\triangle}{0}$ with $B(\tilde{\varrho}, \hat{z}) \sqsubseteq F_1$ and $B(\tilde{\varrho}, \hat{u}) \sqsubseteq F_2$. Let us take a soft complex number $\hat{w} = \widetilde{w_1} + i\,\widetilde{w_2} \succsim \overset{\triangle}{0}$ satisfying

$$\widetilde{w_1}(e) \leq min\{\widetilde{z_1}(e), \widetilde{u_1}(e)\} \ \ and \ \ \widetilde{w_2}(e) \leq min\{\widetilde{z_2}(e), \widetilde{u_2}(e)\}$$

for every $e \in \Xi$. Hence, we get $\hat{w} \precsim \hat{z}$ and $\hat{w} \precsim \hat{u}$. Thus, from the fact that

$$B(\tilde{\varrho}, \hat{w}) \sqsubseteq B(\tilde{\varrho}, \hat{z}) \sqsubseteq F_1 \ \ and \ \ B(\tilde{\varrho}, \hat{w}) \sqsubseteq B(\tilde{\varrho}, \hat{u}) \sqsubseteq F_2$$

it follows that $F_1 \sqcap F_2 \in \tau$. □

Theorem 3.10 *Let* (Θ, Ψ, Ξ) *be a soft complex valued metric space,* $\tilde{\varrho} \in \Theta^{\Xi}$ *and* $\overset{\triangle}{0} \precsim \hat{z}$*. Then, the soft open ball* $B(\tilde{\varrho}, \hat{z})$ *is a soft open set on* Θ*.*

Proof Let $\tilde{\zeta} \,\widetilde{\in}\, B(\tilde{\varrho}, \hat{z})$. Then, we obtain $\overset{\triangle}{0} \precsim \Psi(\tilde{\varrho}, \tilde{\zeta}) \precsim \hat{z}$. Take $\hat{z} - \Psi(\tilde{\varrho}, \tilde{\zeta}) = \hat{u}$. Therefore, if $\tilde{t} \,\widetilde{\in}\, B(\tilde{\zeta}, \hat{u})$, then we have

$$\Psi(\tilde{t}, \tilde{\varrho}) \precsim \Psi(\tilde{t}, \tilde{\zeta}) + \Psi(\tilde{\zeta}, \tilde{\varrho}) \precsim \hat{u} + \Psi(\tilde{\zeta}, \tilde{\varrho}) = \hat{z}.$$

From the fact that $B(\tilde{\zeta}, \hat{u}) \sqsubseteq B(\tilde{\varrho}, \hat{z})$ it follows that $B(\tilde{\varrho}, \hat{z})$ is a soft open set over Θ. □

Corollary 3.11 *Let* (Θ, Ψ, Ξ) *be a soft complex valued metric space. Then,*

$$\mathcal{B} = \{B(\tilde{\varrho}, \hat{z}) : \tilde{\varrho} \in \Theta^{\Xi},\ \overset{\triangle}{0} \precsim \hat{z}\}$$

is a soft base for the soft topology τ *on* Θ*.*

Proof We can easily prove it by Theorem 3.9 and Theorem 3.10. □

Definition 3.12 *Let* (Θ, Ψ, Ξ) *be a soft complex valued metric space. Then,* (Θ, Ψ, Ξ) *is called a soft Hausdorff space if for any two soft elements* $\tilde{\varrho}, \tilde{\zeta} \in \Theta^{\Xi}$ *such that* $\Psi(\tilde{\varrho}, \tilde{\zeta}) \succsim \overset{\triangle}{0}$*, there exist two soft open balls* $B(\tilde{\varrho}, \hat{z})$ *and* $B(\tilde{\zeta}, \hat{u})$ *such that* $B(\tilde{\varrho}, \hat{z}) \sqcap B(\tilde{\zeta}, \hat{u}) = \widetilde{\emptyset}$*.*

Theorem 3.13 *Every soft complex valued metric space is a soft Hausdorff space.*

Proof Let $\tilde{\varrho}, \tilde{\zeta} \in \Theta^{\Xi}$ with $\Psi(\tilde{\varrho}, \tilde{\zeta}) \succsim \overset{\triangle}{0}$. Then, we obtain $|\Psi(\tilde{\varrho}, \tilde{\zeta})| = \tilde{r} \mathrel{\tilde{>}} \overline{0}$. Let us consider $\hat{z} \succsim \overset{\triangle}{0}$ such that $|\hat{z}| = \frac{\tilde{r}}{2}$. It is clear that $\tilde{\varrho} \mathrel{\tilde{\in}} B(\tilde{\varrho}, \hat{z})$ and $\tilde{\zeta} \mathrel{\tilde{\in}} B(\tilde{\zeta}, \hat{z})$. Also, we verify that $B(\tilde{\varrho}, \hat{z}) \sqcap B(\tilde{\zeta}, \hat{z}) = \widetilde{\emptyset}$. Suppose instead that there is a $\tilde{\varrho^*} \mathrel{\tilde{\in}} B(\tilde{\varrho}, \hat{z}) \sqcap B(\tilde{\zeta}, \hat{z})$. Hence,

$$\Psi(\tilde{\varrho}, \tilde{\zeta}) \mathrel{\tilde{\precsim}} \Psi(\tilde{\varrho}, \tilde{\varrho^*}) + \Psi(\tilde{\varrho^*}, \tilde{\zeta}) \precsim 2\hat{z}.$$

So, $|\Psi(\tilde{\varrho}, \tilde{\zeta})| \mathrel{\tilde{<}} 2|\hat{z}| = \tilde{r} = |\Psi(\tilde{\varrho}, \tilde{\zeta})|$ and we get a contradiction. □

Definition 3.14 *Let* $f_{\Xi}, g_{\Xi} : (\Theta, \Psi, \Xi) \to (\Theta, \Psi, \Xi)$ *be two soft mappings, where* (Θ, Ψ, Ξ) *is a soft complex valued metric space.*

(i) A soft element $\tilde{\varrho} \in \Theta^{\Xi}$ *is said to be a fixed soft element of* f_{Ξ} *if* $f_{\Xi}(\tilde{\varrho}) = \tilde{\varrho}$*.*

(ii) A soft element $\tilde{\varrho} \in \Theta^{\Xi}$ *is said to be a common fixed soft element of* f_{Ξ} *and* g_{Ξ} *if* $f_{\Xi}(\tilde{\varrho}) = g_{\Xi}(\tilde{\varrho}) = \tilde{\varrho}$*.*

4. Main results

In this part, proceeding on the lines of [4, 15], we demonstrate some fixed soft element theorems for various soft mappings on soft complex valued metric spaces. Firstly, we give some lemmas which are going to be useful in the proof of our main results.

Throughout this paper, as to move forward effectively, we will define the case of non-equality of two soft elements as follows:

$\tilde{\varrho} \neq \tilde{\zeta} \Leftrightarrow \tilde{\varrho}(e) \neq \tilde{\zeta}(e)$ *for every* $e \in \Xi$.

Lemma 4.1 *Let* (Θ, Ψ, Ξ) *be a soft complex valued metric space and* $\{\widetilde{\varrho_n}\}$ *be a sequence of soft elements in* Θ*. Then,* $\{\widetilde{\varrho_n}\}$ *converges to* $\tilde{\varrho}$ *if and only if* $lim_{n\to\infty}|\Psi(\widetilde{\varrho_n}, \tilde{\varrho})| = \overline{0}$*.*

Proof Let $\{\widetilde{\varrho_n}\}$ converges to $\tilde{\varrho}$ in (Θ, Ψ, Ξ). For a soft real number $\tilde{\epsilon} \mathrel{\tilde{>}} \overline{0}$, let $\hat{c} = \frac{\tilde{\epsilon}}{\sqrt{2}} + i\frac{\tilde{\epsilon}}{\sqrt{2}}$. Since $\overset{\triangle}{0} \precsim \hat{c} \in \mathbb{C}^{\Xi}$, there exists an $n_0 \in \mathbb{N}$ such that $\Psi(\widetilde{\varrho_n}, \tilde{\varrho}) \precsim \hat{c}$ for all $n \geq n_o$. From the fact that

$$|\Psi(\widetilde{\varrho_n}, \tilde{\varrho})| \mathrel{\tilde{<}} |\hat{c}| = \tilde{\epsilon} \quad \textit{for all } n \geq n_o$$

it follows that $lim_{n\to\infty}|\Psi(\widetilde{\varrho_n},\tilde{\varrho})|=\overline{0}$.

For the converse, let us take a $\hat{c}\in\mathbb{C}^{\Xi}$ with $\hat{c}\,\widetilde{\succ}\,\overset{\triangle}{0}$. Then, there is a soft real number $\tilde{r}\,\widetilde{>}\,\overline{0}$ such that for $\hat{z}\in\mathbb{C}^{\Xi}$

$$if\ |\hat{z}|\,\widetilde{<}\,\tilde{r}\ \ then\ \ \hat{z}\,\widetilde{\prec}\,\hat{c}.$$

Since $\tilde{r}\,\widetilde{>}\,\overline{0}$, by hypothesis, there exists a natural number n_0 with $|\Psi(\widetilde{\varrho_n},\tilde{\varrho})|\,\widetilde{<}\,\tilde{r}$ for each $n\geq n_0$. Hence, we have $\Psi(\widetilde{\varrho_n},\tilde{\varrho})\,\widetilde{\prec}\,\hat{c}$ for all $n\geq n_0$, which completes the proof. □

Lemma 4.2 *Let* (Θ,Ψ,Ξ) *be a soft complex valued metric space and* $\{\widetilde{\varrho_n}\}$ *be a sequence of soft elements in* Θ. *Then,* $\{\widetilde{\varrho_n}\}$ *is a Cauchy sequence in* (Θ,Ψ,Ξ) *if and only if* $lim_{n\to\infty}|\Psi(\widetilde{\varrho_n},\widetilde{\varrho_{n+m}})|=\overline{0}$ *with* $m\in\mathbb{N}$.

Proof Let $\{\widetilde{\varrho_n}\}$ be a Cauchy sequence in (Θ,Ψ,Ξ) and take a soft real number $\tilde{\epsilon}\,\widetilde{>}\,\overline{0}$. Consider $\hat{c}=\frac{\tilde{\epsilon}}{\sqrt{2}}+i\frac{\tilde{\epsilon}}{\sqrt{2}}$. From the fact that $\overset{\triangle}{0}\,\widetilde{\prec}\,\hat{c}\in\mathbb{C}^{\Xi}$ it follows that there exists a natural number n_0 satisfying for all $n\geq n_o$, $\Psi(\widetilde{\varrho_n},\widetilde{\varrho_{n+m}})\,\widetilde{\prec}\,\hat{c}$ with $m\in\mathbb{N}$. Since

$$|\Psi(\widetilde{\varrho_n},\widetilde{\varrho_{n+m}})|\,\widetilde{<}\,|\hat{c}|=\tilde{\epsilon}\quad for\ all\ n\geq n_o$$

we get $lim_{n\to\infty}|\Psi(\widetilde{\varrho_n},\widetilde{\varrho_{n+m}})|=\overline{0}$.

To prove the sufficiency, let us take a $\hat{c}\in\mathbb{C}^{\Xi}$ with $\hat{c}\,\widetilde{\succ}\,\overset{\triangle}{0}$. Then, there is a soft real number $\tilde{r}\,\widetilde{>}\,\overline{0}$ such that for $\hat{z}\in\mathbb{C}^{\Xi}$

$$if\ |\hat{z}|\,\widetilde{<}\,\tilde{r}\ \ then\ \ \hat{z}\,\widetilde{\prec}\,\hat{c}.$$

Therefore, by $\tilde{r}\,\widetilde{>}\,\overline{0}$, there exists a natural number n_0 such that $|\Psi(\widetilde{\varrho_n},\widetilde{\varrho_{n+m}})|\,\widetilde{<}\,\tilde{r}$ for all $n\geq n_0$. Thus, we obtain $\Psi(\widetilde{\varrho_n},\widetilde{\varrho_{n+m}})\,\widetilde{\prec}\,\hat{c}$ for all $n\geq n_0$ and the proof is concluded. □

Lemma 4.3 *Let* (Θ,Ψ,Ξ) *be a soft complex valued metric space and* $\{\widetilde{\varrho_n}\}$, $\{\widetilde{\zeta_n}\}$ *be two sequences in* Θ^{Ξ}. *If there exist soft elements* $\tilde{\varrho},\tilde{\zeta}\in\Theta^{\Xi}$ *such that* $lim_{n\to\infty}\widetilde{\varrho_n}=\tilde{\varrho}$ *and* $lim_{n\to\infty}\widetilde{\zeta_n}=\tilde{\zeta}$, *then we have* $lim_{n\to\infty}\Psi(\widetilde{\varrho_n},\widetilde{\zeta_n})=\Psi(\tilde{\varrho},\tilde{\zeta})$.

Proof Let $\widetilde{\epsilon}\,\widetilde{>}\,\overline{0}$. Then, by Lemma 4.1, there exist $m_1,m_2\in\mathbb{N}$ satisfying

$$|\Psi(\widetilde{\varrho_n},\tilde{\varrho})|\,\widetilde{<}\,\frac{\widetilde{\epsilon}}{2}\ \ for\ all\ n\geq m_1\quad and\quad |\Psi(\widetilde{\zeta_n},\tilde{\zeta})|\,\widetilde{<}\,\frac{\widetilde{\epsilon}}{2}\ \ for\ all\ n\geq m_2.$$

If we get $m=max\{m_1,m_2\}$, then for all $n\geq m$, we obtain

$$\begin{aligned}|\Psi(\widetilde{\varrho_n},\widetilde{\zeta_n})-\Psi(\tilde{\varrho},\tilde{\zeta})|&\,\widetilde{\leq}\,|\Psi(\widetilde{\varrho_n},\widetilde{\zeta_n})-\Psi(\widetilde{\varrho_n},\tilde{\zeta})|+|\Psi(\widetilde{\varrho_n},\tilde{\zeta})-\Psi(\tilde{\varrho},\tilde{\zeta})|\\&\,\widetilde{\leq}\,|\Psi(\widetilde{\zeta_n},\tilde{\zeta})|+|\Psi(\widetilde{\varrho_n},\tilde{\varrho})|\\&\,\widetilde{<}\,\frac{\widetilde{\epsilon}}{2}+\frac{\widetilde{\epsilon}}{2}=\widetilde{\epsilon}.\end{aligned}$$

Thus, we infer that $lim_{n\to\infty}\Psi(\widetilde{\varrho_n},\widetilde{\zeta_n})=\Psi(\tilde{\varrho},\tilde{\zeta})$. □

Theorem 4.4 *Let* (Θ,Ψ,Ξ) *be a complete soft complex valued metric space with* Ξ *a finite set and let* $f_\Xi, g_\Xi : (\Theta,\Psi,\Xi) \to (\Theta,\Psi,\Xi)$ *be soft mappings satisfying*

$$\Psi(f_\Xi(\tilde{\varrho}), g_\Xi(\tilde{\zeta})) \,\widetilde{\precsim}\, \tilde{r}\,\Psi(\tilde{\varrho},\tilde{\zeta}) + \frac{\tilde{s}\,\Psi(\tilde{\varrho}, f_\Xi(\tilde{\varrho}))\,\Psi(\tilde{\zeta}, g_\Xi(\tilde{\zeta}))}{\overset{\triangle}{1} + \Psi(\tilde{\varrho},\tilde{\zeta})}$$

for all $\tilde{\varrho},\tilde{\zeta} \in \Theta^\Xi$*, where* $\tilde{r},\tilde{s}$ *are nonnegative soft real numbers with* $\tilde{r}+\tilde{s} \,\widetilde{<}\, \overline{1}$*. Then,* f_Ξ *and* g_Ξ *have a unique common fixed soft element.*

Proof Consider $\widetilde{\varrho_0} \in \Theta^\Xi$ and take a sequence $\{\widetilde{\varrho_n}\}$ in Θ^Ξ such that

$$f_\Xi(\widetilde{\varrho_{2n}}) = \widetilde{\varrho_{2n+1}} \;\; and \;\; g_\Xi(\widetilde{\varrho_{2n+1}}) = \widetilde{\varrho_{2n+2}} \;\; for\; n \in \{0,1,...\}.$$

Then, we have

$$\begin{aligned}\Psi(\widetilde{\varrho_{2n+1}},\widetilde{\varrho_{2n+2}}) &= \Psi(f_\Xi(\widetilde{\varrho_{2n}}), g_\Xi(\widetilde{\varrho_{2n+1}}))\\ &\widetilde{\precsim}\, \tilde{r}\,\Psi(\widetilde{\varrho_{2n}},\widetilde{\varrho_{2n+1}}) + \frac{\tilde{s}\,\Psi(\widetilde{\varrho_{2n+1}}, g_\Xi(\widetilde{\varrho_{2n+1}}))\,\Psi(\widetilde{\varrho_{2n}}, f_\Xi(\widetilde{\varrho_{2n}}))}{\overset{\triangle}{1} + \Psi(\widetilde{\varrho_{2n}},\widetilde{\varrho_{2n+1}})}\\ &\widetilde{\precsim}\, \tilde{r}\,\Psi(\widetilde{\varrho_{2n}},\widetilde{\varrho_{2n+1}}) + \frac{\tilde{s}\,\Psi(\widetilde{\varrho_{2n+1}},\widetilde{\varrho_{2n+2}})\,\Psi(\widetilde{\varrho_{2n}},\widetilde{\varrho_{2n+1}})}{\overset{\triangle}{1} + \Psi(\widetilde{\varrho_{2n}},\widetilde{\varrho_{2n+1}})}.\end{aligned}$$

From the fact that

$$\Psi(\widetilde{\varrho_{2n}},\widetilde{\varrho_{2n+1}}) \,\widetilde{\precsim}\, \overset{\triangle}{1} + \Psi(\widetilde{\varrho_{2n}},\widetilde{\varrho_{2n+1}})$$

it follows that

$$\Psi(\widetilde{\varrho_{2n+1}},\widetilde{\varrho_{2n+2}}) \,\widetilde{\precsim}\, \frac{\tilde{r}}{\overline{1}-\tilde{s}}\Psi(\widetilde{\varrho_{2n}},\widetilde{\varrho_{2n+1}}).$$

Similarly, we obtain

$$\begin{aligned}\Psi(\widetilde{\varrho_{2n+2}},\widetilde{\varrho_{2n+3}}) &= \Psi(f_\Xi(\widetilde{\varrho_{2n+2}}), g_\Xi(\widetilde{\varrho_{2n+1}}))\\ &\widetilde{\precsim}\, \tilde{r}\,\Psi(\widetilde{\varrho_{2n+2}},\widetilde{\varrho_{2n+1}}) + \frac{\tilde{s}\,\Psi(\widetilde{\varrho_{2n+1}}, g_\Xi(\widetilde{\varrho_{2n+1}}))\,\Psi(\widetilde{\varrho_{2n+2}}, f_\Xi(\widetilde{\varrho_{2n+2}}))}{\overset{\triangle}{1} + \Psi(\widetilde{\varrho_{2n+2}},\widetilde{\varrho_{2n+1}})}\\ &\widetilde{\precsim}\, \tilde{r}\,\Psi(\widetilde{\varrho_{2n+2}},\widetilde{\varrho_{2n+1}}) + \frac{\tilde{s}\,\Psi(\widetilde{\varrho_{2n+1}},\widetilde{\varrho_{2n+2}})\,\Psi(\widetilde{\varrho_{2n+2}},\widetilde{\varrho_{2n+3}})}{\overset{\triangle}{1} + \Psi(\widetilde{\varrho_{2n+1}},\widetilde{\varrho_{2n+2}})}.\end{aligned}$$

Since

$$\Psi(\widetilde{\varrho_{2n+1}},\widetilde{\varrho_{2n+2}}) \,\widetilde{\precsim}\, \overset{\triangle}{1} + \Psi(\widetilde{\varrho_{2n+1}},\widetilde{\varrho_{2n+2}})$$

we get

$$\Psi(\widetilde{\varrho_{2n+2}},\widetilde{\varrho_{2n+3}}) \,\widetilde{\precsim}\, \frac{\tilde{r}}{\overline{1}-\tilde{s}}\Psi(\widetilde{\varrho_{2n+2}},\widetilde{\varrho_{2n+1}}).$$

Let us take $\tilde{h} = \frac{\tilde{r}}{\bar{1}-\tilde{s}} \precsim \bar{1}$. Then, for all $n \in \mathbb{N}$, we have

$$\Psi(\widetilde{\varrho_{n+1}}, \widetilde{\varrho_{n+2}}) \precsim \tilde{h}\,\Psi(\widetilde{\varrho_n}, \widetilde{\varrho_{n+1}}) \precsim \ldots \precsim (\tilde{h})^{n+1}\,\Psi(\widetilde{\varrho_0}, \widetilde{\varrho_1}).$$

So, for any $m > n$,

$$\begin{aligned}\Psi(\widetilde{\varrho_n}, \widetilde{\varrho_m}) &\precsim \Psi(\widetilde{\varrho_n}, \widetilde{\varrho_{n+1}}) + \Psi(\widetilde{\varrho_{n+1}}, \widetilde{\varrho_{n+2}}) + \ldots + \Psi(\widetilde{\varrho_{m-1}}, \widetilde{\varrho_m})\\ &\precsim \left((\tilde{h})^n + (\tilde{h})^{n+1} + \ldots + (\tilde{h})^{m-1}\right)\Psi(\widetilde{\varrho_0}, \widetilde{\varrho_1})\\ &\precsim \frac{(\tilde{h})^n}{\bar{1}-\tilde{h}}\Psi(\widetilde{\varrho_0}, \widetilde{\varrho_1}).\end{aligned}$$

Therefore, we obtain $|\Psi(\widetilde{\varrho_n}, \widetilde{\varrho_m})| \leqq \frac{(\tilde{h})^n}{\bar{1}-\tilde{h}}|\Psi(\widetilde{\varrho_0}, \widetilde{\varrho_1})|$. As Ξ is a finite set, $lim_{n\to\infty}(\tilde{h})^n = \bar{0}$ and so, by Lemma 4.2, it follows that $\{\widetilde{\varrho_n}\}$ is a Cauchy sequence in (Θ, Ψ, Ξ). Hence, since (Θ, Ψ, Ξ) is a complete space, there exists a $\tilde{\varrho} \in \Theta^{\Xi}$ such that $lim_{n\to\infty}\widetilde{\varrho_n} = \tilde{\varrho}$. Now, let $\Psi(\tilde{\varrho}, f_{\Xi}(\tilde{\varrho})) = \hat{z}$. Then, we obtain

$$\begin{aligned}\hat{z} &\precsim \Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+2}}) + \Psi(\widetilde{\varrho_{2n+2}}, f_{\Xi}(\tilde{\varrho}))\\ &= \Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+2}}) + \Psi(g_{\Xi}(\widetilde{\varrho_{2n+1}}), f_{\Xi}(\tilde{\varrho}))\\ &\precsim \Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+2}}) + \tilde{r}\,\Psi(\widetilde{\varrho_{2n+1}}, \tilde{\varrho}) + \frac{\tilde{s}\,\Psi(\tilde{\varrho}, f_{\Xi}(\tilde{\varrho}))\,\Psi(\widetilde{\varrho_{2n+1}}, g_{\Xi}(\widetilde{\varrho_{2n+1}}))}{\overset{\triangle}{1} + \Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+1}})}\\ &= \Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+2}}) + \tilde{r}\,\Psi(\widetilde{\varrho_{2n+1}}, \tilde{\varrho}) + \frac{\tilde{s}\,\Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+2}})\,\hat{z}}{\overset{\triangle}{1} + \Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+1}})}.\end{aligned}$$

So, from Theorem 2.14, we have

$$|\hat{z}| \leqq |\Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+2}})| + \tilde{r}\,|\Psi(\widetilde{\varrho_{2n+1}}, \tilde{\varrho})| + \frac{\tilde{s}\,|\Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+2}})|\,|\hat{z}|}{|\overset{\triangle}{1} + \Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+1}})|}.$$

Taking the limit of this inequality as $n \to \infty$ gives $|\hat{z}| = \bar{0}$. Therefore, we get $f_{\Xi}(\tilde{\varrho}) = \tilde{\varrho}$. Arguing the same, one can show that $\tilde{\varrho}$ is a fixed soft element of g_{Ξ}.

For the uniqueness result, assume that $\tilde{\varrho^*}$ is another common fixed soft element of f_{Ξ} and g_{Ξ}. Then, $\Psi(\tilde{\varrho}, \tilde{\varrho^*}) = \Psi(f_{\Xi}(\tilde{\varrho}), g_{\Xi}(\tilde{\varrho^*})) \precsim \tilde{r}\Psi(\tilde{\varrho}, \tilde{\varrho^*}) + \frac{\tilde{s}\Psi(\tilde{\varrho}, f_{\Xi}(\tilde{\varrho}))\,\Psi(\tilde{\varrho}, g_{\Xi}(\tilde{\varrho^*}))}{\overset{\triangle}{1}+\Psi(\tilde{\varrho}, \tilde{\varrho^*})} = \tilde{r}\Psi(\tilde{\varrho}, \tilde{\varrho^*}) \not\precsim \Psi(\tilde{\varrho}, \tilde{\varrho^*})$.

Thus, we arrive at a contradiction, so that $\tilde{\varrho} = \tilde{\varrho^*}$, which proves the uniqueness of common fixed soft element in (Θ, Ψ, Ξ). □

Corollary 4.5 *Let (Θ, Ψ, Ξ) be a complete soft complex valued metric space with Ξ a finite set and let $f_{\Xi} : (\Theta, \Psi, \Xi) \to (\Theta, \Psi, \Xi)$ be a soft mapping such that*

$$\Psi(f_{\Xi}(\tilde{\varrho}), f_{\Xi}(\tilde{\zeta})) \precsim \tilde{r}\,\Psi(\tilde{\varrho}, \tilde{\zeta}) + \frac{\tilde{s}\,\Psi(\tilde{\varrho}, f_{\Xi}(\tilde{\varrho}))\,\Psi(\tilde{\zeta}, f_{\Xi}(\tilde{\zeta}))}{\overset{\triangle}{1} + \Psi(\tilde{\varrho}, \tilde{\zeta})}$$

for all $\tilde{\varrho}, \tilde{\zeta} \in \Theta^{\Xi}$, where $\tilde{r}, \tilde{s}$ are nonnegative soft real numbers satisfying $\tilde{r} + \tilde{s} \precsim \bar{1}$. Then, f_{Ξ} has a unique fixed soft element.

Proof This result can be proved by utilizing Theorem 4.4 with $f_\Xi = g_\Xi$. □

Corollary 4.6 *Let* (Θ, Ψ, Ξ) *be a complete soft complex valued metric space with* Ξ *a finite set. Consider a soft mapping* $f_\Xi : (\Theta, \Psi, \Xi) \to (\Theta, \Psi, \Xi)$ *such that*

$$\Psi(f^n_\Xi(\tilde{\varrho}), f^n_\Xi(\tilde{\zeta})) \,\widetilde{\precsim}\, \tilde{r}\,\Psi(\tilde{\varrho}, \tilde{\zeta}) + \frac{\tilde{s}\,\Psi(\tilde{\varrho}, f^n_\Xi(\tilde{\varrho}))\,\Psi(\tilde{\zeta}, f^n_\Xi(\tilde{\zeta}))}{\overset{\triangle}{1} + \Psi(\tilde{\varrho}, \tilde{\zeta})}$$

for all $\tilde{\varrho}, \tilde{\zeta} \in \Theta^\Xi$*, where* $\tilde{r}, \tilde{s}$ *are nonnegative soft real numbers with* $\tilde{r} + \tilde{s} \,\widetilde{<}\, \overline{1}$*. Then, there exists a unique fixed soft element of* f_Ξ *(Here,* f^n_Ξ *is the* n*th iterate of* f_Ξ*).*

Proof By Corollary 4.5, we have $\tilde{\varrho} \in \Theta^\Xi$ such that $f^n_\Xi(\tilde{\varrho}) = \tilde{\varrho}$. Now, suppose that $f_\Xi(\tilde{\varrho}) \neq \tilde{\varrho}$. Because of Definition 3.5 (scm_2),

$$\Psi(f_\Xi(\tilde{\varrho}), \tilde{\varrho})(e) \neq \overset{\triangle}{0}(e) = \overline{0}(e) + i\overline{0}(e) = 0 + i0 \;\; for\ every\ e \in \Xi.$$

Then, we have

$$\begin{aligned}
\Psi(f_\Xi(\tilde{\varrho}), \tilde{\varrho}) &= \Psi(f_\Xi(f^n_\Xi(\tilde{\varrho})), f^n_\Xi(\tilde{\varrho})) = \Psi(f^n_\Xi(f_\Xi(\tilde{\varrho})), f^n_\Xi(\tilde{\varrho})) \\
&\widetilde{\precsim}\, \tilde{r}\,\Psi(f_\Xi(\tilde{\varrho}), \tilde{\varrho}) + \frac{\tilde{s}\,\Psi(f_\Xi(\tilde{\varrho}), f^n_\Xi(f_\Xi(\tilde{\varrho})))\,\Psi(\tilde{\varrho}, f^n_\Xi(\tilde{\varrho}))}{\overset{\triangle}{1} + \Psi(f_\Xi(\tilde{\varrho}), \tilde{\varrho})} \\
&= \tilde{r}\,\Psi(f_\Xi(\tilde{\varrho}), \tilde{\varrho}) + \frac{\tilde{s}\,\Psi(f_\Xi(\tilde{\varrho}), f_\Xi(f^n_\Xi(\tilde{\varrho})))\,\Psi(\tilde{\varrho}, \tilde{\varrho})}{\overset{\triangle}{1} + \Psi(f_\Xi(\tilde{\varrho}), \tilde{\varrho})} \\
&= \tilde{r}\,\Psi(f_\Xi(\tilde{\varrho}), \tilde{\varrho}) \\
&\widetilde{\precnsim}\, \Psi(f_\Xi(\tilde{\varrho}), \tilde{\varrho}),
\end{aligned}$$

a contradiction. Hence, we get $f_\Xi(\tilde{\varrho}) = \tilde{\varrho}$. Thus, since

$$f_\Xi(\tilde{\varrho}) = f^n_\Xi(\tilde{\varrho}) = \tilde{\varrho},$$

the fixed soft element of f_Ξ is unique. □

We adopt an example to demonstrate the validity of hypotheses of Corollary 4.6.

Example 4.7 *Let us take Example 3.7 as complete soft complex valued metric space. Consider a mapping* $f : \mathbb{C} \to \mathbb{C}$ *by*

$$f(z) = \begin{cases} 0, & if\ \ x, y \in \mathbb{Q}, \\ 3, & if\ \ x \in \mathbb{Q}^c, y \in \mathbb{Q}, \\ i3, & if\ \ x \in \mathbb{Q}^c, y \in \mathbb{Q}^c, \\ 3 + i3, & if\ \ x \in \mathbb{Q}, y \in \mathbb{Q}^c, \end{cases}$$

where $z = x + iy \in \mathbb{C}$*. Now, for* $\hat{z} = \overset{\triangle}{\left(\frac{1}{\sqrt{3}}\right)}$ *and* $\hat{u} = \overset{\triangle}{0}$*, we get*

$$\Psi(f_\Xi(\hat{z}), f_\Xi(\hat{u})) = \overline{0} + i\overline{3} \precsim \tilde{r}\,\Psi(\hat{z},\hat{u}) + \frac{\tilde{s}\,\Psi(\hat{z}, f_\Xi(\hat{z}))\,\Psi(\hat{u}, f_\Xi(\hat{u}))}{\overset{\triangle}{1} + \Psi(\hat{z},\hat{u})} = \tilde{r}\left(\overset{\triangle}{\frac{1}{\sqrt{3}}}\right) + \overset{\triangle}{0} = \tilde{r}\left(\overset{\triangle}{\frac{1}{\sqrt{3}}}\right)$$

which is a contradiction for every choice of $\tilde{r}$ *satisfying* $\overline{0} \widetilde{\leq} \tilde{r} \widetilde{<} \overline{1}$. *However, notice that* $f_\Xi^n(\hat{z}) = \overset{\triangle}{0}$ *for every* $\hat{z} \in \mathbb{C}^\Xi$ *and* $n > 1$, *so that*

$$\overset{\triangle}{0} = \Psi(f_\Xi^n(\hat{z}), f_\Xi^n(\hat{u})) \precsim \tilde{r}\,\Psi(\hat{z},\hat{u}) + \frac{\tilde{s}\,\Psi(\hat{z}, f_\Xi^n(\hat{z}))\Psi(\hat{u}, f_\Xi^n(\hat{u}))}{\overset{\triangle}{1} + \Psi(\hat{z},\hat{u})}$$

for every $\hat{z}, \hat{u} \in \mathbb{C}^\Xi$ *and* $\tilde{r}, \tilde{s} \widetilde{\geq} \overline{0}$ *with* $\tilde{r} + \tilde{s} \widetilde{<} \overline{1}$. *So, we deduce the existence and uniqueness of the fixed soft element of* f_Ξ. *Here,* $\overset{\triangle}{0}$ *is the unique fixed soft element of* f_Ξ.

Theorem 4.8 *Let* (Θ, Ψ, Ξ) *be a complete soft complex valued metric space with* Ξ *a finite set and let* $f_\Xi, g_\Xi : (\Theta, \Psi, \Xi) \to (\Theta, \Psi, \Xi)$ *be soft mappings satisfying*

$$\Psi(f_\Xi(\tilde{\varrho}), g_\Xi(\tilde{\zeta})) \precsim \tilde{r}\,\Psi(\tilde{\varrho}, \tilde{\zeta}) + \frac{\tilde{s}\,\Psi(\tilde{\varrho}, f_\Xi(\tilde{\varrho}))\,\Psi(\tilde{\zeta}, g_\Xi(\tilde{\zeta}))}{\Psi(\tilde{\varrho}, g_\Xi(\tilde{\zeta})) + \Psi(\tilde{\zeta}, f_\Xi(\tilde{\varrho})) + \Psi(\tilde{\varrho}, \tilde{\zeta})}$$

for all $\tilde{\varrho}, \tilde{\zeta} \in \Theta^\Xi$ *such that* $\Psi(\tilde{\varrho}, g_\Xi(\tilde{\zeta})) + \Psi(\tilde{\zeta}, f_\Xi(\tilde{\varrho})) + \Psi(\tilde{\varrho}, \tilde{\zeta}) \neq \overset{\triangle}{0}$ *where* $\tilde{r}, \tilde{s}$ *are nonnegative soft real numbers with* $\tilde{r} + \tilde{s} \widetilde{<} \overline{1}$ *or* $\Psi(f_\Xi(\tilde{\varrho}), g_\Xi(\tilde{\zeta})) = \overset{\triangle}{0}$ *if* $\Psi(\tilde{\varrho}, g_\Xi(\tilde{\zeta})) + \Psi(\tilde{\zeta}, f_\Xi(\tilde{\varrho})) + \Psi(\tilde{\varrho}, \tilde{\zeta}) = \overset{\triangle}{0}$. *Then,* f_Ξ *and* g_Ξ *have a unique common fixed soft element.*

Proof Take $\widetilde{\varrho_0} \in \Theta^\Xi$. Construct a sequence $\{\widetilde{\varrho_n}\}$ in Θ^Ξ as follows:

$$f_\Xi(\widetilde{\varrho_{2n}}) = \widetilde{\varrho_{2n+1}} \text{ and } g_\Xi(\widetilde{\varrho_{2n+1}}) = \widetilde{\varrho_{2n+2}} \text{ for } n \in \{0, 1, ...\}.$$

Then, we get

$$\begin{aligned}
\Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+2}}) &= \Psi(f_\Xi(\widetilde{\varrho_{2n}}), g_\Xi(\widetilde{\varrho_{2n+1}})) \\
&\precsim \tilde{r}\,\Psi(\widetilde{\varrho_{2n}}, \widetilde{\varrho_{2n+1}}) + \frac{\tilde{s}\,\Psi(\widetilde{\varrho_{2n+1}}, g_\Xi(\widetilde{\varrho_{2n+1}}))\,\Psi(\widetilde{\varrho_{2n}}, f_\Xi(\widetilde{\varrho_{2n}}))}{\Psi(\widetilde{\varrho_{2n}}, g_\Xi(\widetilde{\varrho_{2n+1}})) + \Psi(\widetilde{\varrho_{2n+1}}, f_\Xi(\widetilde{\varrho_{2n}})) + \Psi(\widetilde{\varrho_{2n}}, \widetilde{\varrho_{2n+1}})} \\
&\precsim \tilde{r}\,\Psi(\widetilde{\varrho_{2n}}, \widetilde{\varrho_{2n+1}}) + \frac{\tilde{s}\,\Psi(\widetilde{\varrho_{2n}}, \widetilde{\varrho_{2n+1}})\,\Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+2}})}{\Psi(\widetilde{\varrho_{2n}}, \widetilde{\varrho_{2n+2}}) + \Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+1}}) + \Psi(\widetilde{\varrho_{2n}}, \widetilde{\varrho_{2n+1}})}.
\end{aligned}$$

Because

$$\Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+2}}) \precsim \Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n}}) + \Psi(\widetilde{\varrho_{2n}}, \widetilde{\varrho_{2n+2}})$$

we obtain

$$\Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+2}}) \precsim \tilde{r}\,\Psi(\widetilde{\varrho_{2n}}, \widetilde{\varrho_{2n+1}}) + \tilde{s}\,\Psi(\widetilde{\varrho_{2n}}, \widetilde{\varrho_{2n+1}}) = (\tilde{r} + \tilde{s})\,\Psi(\widetilde{\varrho_{2n}}, \widetilde{\varrho_{2n+1}}).$$

Similarly, replacing $\tilde{\varrho}$ by $\widetilde{\varrho_{2n+2}}$ and $\tilde{\zeta}$ by $\widetilde{\varrho_{2n+1}}$, we get

$$\begin{aligned}\Psi(\widetilde{\varrho_{2n+2}}, \widetilde{\varrho_{2n+3}}) &= \Psi(f_\Xi(\widetilde{\varrho_{2n+2}}), g_\Xi(\widetilde{\varrho_{2n+1}}))\\ &\widetilde{\precsim} \tilde{r}\,\Psi(\widetilde{\varrho_{2n+2}}, \widetilde{\varrho_{2n+1}}) + \frac{\tilde{s}\,\Psi(\widetilde{\varrho_{2n+2}}, f_\Xi(\widetilde{\varrho_{2n+2}}))\,\Psi(\widetilde{\varrho_{2n+1}}, g_\Xi(\widetilde{\varrho_{2n+1}}))}{\Psi(\widetilde{\varrho_{2n+1}}, f_\Xi(\widetilde{\varrho_{2n+2}})) + \Psi(\widetilde{\varrho_{2n+2}}, g_\Xi(\widetilde{\varrho_{2n+1}})) + \Psi(\widetilde{\varrho_{2n+2}}, \widetilde{\varrho_{2n+1}})}\\ &\widetilde{\precsim} \tilde{r}\,\Psi(\widetilde{\varrho_{2n+2}}, \widetilde{\varrho_{2n+1}}) + \frac{\tilde{s}\,\Psi(\widetilde{\varrho_{2n+2}}, \widetilde{\varrho_{2n+3}})\,\Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+2}})}{\Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+3}}) + \Psi(\widetilde{\varrho_{2n+2}}, \widetilde{\varrho_{2n+2}}) + \Psi(\widetilde{\varrho_{2n+2}}, \widetilde{\varrho_{2n+1}})}.\end{aligned}$$

From the fact that

$$\Psi(\widetilde{\varrho_{2n+2}}, \widetilde{\varrho_{2n+3}}) \widetilde{\precsim} \Psi(\widetilde{\varrho_{2n+2}}, \widetilde{\varrho_{2n+1}}) + \Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+3}})$$

it follows that

$$\Psi(\widetilde{\varrho_{2n+2}}, \widetilde{\varrho_{2n+3}}) \widetilde{\precsim} \tilde{r}\,\Psi(\widetilde{\varrho_{2n+2}}, \widetilde{\varrho_{2n+1}}) + \tilde{s}\,\Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+2}}) = (\tilde{r}+\tilde{s})\,\Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+2}}).$$

Let us take $\tilde{h} = (\tilde{r}+\tilde{s}) \widetilde{<} \overline{1}$. Therefore, for all $n \in \mathbb{N}$, we have

$$\Psi(\widetilde{\varrho_{n+1}}, \widetilde{\varrho_{n+2}}) \widetilde{\precsim} \tilde{h}\,\Psi(\widetilde{\varrho_n}, \widetilde{\varrho_{n+1}}) \widetilde{\precsim} \dots \widetilde{\precsim} (\tilde{h})^{n+1}\,\Psi(\widetilde{\varrho_0}, \widetilde{\varrho_1}).$$

Now, we shall verify that $\{\widetilde{\varrho_n}\}$ is a Cauchy sequence in (Θ, Ψ, Ξ). For any $m > n$,

$$\begin{aligned}\Psi(\widetilde{\varrho_n}, \widetilde{\varrho_m}) &\widetilde{\precsim} \Psi(\widetilde{\varrho_n}, \widetilde{\varrho_{n+1}}) + \Psi(\widetilde{\varrho_{n+1}}, \widetilde{\varrho_{n+2}}) + \dots + \Psi(\widetilde{\varrho_{m-1}}, \widetilde{\varrho_m})\\ &\widetilde{\precsim} \left((\tilde{h})^n + (\tilde{h})^{n+1} + \dots + (\tilde{h})^{m-1}\right)\Psi(\widetilde{\varrho_0}, \widetilde{\varrho_1})\\ &\widetilde{\precsim} \tfrac{(\tilde{h})^n}{\overline{1}-\tilde{h}}\Psi(\widetilde{\varrho_0}, \widetilde{\varrho_1}).\end{aligned}$$

Hence, we have $|\Psi(\widetilde{\varrho_n}, \widetilde{\varrho_m})| \widetilde{\leq} \frac{(\tilde{h})^n}{\overline{1}-\tilde{h}}|\Psi(\widetilde{\varrho_0}, \widetilde{\varrho_1})|$. Because Ξ is a finite set, $lim_{n\to\infty}(\tilde{h})^n = \overline{0}$ and therefore, by Lemma 4.2, $\{\widetilde{\varrho_n}\}$ is a Cauchy sequence in (Θ, Ψ, Ξ). Because of completeness, there is a $\tilde{\varrho} \in \Theta^\Xi$ such that $lim_{n\to\infty}\widetilde{\varrho_n} = \tilde{\varrho}$.

After that, we shall demonstrate that $\tilde{\varrho}$ is a fixed soft element of f_Ξ. Therefore, let $\Psi(\tilde{\varrho}, f_\Xi(\tilde{\varrho})) = \hat{z}$. By using the triangular inequality, we have

$$\begin{aligned}\hat{z} &\widetilde{\precsim} \Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+2}}) + \Psi(\widetilde{\varrho_{2n+2}}, f_\Xi(\tilde{\varrho}))\\ &= \Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+2}}) + \Psi(g_\Xi(\widetilde{\varrho_{2n+1}}), f_\Xi(\tilde{\varrho}))\\ &\widetilde{\precsim} \Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+2}}) + \tilde{r}\,\Psi(\widetilde{\varrho_{2n+1}}, \tilde{\varrho}) + \frac{\tilde{s}\,\Psi(\tilde{\varrho}, f_\Xi(\tilde{\varrho}))\,\Psi(\widetilde{\varrho_{2n+1}}, g_\Xi(\widetilde{\varrho_{2n+1}}))}{\Psi(\tilde{\varrho}, g_\Xi(\widetilde{\varrho_{2n+1}})) + \Psi(\widetilde{\varrho_{2n+1}}, f_\Xi(\tilde{\varrho})) + \Psi(\tilde{\varrho}, \widetilde{x_{2n+1}})}\\ &= \Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+2}}) + \tilde{r}\,\Psi(\widetilde{\varrho_{2n+1}}, \tilde{\varrho}) + \frac{\tilde{s}\,\Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+2}})\,\hat{z}}{\Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+2}}) + \Psi(\widetilde{\varrho_{2n+1}}, f_\Xi(\tilde{\varrho})) + \Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+1}})}.\end{aligned}$$

Then, by Theorem 2.14, we get

$$|\hat{z}| \widetilde{\leq} |\Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+2}})| + \tilde{r}\,|\Psi(\widetilde{\varrho_{2n+1}}, \tilde{\varrho})| + \frac{\tilde{s}\,|\Psi(\widetilde{\varrho_{2n+1}}, \widetilde{\varrho_{2n+2}})|\,|\hat{z}|}{|\Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+2}})| + |\Psi(\widetilde{\varrho_{2n+1}}, f_\Xi(\tilde{\varrho})| + |\Psi(\tilde{\varrho}, \widetilde{\varrho_{2n+1}})|}.$$

Taking the limit of this inequality as $n \to \infty$ proves that $|\tilde{z}| = \overline{0}$. Thus, we obtain $f_\Xi(\tilde{\varrho}) = \tilde{\varrho}$. Similarly, we obtain $g_\Xi(\tilde{\varrho}) = \tilde{\varrho}$.

Now, let us take another common fixed soft element $\tilde{\varrho^*}$ of f_Ξ and g_Ξ to check the uniqueness. Hence,

$$\begin{aligned}\Psi(\tilde{\varrho}, \tilde{\varrho^*}) &= \Psi(f_\Xi(\tilde{\varrho}), g_\Xi(\tilde{\varrho^*})) \\ &\widetilde{\precsim} \tilde{r}\, \Psi(\tilde{\varrho}, \tilde{\varrho^*}) + \frac{\tilde{s}\, \Psi(\tilde{\varrho}, f_\Xi(\tilde{\varrho}))\, \Psi(\tilde{\varrho}, g_\Xi(\tilde{\varrho^*}))}{\Psi(\tilde{\varrho}, g_\Xi(\tilde{\varrho^*})) + \Psi(\tilde{\varrho^*}, f_\Xi(\tilde{\varrho})) + \Psi(\tilde{\varrho}, \tilde{\varrho^*})} \\ &= \tilde{r}\Psi(\tilde{\varrho}, \tilde{\varrho^*}) \\ &\widetilde{\precnsim} \Psi(\tilde{\varrho}, \tilde{\varrho^*}),\end{aligned}$$

which is a contradiction. Thus, we get $\tilde{\varrho} = \tilde{\varrho^*}$ and so $\tilde{\varrho}$ is a unique common fixed soft element of f_Ξ and g_Ξ.

For the other case, if $\Psi(\widetilde{\varrho_{2n}}, g_\Xi(\widetilde{\varrho_{2n+1}})) + \Psi(\widetilde{\varrho_{2n+1}}, f_\Xi(\widetilde{\varrho_{2n}})) + \Psi(\widetilde{\varrho_{2n}}, \widetilde{\varrho_{2n+1}}) = \overset{\triangle}{0}$ for any $n \in \mathbb{N}$, then we get $\Psi(f_\Xi(\widetilde{\varrho_{2n}}), g_\Xi(\widetilde{\varrho_{2n+1}})) = \overset{\triangle}{0}$. So, we obtain

$$\widetilde{\varrho_{2n}} = f_\Xi(\widetilde{\varrho_{2n}}) = \widetilde{\varrho_{2n+1}} = g_\Xi(\widetilde{\varrho_{2n+1}}) = \widetilde{\varrho_{2n+2}}.$$

Therefore, from the fact that $\widetilde{\varrho_{2n}} = f_\Xi(\widetilde{\varrho_{2n}}) = \widetilde{\varrho_{2n+1}}$ it follows that there exist two soft elements $\widetilde{r_1}$ and $\widetilde{s_1}$ such that $\widetilde{r_1} = f_\Xi(\widetilde{r_1}) = \widetilde{s_1}$. Using foregoing arguments, we see that there exist two soft elements $\widetilde{r_2}$ and $\widetilde{s_2}$ such that $\widetilde{r_2} = g_\Xi(\widetilde{r_2}) = \widetilde{s_2}$. Because

$$\Psi(\widetilde{r_1}, g_\Xi(\widetilde{r_2})) + \Psi(\widetilde{r_2}, f_\Xi(\widetilde{r_1})) + \Psi(\widetilde{r_1}, \widetilde{r_2}) = \overset{\triangle}{0}$$

we have $\widetilde{s_1} = f_\Xi(\widetilde{r_1}) = g_\Xi(\widetilde{r_2}) = \widetilde{s_2}$, which deduce the equalities $\widetilde{s_1} = f_\Xi(\widetilde{r_1}) = f_\Xi(\widetilde{s_1})$ and $\widetilde{s_2} = g_\Xi(\widetilde{r_2}) = g_\Xi(\widetilde{s_2})$. Since $\widetilde{s_1} = \widetilde{s_2}$, we get $f_\Xi(\widetilde{s_1}) = g_\Xi(\widetilde{s_1}) = \widetilde{s_1}$. Thus, $\widetilde{s_1} = \widetilde{s_2}$ is common fixed soft element of f_Ξ and g_Ξ.

For uniqueness, let $\widetilde{s_1^*} \in \Theta^\Xi$ be another common fixed soft element of f_Ξ and g_Ξ, i.e. $f_\Xi(\widetilde{s_1^*}) = g_\Xi(\widetilde{s_1^*}) = \widetilde{s_1^*}$. Since

$$\Psi(\widetilde{s_1}, g_\Xi(\widetilde{s_1^*})) + \Psi(\widetilde{s_1^*}, f_\Xi(\widetilde{s_1})) + \Psi(\widetilde{s_1}, \widetilde{s_1^*}) = \overset{\triangle}{0}$$

we have $\widetilde{s_1^*} = g_\Xi(\widetilde{s_1^*}) = f_\Xi(\widetilde{s_1}) = \widetilde{s_1}$, which shows that $\widetilde{s_1^*} = \widetilde{s_1}$. This completes the proof. □

From this theorem it is easy to deduce the following two corollaries.

Corollary 4.9 *Let* (Θ, Ψ, Ξ) *be a complete soft complex valued metric space with* Ξ *a finite set and let* $f_\Xi : (\Theta, \Psi, \Xi) \to (\Theta, \Psi, \Xi)$ *be a soft mapping satisfying*

$$\Psi(f_\Xi(\tilde{\varrho}), f_\Xi(\tilde{\zeta})) \widetilde{\precsim} \tilde{r}\, \Psi(\tilde{\varrho}, \tilde{\zeta}) + \frac{\tilde{s}\, \Psi(\tilde{\varrho}, f_\Xi(\tilde{\varrho}))\, \Psi(\tilde{\zeta}, f_\Xi(\tilde{\zeta}))}{\Psi(\tilde{\varrho}, f_\Xi(\tilde{\zeta})) + \Psi(\tilde{\zeta}, f_\Xi(\tilde{\varrho})) + \Psi(\tilde{\varrho}, \tilde{\zeta})}$$

for all $\tilde{\varrho}, \tilde{\zeta} \in \Theta^\Xi$ *such that* $\Psi(\tilde{\varrho}, f_\Xi(\tilde{\zeta})) + \Psi(\tilde{\zeta}, f_\Xi(\tilde{\varrho})) + \Psi(\tilde{\varrho}, \tilde{\zeta}) \neq \overset{\triangle}{0}$ *where* $\tilde{r}, \tilde{s}$ *are nonnegative soft real numbers with* $\tilde{r} + \tilde{s} \,\widetilde{<}\, \overline{1}$ *or* $\Psi(f_\Xi(\tilde{\varrho}), f_\Xi(\tilde{\zeta})) = \overset{\triangle}{0}$ *if* $\Psi(\tilde{\varrho}, f_\Xi(\tilde{\zeta})) + \Psi(\tilde{\zeta}, f_\Xi(\tilde{\varrho})) + \Psi(\tilde{\varrho}, \tilde{\zeta}) = \overset{\triangle}{0}$. *Then,* f_Ξ *has a unique fixed soft element.*

Proof By setting $f_\Xi = g_\Xi$ in Theorem 4.8, we obtain this proof. □

Corollary 4.10 *Let (Θ, Ψ, Ξ) be a complete soft complex valued metric space with Ξ a finite set and let $f_\Xi : (\Theta, \Psi, \Xi) \to (\Theta, \Psi, \Xi)$ be a soft mapping such that*

$$\Psi(f_\Xi^n(\tilde{\varrho}), f_\Xi^n(\tilde{\zeta})) \widetilde{\precsim} \tilde{r}\, \Psi(\tilde{\varrho}, \tilde{\zeta}) + \frac{\tilde{s}\, \Psi(\tilde{\varrho}, f_\Xi^n(\tilde{\varrho}))\, \Psi(\tilde{\zeta}, f_\Xi^n(\tilde{\zeta}))}{\Psi(\tilde{\varrho}, f_\Xi^n(\tilde{\zeta})) + \Psi(\tilde{\zeta}, f_\Xi^n(\tilde{\varrho})) + \Psi(\tilde{\varrho}, \tilde{\zeta})}$$

for all $\tilde{\varrho}, \tilde{\zeta} \in \Theta^\Xi$ such that $\Psi(\tilde{\varrho}, f_\Xi^n(\tilde{\zeta})) + \Psi(\tilde{\zeta}, f_\Xi^n(\tilde{\varrho})) + \Psi(\tilde{\varrho}, \tilde{\zeta}) \neq \overset{\triangle}{0}$ where $\tilde{r}, \tilde{s}$ are nonnegative soft real numbers with $\tilde{r} + \tilde{s} \widetilde{<} \bar{1}$ or $\Psi(f_\Xi^n(\tilde{\varrho}), f_\Xi^n(\tilde{\zeta})) = \overset{\triangle}{0}$ if $\Psi(\tilde{\varrho}, f_\Xi^n(\tilde{\zeta})) + \Psi(\tilde{\zeta}, f_\Xi^n(\tilde{\varrho})) + \Psi(\tilde{\varrho}, \tilde{\zeta}) = \overset{\triangle}{0}$. Then, f_Ξ has a unique fixed soft element.

Proof It is similar to proof of Corollary 4.6 and is easily obtained. □

The validity of Corollary 4.9 is substantiated by the following example.

Example 4.11 *Let $\Theta = \Theta_1 \cup \Theta_2$ where*

$$\Theta_1 = \{z = x + iy \in \mathbb{C} : x \geq 0,\ y = 0\} \text{ and } \Theta_2 = \{z = x + iy \in \mathbb{C} : x = 0,\ y \geq 0\}.$$

Take $\Theta^\Xi = (\Theta_1)^\Xi \cup (\Theta_2)^\Xi$ with Ξ is a finite set of parameters and let us consider a mapping $d : \Theta^\Xi \times \Theta^\Xi \to \mathbb{C}^\Xi$ as follows:

$$\Psi(\hat{z}, \hat{u}) = \begin{cases} max\{\widetilde{z_1}, \widetilde{u_1}\} + i\, max\{\widetilde{z_1}, \widetilde{u_1}\}, & \text{if } \hat{z}, \hat{u} \in (\Theta_1)^\Xi, \\ max\{\widetilde{z_2}, \widetilde{u_2}\} + i\, max\{\widetilde{z_2}, \widetilde{u_2}\}, & \text{if } \hat{z}, \hat{u} \in (\Theta_2)^\Xi, \\ (\widetilde{z_1} + \widetilde{u_2}) + i(\widetilde{z_1} + \widetilde{u_2}), & \text{if } \hat{z} \in (\Theta_1)^\Xi, \hat{u} \in (\Theta_2)^\Xi, \\ (\widetilde{z_2} + \widetilde{u_1}) + i(\widetilde{z_2} + \widetilde{u_1}), & \text{if } \hat{z} \in (\Theta_2)^\Xi, \hat{u} \in (\Theta_1)^\Xi, \end{cases}$$

where $\hat{z}, \hat{u} \in \Theta^\Xi$ with $\hat{z} = \widetilde{z_1} + i\widetilde{z_2}$ and $\hat{u} = \widetilde{u_1} + i\widetilde{u_2}$ (Here, $max\{\widetilde{z_1}, \widetilde{u_1}\}$ is a soft real number defined by $(max\{\widetilde{z_1}, \widetilde{u_1}\})(e) = max\{\widetilde{z_1}(e), \widetilde{u_1}(e)\}$ for all $e \in \Xi$). Clearly, (Θ, Ψ, Ξ) is a complete soft complex valued metric space. Now, define a mapping $f : \Theta \to \Theta$ as

$$f(z) = \begin{cases} \frac{ix}{3}, & \text{if } z \in \Theta_1, \\ \frac{y}{3}, & \text{if } z \in \Theta_2. \end{cases}$$

Therefore, the condition in Corollary 4.9 holds for all $\hat{z}, \hat{u} \in \Theta^\Xi$ with $\tilde{r} = \overline{\left(\frac{1}{3}\right)}$ and $\bar{0} \widetilde{<} \tilde{s} \widetilde{<} \overline{\left(\frac{2}{3}\right)}$. Thus, all the conditions of Corollary 4.9 are satisfied and $\overset{\triangle}{0} \in \Theta^\Xi$ is a unique fixed soft element of f_Ξ.

5. Conclusion

In this paper, by utilizing the soft complex numbers given by Das and Samanta [8], we establish the idea of a soft complex valued metric space and study its topological aspects. Then, with the help of the soft mappings, we obtain certain fixed soft element theorems on soft complex valued metric spaces. Therefore, it is expected that this paper will help researches in many areas of application as well as in theoretical works. One can establish another fixed soft element theorems on soft complex valued metric spaces. Also, one can endeavour to extend soft complex valued metric spaces to the cases of fuzzy soft complex valued metric spaces, hesitant soft complex valued metric spaces, and neutrosophic soft complex valued metric spaces.

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