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Slow Flow of The Reiner-Rivlin Fluid in a Coverging or Diverging Channel with Suction and Injection

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Abstract

Two dimensional slow flow of an viscoinelastic fluid in a converging or diverging channel with porous walls has been studied. It is assumed that, the magnitude of the velocity at the center line of the flow domain is equal to unity. The flow phenomenon has been characterized by two parameters, R (suction Reynolds number) and N (inelastic number). Effects of these numbers are carefully delineated.

Key Words: Diverging or converging channels, porous media, viscoelastic fluid, intersecting planes

Reiner-Rivlin Akışkanının Daralan veya Genişleyen Gözenekli Kanal İçerisindeki Yavaş Akımı

Özet

Daralan veya genişleyen gözenekli kanalda viskoinelastik akışkanın iki boyutlu yavaş akımı incelenmiştir. Akım bölgesinin simetri ekseni üzerinde skaler hızın birim büyüklükte olduğu varsayımı yapılmıştır. Bu varsayımın ışığında hareketin diferansiyel denklemi analitik olarak çözülmüştür. Akım, R emme Reynods sayısı ve N inelastis sayısı cinsinden karakterize edilmiştir. Bu sayıların etkileri özenle incelenmiştir.

Anahtar Sözcükler: genişleyen veya daralan kanallar, gözenekli ortam, viskoelastik akışkan, kesişen düzlemler

Introduction

The steady flow solution of viscous incompressible fluids in converging or diverging channel is expressed exactly in terms of elliptical functions (Pai 1956). Rosenhead (1940) has worked on laminar two dimensional radial flow of an incompressible viscous fluid between two impermeable intersecting planes. If the Reynolds number is large and there is suction and blowing at the walls, whose magnitude is inversely proportional to the distance along the wall from the origin of the channel, a solution of the laminar boundary layer equation can be obtained (Rosenhead 1982). Terril (1965) obtained analytical solution for the slow flow as the viscous fluid running through the channel for the case of suction at one wall and equal blowing at the other walll. Sinha and Nayak (1982) have obtained a solution by using series for the steady two dimensional incompressible laminar slow flow of a visco-elastic (Walters B') fluid in a converging or diverging channel with suction and injection. In the case of impermeable walls, R.K. Bhatnagar et al. (1993) solved the same problem serially for Oldroyd-B fluid. Öztürk et al (1995) also worked on the same problem for a special form of Oldroyd-B fluid and Reiner-Rivlin (visco-inelastic) fluid, in diverging or converging channel. The main purpose of this study is to solve the same problem for Reiner-Rivlin fluid in the steady-state case and slow motion of the fluid with suction and injection. Constitutive equation of the Reiner-Rivlin fluid is,

$$T = -pI + \mu_0 A_1 + \mu_1 A_1^2, \tag{1}$$

where μ_0 and μ_1 are viscosity and cross-viscosity respectively, and

$$A_1 = \nabla \mathbf{v} + (\nabla \mathbf{v})^T \tag{2}$$

The equation of motion, in the absence of body forces is

$$\rho \frac{D\mathbf{v}}{Dt} = \nabla . T \tag{3}$$

The continuity equation for velocity fields is

$$\nabla . \mathbf{v} = 0 \tag{4}$$

As it is seen in Fig.1 the flow field assumed to be in (r, θ, z) cylindrical polar coordinates and components of the velocity vector of the flow are given as follows

$$v_r = u(r, \theta), \quad v_\theta = v(r, \theta), \quad v_z = 0. \tag{5}$$

Let the boundaries of the channel be given by $\theta = \mp\alpha$ and assume that the velocities of blowing at $\theta = -\alpha$ and of suction at $\theta = \alpha$ are both $v_\theta = \frac{Vr_0}{r}$, where, r_0 is a typical length and V is independent of θ. Then from the continuity equation $u = \frac{kF(\theta)}{r}$, where k is a constant which we will write equal to $v_0 R_1$, where, R_1 is the Reynolds number $\frac{Ur_0}{v}$ of the flow. Using $u = \frac{kF(\theta)}{r}$ and $v = \frac{Vr_0}{r}$ in A_1 and A_1^2 we get the components of the stress

$$T_{rr} = -p - 2k\mu_0 \frac{F}{r^2} + \mu_1 \left[\frac{4k^2F^2}{r^4} + \left(\frac{kF'}{r^2} - \frac{2Vr_0}{r^2} \right)^2 \right],$$

$$T_{r\theta} = T_{\theta r} = \mu_0 \left(\frac{k}{r^2}F' - \frac{2Vr_0}{r^2} \right),$$

$$T_{rz} = T_{zr} = 0,$$

$$T_{\theta\theta} = -p + 2k\mu_0 \frac{F}{r^2} + \mu_1 \left[\left(\frac{k}{r^2}F' - \frac{2Vr_0}{r^2} \right)^2 + \frac{4k^2F^2}{r^4} \right],$$

$$T_{\theta z} = T_{z\theta} = 0,$$

$$T_{zz} = -p.$$

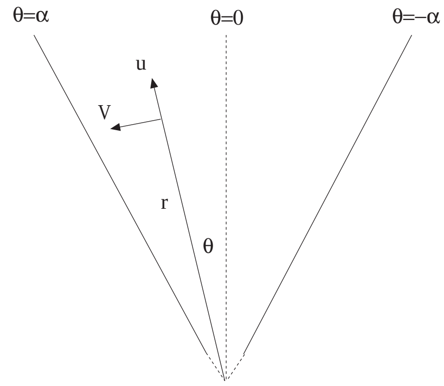

Figure 1. Geometry of the flow

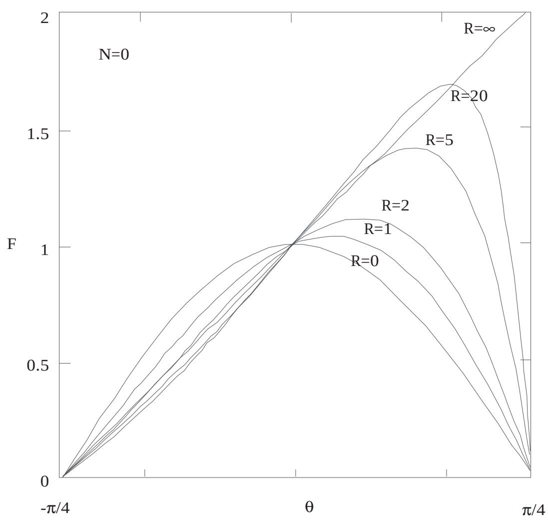

Figure 2. Effect of R on $F(\theta)$

Prime denotes differentiation with respect to θ. Using these stress components in equation (1.3) we have

$$-\frac{\partial p}{\partial r} + \frac{k\mu_0}{r^3}F'' + \mu_1 \left[-\frac{16k^2F^2}{r^5} \right. \tag{6a}$$

$$\left. +2\left(\frac{k}{r^2}F' \frac{2Vr_0}{r^2} \right)\left(-\frac{2kF'}{r^3} + \frac{4Vr_0}{r^3} \right) \right] \tag{6b}$$

$$= \rho \left(-\frac{k^2F^2}{r^3} + \frac{Vr_0k}{r^3}F' - \frac{V^2r_0^2}{r^3} \right) \tag{6c}$$

$$-\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{2k\mu_0}{r^3}F' + \mu_1 \left[\frac{2k}{r^5}(kF' - 2Vr_0)F'' \right.$$

$$\left. + \frac{8k^2FF'}{r^5} \right] = 0. \tag{6d}$$

By cross-differentiating (1.6) and (1.7) and using the parameters which are listed below, we have

$$2R_1FF' - RF''+F'''+4F'-16RNF''=0, \qquad (7)$$

where

$$R = \frac{Vr_0}{v_0}, \quad R_1 = \frac{Ur_0}{v_0}, \quad N = \frac{v_1}{r^2}, \qquad (8a)$$

$$v_1 = \frac{\mu_1}{\rho}, \quad v_0 = \frac{\mu_0}{\rho}, \quad k = v_0 r. \qquad (8b)$$

The boundary conditions are

$$F(\mp\alpha) = 0. \qquad (9a)$$

Term $2R_1FF'$ in equation (1.8) is negligible for a very slow flow. Then, the equation is reduced to

$$F'' - R(16N+1)F'' + 4F' = 0. \qquad (10a)$$

Solution of Equation

It is difficult to have a closed form solution of equation (1.11) with two boundary conditions. Then one more boundary condition is needed, for this reason, we have chosen the convention by taking the magnitude of the velocity at the center line to be unity. Then for diverging flow F(θ)=1 and for converging flow F(θ)=-1. We have chosen the case of diverging. Then, we will solve the equation (1.11) by means of the boundary conditions

$$F(\mp\alpha) = 0 \quad \text{and} \quad F(0) = 1. \qquad (11a)$$

Solving eq. (1.11) with (2.1), the following is obtained.

$$F = K_1 e^{(\beta-\sqrt{\beta^2-4})\theta} + K_2 e^{(\beta+\sqrt{\beta^2-4})\theta} + K_3, \qquad (12a)$$

where, K_1, K_2 and K_3 are constants of integration, and $\beta = \frac{1}{2}R(1+16N)$. Using the eq.(2.1) and eq.(2.2), K_1, K_2 and K_3 are obtained as follows;

$$K_1 = \frac{-K_3\left[e^{-\alpha(\beta+\sqrt{\beta^2-4})} - e^{\alpha(\beta+\sqrt{\beta^2-4})}\right]}{e^{-2\alpha(\sqrt{\beta^2-4})} - e^{2\alpha(\sqrt{\beta^2-4})}}, \qquad (13a)$$

$$K_2 = \frac{-K_3\left[e^{\alpha(\beta-\sqrt{\beta^2-4})} - e^{\alpha(-\beta+\sqrt{\beta^2-4})}\right]}{e^{-2\alpha\sqrt{\beta^2-4}} - e^{2\alpha\sqrt{\beta^2-4}}}, \qquad (14a)$$

$$K_3 = \frac{\cosh(\alpha\sqrt{\beta^2-4})}{\cosh(\alpha\beta) - \cosh(\alpha\sqrt{\beta^2-4})}. \qquad (15a)$$

The solution is not valid for $\beta = 2$. When $\beta = 2$, the required solution is

$$F = (\gamma_1 + \gamma_2\theta)e^{20} + \gamma_3, \qquad (16a)$$

where γ_1, γ_2 and γ_3 are constants of integration. They are calculated by means of boundary condition (2.1) as follows;

$$\gamma_1 = \frac{e^{4\alpha}+1}{(1-e^{2\alpha})^2}, \gamma_2 = \frac{e^{-2\alpha} - e^{2\alpha}}{\alpha e^{-2\alpha}(1-e^{2\alpha})^2},$$
$$\gamma_3 = -\frac{2e^{2\alpha}}{(1-e^{2\alpha})^2}. \qquad (17a)$$

Discussion of Results

As it is seen in eq.(2.2), we have an oscillatory solution for $\beta < 2$. When $\beta < 2$, the solution is exponential. In the case of $R = 0$ (that means there is no suction and injection) the inelastic number N does not play any role.

In order to discuss the effects of R on radial component of the velocity field, we have plotted F(θ) $-$ θ graphics for diverging flow and various R (Fig.1). It is clear that the solution depends on the angle of the channel walls $\theta = 2\alpha$. To see what happens in a right angle and an obtuse angle, we worked with $\alpha = \pi/4$ and $\alpha = 3\pi/8$. It is also obvious that, all profiles for negative R, are of the same shape but reflected in the axis θ=0. The solution is obtained for slow converging flow by reflecting the profiles in the $F(\theta) = 0$ axis. In both cases of the channel angles, in the regions $[-\pi/4, 0]$ and $[-3\pi/8, 0, F(\theta)]$ decreases with an increase in R. In regions $[0, \pi/4]$ and $[0, 3\pi/8], F(\theta)$ increases with increase in R. These increases and decreases are stronger in the case of obtuse angle than in the case of right angle. (See Fig.1 and Fig.2.)

In order to study effects of the inelastic number N on velocity component u, we have plotted $F(\theta)$ for some values of R and various values of N. Fig.3 and Fig.4 represent the changes in $F(\theta)$ for $R = 1$, and Fig.5 and Fig.6 for $R = 5$. As it is seen in these figures and in figures 7 and 8, in the regions $(0, \pi/4)$ and $(0, 3\pi/8)$ the inelastic number N cause to increase in $F(\theta)$. But, in the case of obsute angle, differences in $F(\theta)$ stronger than in the case of right angle. For large values of R, big changes in $F(\theta)$ occur in a thin layer near one wall.

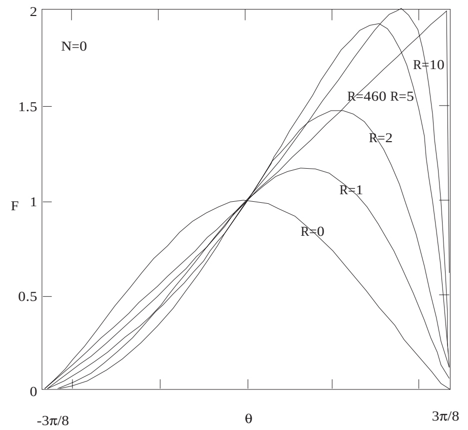

Figure 3. Effect of R on $F(\theta)$

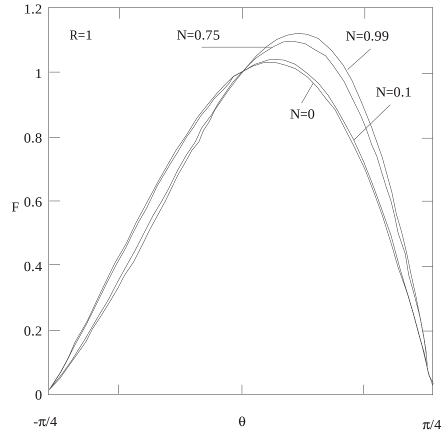

Figure 4. Effect of R on $F(\theta)$

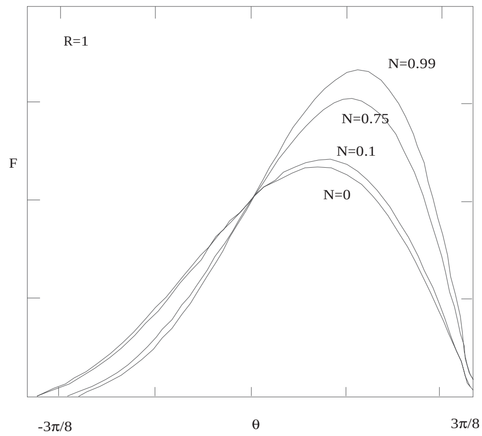

Figure 5. Effect of R on $F(\theta)$

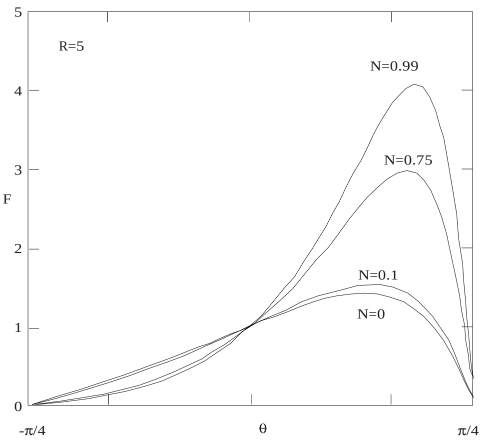

Figure 6. Effect of R on $F(\theta)$

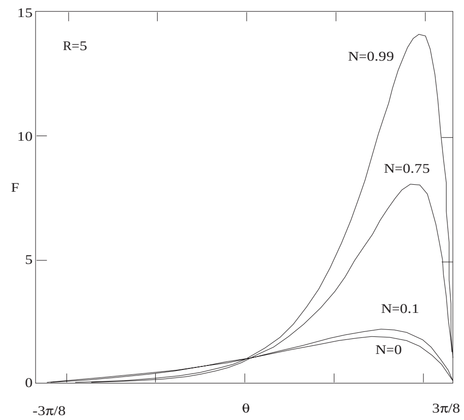

Figure 7. Effect of R on $F(\theta)$

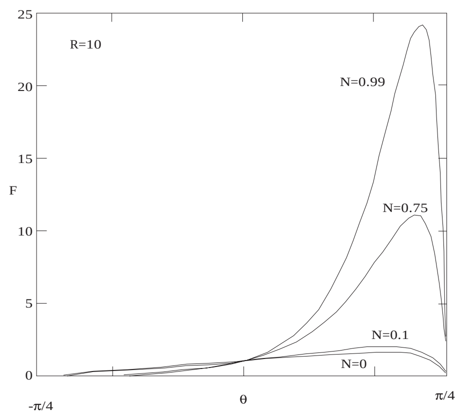

Figure 8. Effect of R on $F(\theta)$

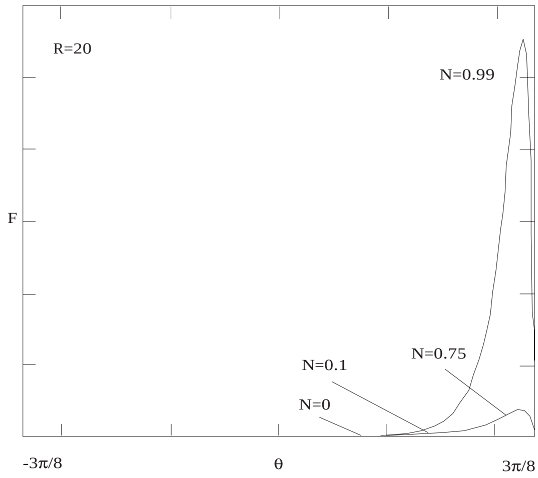

Figure 9. Effect of R on $F(\theta)$

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