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An analytical study of MHD heat and mass transfer oscillatory flow of a micropolar fluid over a vertical permeable plate in a porous medium

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Abstract

An analytical solution is presented for the problem of heat and mass transfer of an oscillatory 2-dimensional viscous, electrically conducting micropolar fluid over an infinite moving permeable plate in a saturated porous medium in the presence of a transverse magnetic field. Numerical solutions are given for the governing momentum, angular momentum, energy, and concentration equations. The effects of permeability and chemical reaction parameters are presented, graphically or in tables, for the velocity profiles, microrotation profiles, skin friction coefficient, and wall couple stress coefficient. The results indicate that increasing the chemical reaction parameter produces a decreasing effect on the skin friction coefficient and the couple stress coefficient at the wall, while the opposite is true when the permeability parameter is increased.

Key Words: Analytical solution; Heat and mass transfer; Chemical reaction; Micropolar fluid; Magneto-hydrodynamics; Porous medium.

Introduction

Chemical reactions are classified as either heterogeneous or homogeneous processes depending on whether they occur at an interface or as a single-phase volume reaction. A reaction is said to be first-order if the rate of reaction is directly proportional to the concentration itself. In many chemical processes, a chemical reaction occurs between a foreign mass and a fluid in which a plate is moving. These processes take place in numerous industrial applications, e.g., polymer production, manufacturing of ceramics or glassware, and food processing (Cussler, 1998). Chambre and Young (1958) analyzed the diffusion of chemically reactive species in a laminar boundary layer flow. Vajravelu (1986) studied the exact solution for hydrodynamic boundary layer flow and heat transfer over a continuous, moving horizontal flat surface with uniform suction and internal heat generation/absorption. Das et al. (1994) studied the effect of a homogeneous first-order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Muthucumaraswamy (2001) studied a first-order chemical reaction on the flow past an impulsively started vertical plate with uniform heat and mass flux. The same author (2002) studied the effects of a chemical reaction on a moving isothermal vertical infinitely long surface with suction. Anjali Devi and Kandasamy (2002) studied the effects of chemical reaction and heat and mass transfer on nonlinear MHD laminar boundary layer flow over a wedge with suction and injection. Chamkha (2003) presented an analytical solution for heat and mass transfer by laminar flow of a Newtonian, viscous, electrically conducting fluid and heat generation/absorption. Kandasamy et al. (2005) studied the nonlinear MHD flow, with heat and mass transfer characteristics, of an incompressible, viscous, electrically conducting, Boussinesq fluid on a vertical stretching surface with chemical reaction and thermal stratification effects.

The theory of micropolar fluids originally developed by Eringen (1966) has been a popular field of research in recent years. Micropolar fluids are those consisting of randomly oriented particles suspended in a viscous medium, which can undergo a rotation that can affect the hydrodynamics of the flow, making it a distinctly non-Newtonian fluid. Eringen's theory has provided a good model for studying a number of complicated fluids, such as colloidal fluids, polymeric fluids, and blood; they have a non-symmetrical stress tensor. The analysis of mixed convection heat transfer for an electrically conducting micropolar fluid over a vertical plate embedded in non-Darcian porous medium has important applications for several geophysical and engineering fields. These applications include magnetohydrodynamic (MHD) generators, geothermal resources extraction, petroleum resources, nuclear reactors, and the boundary layer control in the field of aerodynamics. Raptis (2000) analyzed the boundary layer of a micropolar fluid through a porous medium. Sharma and Gupta (1995) presented the effect of medium permeability on thermal convection in micropolar fluids. Hassanien et al. (2004) studied the natural convection flow of a micropolar fluid from a permeable uniform heat flux surface in a porous medium. Zakaria (2004) studied the problem of electromagnetic free convection flow of a micropolar fluid with relaxation time through a porous medium. Abo-Eldahab and El Aziz (2005) presented flow and heat transfer in a micropolar fluid past a stretching surface embedded in a non-Darcian porous medium with uniform free stream. Kim and Lee (2003) reported an analytical study on the MHD oscillatory flow of a micropolar fluid over a vertical porous plate.

In this paper, we consider the effect of chemical reaction on the heat and mass transfer of micropolar fluids in a saturated porous medium over an infinite moving permeable plate. The magnetic field is imposed transversely to the plate. The temperature and concentration of the plate is oscillating with time about a constant non-zero mean value.

Mathematical Formulation

Consider the unsteady, 2-dimensional laminar non-Darcian mixed convection flow of a viscous, incompressible, electrically-conducting micropolar fluid over an infinite vertical porous moving permeable plate in a saturated porous medium. A magnetic field of strength B_0 is applied perpendicular to the surface and the effect of the induced magnetic field is neglected. The x^*-axis is taken along the planar surface in the upward direction and the y^*-axis is taken to be normal to it. Due to the infinite plane surface assumption, the flow variables are functions of y^* and the time t^* only. Initially, the fluid as well as the plate is at rest, but for time $t \quad > 0$ the whole system is allowed to move with a constant velocity. At t = 0, the plate temperature is suddenly raised to T_w and maintained constant thereafter.

The governing equations for such a motion are given by:

$$\frac{\partial v^*}{\partial y^*} = 0, \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = (\upsilon + \upsilon_r)\frac{\partial^2 u^*}{\partial y^{*2}} + 2\upsilon_r\frac{\partial \omega^*}{\partial y^*} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) \\ -\frac{\sigma B_0^2}{\rho}u^* - \frac{\upsilon+\upsilon_r}{K}u^* \tag{2}$$

$$\rho j^*\left(\frac{\partial \omega^*}{\partial t^*} + v^*\frac{\partial \omega^*}{\partial y^*}\right) = \gamma\frac{\partial^2 \omega^*}{\partial y^{*2}}, \tag{3}$$

$$\frac{\partial T}{\partial t^*} + v^*\frac{\partial T}{\partial y^*} = \alpha\frac{\partial^2 T}{\partial y^{*2}}, \tag{4}$$

$$\frac{\partial C}{\partial t^*} + v^*\frac{\partial C}{\partial y^*} = D\frac{\partial^2 C}{\partial y^{*2}} + \gamma_1^*(C - C_\infty), \tag{5}$$

where (u^*,v^*) are the components of velocity at any point (x^*,y^*); ω^* is the component of the angular velocity normal to the x^*y^* plane; T is temperature of the fluid; and C is the mass concentration of the species in the flow. $\rho,\upsilon,\upsilon_r,g,\beta_T,\beta_C,\sigma,K,j^*,\gamma,\alpha,D$, and γ_1^* are the density, kinematic viscosity, kinematic rotational viscosity, acceleration of gravity, coefficient of volumetric thermal expansion of the fluid, coefficient of volumetric mass expansion of the fluid, electrical conductivity of the fluid, permeability of the medium, microinertia per unit mass, spin gradient viscosity, thermal diffusivity, molecular diffusivity, and the dimensional chemical reaction parameter, respectively.

The appropriate boundary conditions for the problem are:

$$u^* = u_p^* \quad , \quad \omega^* = -n_1\frac{\partial u^*}{\partial y^*} \quad , \quad T = T_\infty + \varepsilon(T_w - T_\infty)e^{n^*t^*},$$

$$C = C_\infty + \varepsilon(C_w - C_\infty)e^{n^*t^*} \text{ at } y^* = 0 \tag{6}$$

$$u^* \to 0, \quad \omega^* \to 0, \quad T \to T_\infty \quad , \quad C \to C_\infty \text{ as } y^* \to \infty$$

The following comment should be made about the boundary condition used for the microrotation term: when $n_1 = 0$, we obtain from the boundary condition stated in Eq. (6), for the microrotation, $\omega^* = 0$. This represents the case of concentrated particle flows in which the microelements close to the wall are not able to rotate (Jena and Mathur, 1982). The case corresponding to $n_1 = 0.5$ results in the vanishing of the antisymmetric part of the stress tensor and represents weak concentrations (Ahmadi, 1976). Ahmadi (1976) suggested that the particle spin is equal to the fluid vorticity at the boundary for fine particle suspensions. As suggested by Peddieson (1972), the case corresponding to $n_1 = 1$ is representative of turbulent boundary layer flows. Thus, for $n_1 = 0$, the particles are not free to rotate near the surface. However, as $n_1 = 0.5$ and 1, the microrotation term gets augmented and induces flow enhancement.

Integrating the continuity Eq. (1), we get

$$v^* = -V_0, \tag{7}$$

where V_0 is a scale of suction velocity, which has a non-zero positive constant.

It is convenient to employ the following dimensionless variables:

$$u^* = U_0 u, v^* = V_0 v, y^* = \frac{v}{V_0} y, u_p^* = U_0 U_p, \omega^* = \frac{U_0 V_0}{v}\omega,$$

$$t^* = \frac{v}{V_0^2} t, T - T_\infty = (T_w - T_\infty)\theta, C - C_\infty = (C_w - C_\infty)\phi, n^* = \frac{V_0^2}{v} n,$$

$$j^* = \frac{v^2}{V_0^2} j, \Pr = \frac{v}{\alpha}, Sc = \frac{v}{D}, M = \frac{\sigma B_0^2 v}{\rho V_0^2}, Gr_T = \frac{v g \beta_T (T_w - T_\infty)}{U_0 V_0^2}, \tag{8}$$

$$Gr_C = \frac{v g \beta_C (C_w - C_\infty)}{U_0 V_0^2}, \gamma = (\mu + \frac{\Lambda}{2}) j^* = \mu j^* (1 + \frac{\beta}{2}), \beta = \frac{\Lambda}{\mu} = \frac{v_r}{v},$$

$$K' = \frac{K U_0 V_0^2}{v^2}, \eta = \frac{\mu j^*}{\gamma} = \frac{2}{2+\beta}, \gamma_1 = \frac{v \gamma_1^*}{V V_0^2},$$

where U_0 is a scale of free stream velocity and β denotes the dimensionless viscosity ratio in which Λ is the coefficient of vortex viscosity. Pr, Sc, M, Gr, K', and γ_1 are the Prandtl number, Schmidt number, magnetic field parameter, Grashof number, permeability parameter, and the dimensionless chemical reaction parameter, respectively.

With the help of Eq. (6), Eqs. (1)-(7) reduce to the following initial-value problem:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = (1+\beta)\frac{\partial^2 u}{\partial y^2} + 2\beta \frac{\partial \omega}{\partial y} + Gr_T \theta + Gr_c \phi - Mu - \frac{1+\beta}{K'} u \tag{9}$$

$$\frac{\partial \omega}{\partial t} - \frac{\partial \omega}{\partial y} = \frac{1}{\eta}\frac{\partial^2 \omega}{\partial y^2}, \tag{10}$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}, \tag{11}$$

$$\frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} + \gamma_1 \phi, \tag{12}$$

with the following dimensionless boundary conditions:

$$\begin{aligned} & u = U_p \omega = -n_1 \frac{\partial u}{\partial y}, \theta = 1 + \varepsilon e^{nt}, \phi = 1 + \varepsilon e^{nt}, at y = 0 \\ & u \to 0, \omega \to 0, \theta \to 0, \phi \to 0, as y \to \infty \end{aligned} \tag{13}$$

To solve Eqs. (9)-(12) subject to the boundary conditions (13), we may use the following linear transformations for low values of ε (Kim and Lee, 2003):

$$\begin{aligned} u(y,t) &= u_0(y) + \varepsilon e^{nt} u_1(y) + O(\varepsilon^2) \\ \omega(y,t) &= \omega_0(y) + \varepsilon e^{nt} \omega_1(y) + O(\varepsilon^2) \\ \theta(y,t) &= \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + O(\varepsilon^2) \\ \phi(y,t) &= \phi_0(y) + \varepsilon e^{nt} \phi_1(y) + O(\varepsilon^2) \end{aligned} \tag{14}$$

After substituting the expressions (14) into Eqs. (9)-(13), we have

$$(1+\beta) u_0'' + u_0' - \left(M + \frac{1+\beta}{K'} \right) u_0 = -Gr_T \theta_0 - Gr_C \phi_0 - 2\beta \omega_0', \tag{15}$$

$$(1+\beta)u_1'' + u_1' + (n- \ M - \frac{1+\beta}{K'}\ )\, u_1 = -Gr_T\theta_1 - Gr_C\phi_1 - 2\beta\omega_1', \tag{16}$$

$$\omega_0'' + \eta\omega_0' = 0, \tag{17}$$

$$\omega_1'' + \eta\omega_1' + n\eta\omega_1 = 0, \tag{18}$$

$$\theta_0'' + \Pr\theta_0' = 0, \tag{19}$$

$$\theta_1'' + \Pr\theta_1' + n\Pr\theta_1 = 0, \tag{20}$$

$$\phi_0'' + Sc\phi_0' + Sc\gamma_1\phi_0 = 0, \tag{21}$$

$$\phi_1'' + Sc\phi_1' + Sc(n+\gamma_1)\phi_1 = 0, \tag{22}$$

with the following boundary conditions:

$$\begin{array}{l} u_0 = U_p, u_1 = 0, \omega_0 = -n_1 u_0', \omega_1 = -n_1 u_1' \\ \theta_0 = 1, \theta_1 = 1, \phi_0 = 1, \phi_1 = 1 aty = 0, \\ u_0 = 0, u_1 = 0, \omega_0 = 0, \omega_1 = 0 \\ \theta_0 = 0, \theta_1 = 0, \phi_0 = 0, \phi_1 = 0 asy \to \infty. \end{array} \tag{23}$$

Solving Eqs. (15)-(22) with the boundary conditions (23) and substituting the solutions into Eq. (14), we get

$$u = a_1e^{-h_2y} + a_2e^{-\Pr y} + a_4e^{-h_5y} + a_3e^{-\eta y} + \varepsilon(b_1e^{-h_1y} + b_2e^{-h_3y} + b_3e^{-h_4y} + b_4e^{-h_6y})e^{nt}, \tag{24}$$

$$\omega = c_1e^{-\eta y} + \varepsilon(c_2e^{-h_1y})e^{nt}, \tag{25}$$

$$\theta = e^{-\Pr y} + \varepsilon(e^{-h_4y})e^{nt}, \tag{26}$$

$$\phi = e^{-h_5y} + \varepsilon(e^{-h_6y})e^{nt}, \tag{27}$$

where

$$h_1 = \frac{\eta}{2}\left[1 + \sqrt{1 - \frac{4n}{\eta}}\right]$$

$$h_2 = \frac{1}{2(1+\beta)}\left[1 + \sqrt{1 + 4(M + \frac{1+\beta}{K'})(1+\beta)}\right]$$

$$h_3 = \frac{1}{2(1+\beta)}\left[1 + \sqrt{1 - 4(n - M - \frac{1+\beta}{K'})(1+\beta)}\right]$$

$$h_4 = \frac{\Pr}{2}\left[1 + \sqrt{1 - \frac{4n}{\Pr}}\right]$$

$$h_5 = \frac{Sc}{2}\left[1 + \sqrt{1 - \frac{4\gamma_1}{Sc}}\right]$$

$$h_6 = \frac{Sc}{2}\left[1 + \sqrt{1 - \frac{4(n+\gamma_1)}{Sc}}\right]$$

$$a_1 = U_p - a_2 - a_3 - a_4$$

$$a_2 = -\frac{Gr_T}{(1+\beta)\Pr^2 - \Pr - (M + \frac{1+\beta}{K'})}$$

$$a_3 = \frac{2\beta\eta}{(1+\beta)\eta^2 - \eta - (M + \frac{1+\beta}{K'})} c_1 = \lambda c_1$$

$$a_4 = -\frac{Gr_c}{(1+\beta)h_5^2 - h_5 - (M + \frac{1+\beta}{K'})}$$

$$b_1 = \frac{2\beta h_1}{(1+\beta)h_1^2 - h_1 + (n - M - \frac{1+\beta}{K'})} c_2 = \xi c_2$$

$$b_2 = -(b_1 + b_3 + b_4)$$

$$b_3 = -\frac{Gr_T}{(1+\beta)h_4^2 - h_4 + (n - M - \frac{1+\beta}{K'})}$$

$$b_4 = -\frac{Gr_C}{(1+\beta)h_6^2 - h_6 + (n - M - \frac{1+\beta}{K'})}$$

$$c_1 = \frac{n_1[h_2 U_p - h_2 a_2 - h_2 a_4 + \Pr a_2 + h_5 a_4]}{1 + n_1 \lambda (h_2 - \eta)}$$

$$c_2 = \frac{n_1 b_3 (h_4 - h_3) + n_1 b_4 (h_6 - h_3)}{1 + n_1 \xi (h_3 - h_1)}$$

The local skin friction coefficient, local wall couple stress coefficient, local Nusselt number, and local Sherwood number are important physical quantities for this type of heat and mass transfer problem. These are defined as follows:

The wall shear stress may be written as:

$$\begin{aligned} \tau_w^* &= (\mu + \Lambda)\tfrac{\partial u^*}{\partial y^*} \mid_{y^*=0} + \Lambda \omega^* \mid_{y^*=0} \\ &= \rho U_0 V_0 [1 + (1 - n_1)\beta] u'(0) \end{aligned} \tag{28}$$

Therefore, the local skin-friction factor is given by:

$$C_f = \frac{2\tau_w^*}{\rho U_0 V_0} = 2[1 + (1 - n_1)\beta] u'(0), \tag{29}$$

The wall couple stress may be written as:

$$M_w = \gamma \frac{\partial \omega^*}{\partial y^*} \mid_{y=0}, \tag{30}$$

Therefore, the local couple stress coefficient is given by:

$$C'_w = \frac{M_w \upsilon^2}{\gamma U_0 V_0^2} = \omega'(0). \tag{31}$$

The rate of heat transfer at the surface in terms of the local Nusselt number can be written as:

$$Nu = x\frac{(\partial T/\partial y^*)_{y^*=0}}{T_\infty - T_w}, \tag{32}$$

$$NuRe_x^{-1} = -\theta'(0)$$

where $Re_x = \frac{xV_0}{v}$ is the local Reynolds number.

The rate of mass transfer at the surface in terms of the local Sherwood number is given by:

$$Sh = x\frac{(\partial C/\partial y^*)_{y^*=0}}{C_\infty - C_w}, \tag{33}$$

$$ShRe_x^{-1} = -\phi'(0)$$

Results and Discussion

Numerical evaluation of the analytical solutions reported in the previous section was performed and the results are presented in graphical and tabular form. This was done to illustrate the influence of the various parameters involved in the problem on the solutions. In plotting the results, we used the boundary condition for $y \to \infty$ as $y_{\max} = 8$ and step size $\Delta y = 0.001$.

Figures 1-9 display the effects of variations in the flow conditions and the fluid properties on the velocity, microrotational velocity, temperature, and concentration profiles. The effects due to the chemical reaction γ_1 and permeability K' on the velocity, microrotational velocity, and concentration distributions are shown in Figures 1-3, respectively. It was observed that the microrotational velocity and concentration increase as the chemical reaction parameter γ_1 increases, while the velocity distribution has the opposite behavior. It can be seen from these figures that the velocity and concentration of the fluid decrease with the increase of a nondestructive reaction ($\gamma > 0$) of chemical reaction, whereas the temperature of the fluid is not significant with an increase of a nondestructive reaction. It was further observed that the concentration of the fluid decreases uniformly near the wall of the sphere. On the other hand, as permeability parameter K' increases, the velocity increases along with the boundary layer thickness, while the microrotational velocity decreases due to increases in permeability parameter K'. Physically, the presence of a porous medium in the flow presents resistance to flow (i.e. as K' decreases). Thus, the resulting resistive force tends to slow the motion of the fluid along the plate surface and causes increases in its microrotational velocity. It can also be noted from Figure 3 that the concentration decreases as the Schmidt number increases.

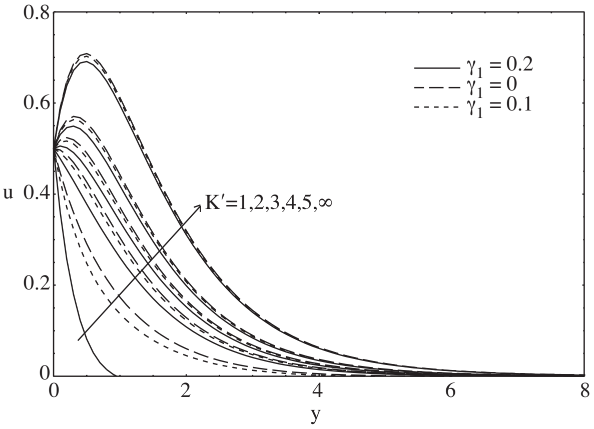

Figure 1. Velocity distribution for various values of permeability parameter K' and chemical reaction parameter γ_1 for $t= 1$, $\varepsilon = 0.01$, $n_1 = 0.5$, $n= 0.1$, $\beta = 1$, $M= 2$, $Gr_T = 2$, $Gr_C = 1$, $\Pr = 1$, $Sc = 2$, and $U_p = 0.5$.

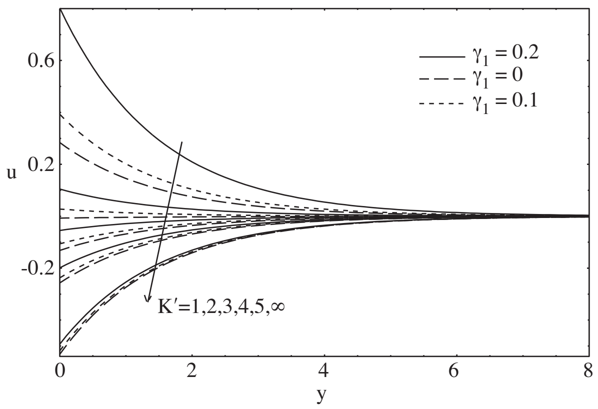

Figure 2. Microrotational velocity distribution for various values of permeability parameter K' and chemical reaction parameter γ_1 for $t= 1$, $\varepsilon = 0.01$, $n_1 = 0.5$, $n= 0.1$, $\beta = 1$, $M= 2$, $Gr_T = 2$, $Gr_C = 1$, $\Pr = 1$, $Sc = 2$, and $U_p = 0.5$.

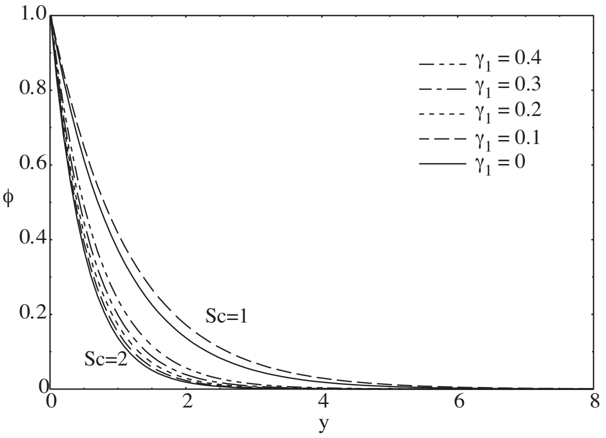

Figure 3. Concentration distribution for various values of Schmidt number Sc and chemical reaction parameter γ_1 for $t= 1$, $\varepsilon = 0.01$, $n_1 = 0.5$, $n= 0.1$, $\beta = 1$, $M= 2$, $Gr_T = 2$, $Gr_C = 1$, $\Pr = 1$, $K' = 5$, and $U_p = 0.5$.

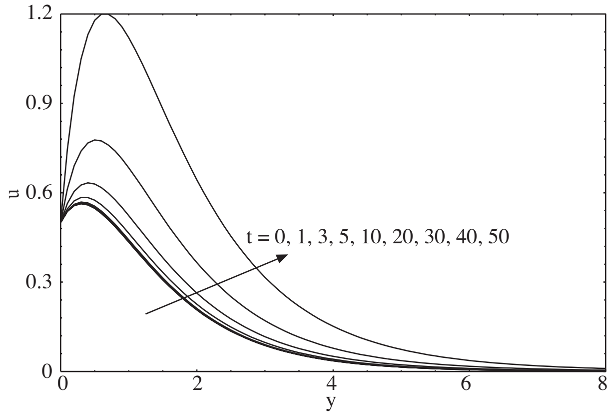

Figure 4. Unsteady velocity distribution for $\varepsilon = 0.01$, $n_1 = 0.5$, $n= 0.1$, $\beta = 1$, $M= 2$, $Gr_T = 2$, $Gr_C = 1$, $\Pr = 1$, $Sc = 2$, $\gamma_1 = 0.1$, $K' = 5$, and $U_p = 0.5$.

The temporal development of the velocity, temperature, concentration, and microrotational velocity with different values of time t are elucidated in Figures 4-7, respectively. It is clear that the fluid velocity and temperature and the solute concentration in the fluid increase as time increases, while the microrotational velocity of the fluid reduces as time progresses or increases. Moreover, it can be concluded that the velocity increases with the time. Near the surface, the velocity profiles increase to the maximum and then decrease, and finally take an asymptotic value (free steam velocity). In addition, the momentum boundary layer thickness increases as t increases. Moreover, the thermal boundary layer thickness decreases and the temperature gradient at the wall increases, and hence the heat transfer rate increases as t decreases. The temperature profile is large near the surface of the plate and decreases far away from the plate, finally taking an asymptotic value. All of these behaviors are clear in Figures 4-7.

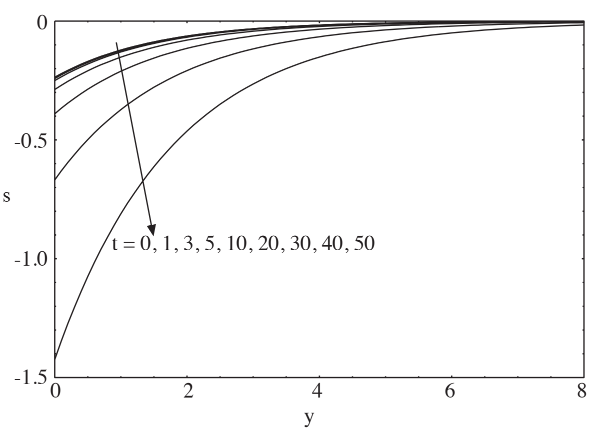

Figure 5. Unsteady microrotational velocity distribution for $\varepsilon = 0.01$, $n_1 = 0.5$, $n= 0.1$, $\beta = 1$, $M= 2$, $Gr_T = 2$, $Gr_C = 1$, $\Pr = 1$, $Sc = 2$, $\gamma_1 = 0.1$, $K' = 5$, and $U_p = 0.5$.

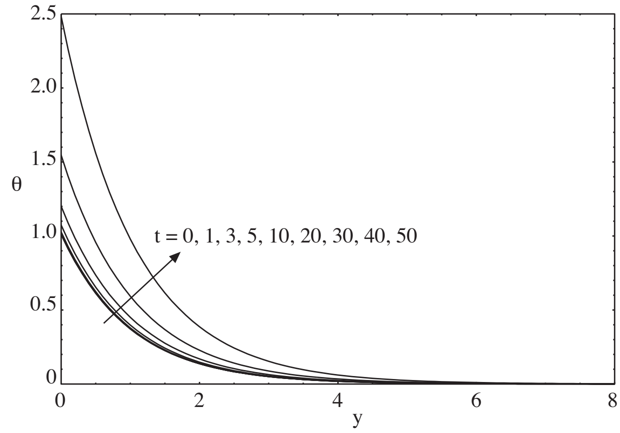

Figure 6. Unsteady temperature distribution for $\varepsilon = 0.01$, $n_1 = 0.5$, $n= 0.1$, $\beta = 1$, $M= 2$, $Gr_T = 2$, $Gr_C = 1$, $\Pr = 1$, $Sc = 2$, $\gamma_1 = 0.1$, $K' = 5$, and $U_p = 0.5$.

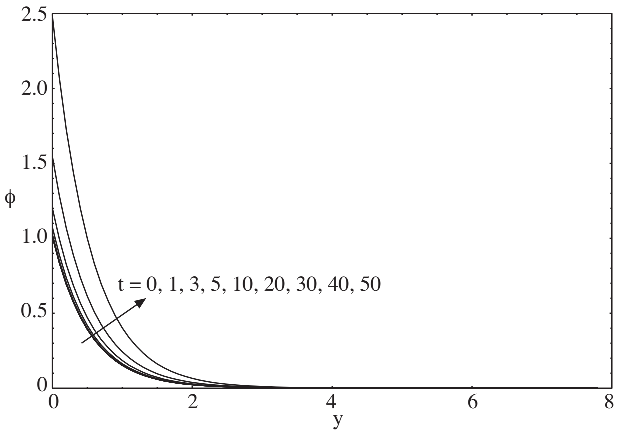

Figure 7. Unsteady concentration distribution for $\varepsilon = 0.01$, $n_1 = 0.5$, $n= 0.1$, $\beta = 1$, $M= 2$, $Gr_T = 2$, $Gr_C = 1$, $\Pr = 1$, $Sc = 2$, $\gamma_1 = 0.1$, $K' = 5$, and $U_p = 0.5$.

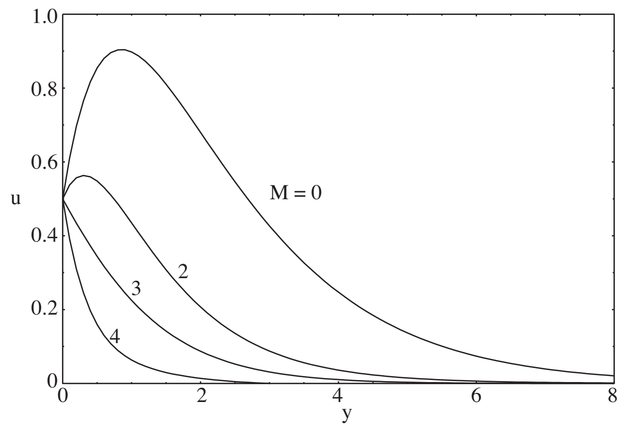

Figure 8. Velocity distribution for various values of magnetic field parameter $M$for $t= 1$, $\varepsilon = 0.01$, $n_1 = 0.5$, $n= 0.1$, $\beta = 1$, $Gr_T = 2$, $Gr_C = 1$, $\Pr = 1$, $Sc = 2$, $\gamma_1 = 0.1$, $K' = 5$, and $U_p = 0.5$.

Figures 8 and 9 depict the velocity and the microrotational velocity profiles for different values of the magnetic field parameter M. It is clear that the velocity decreases with increases in the strength of the magnetic field. In contrast, the microrotational velocity distribution increases with increases in the strength of the magnetic field. That is because the application of a magnetic field in the y-direction to an electrically conducting fluid gives rise to a flow resistive force called the Lorentz force.

Table 1 shows the effects of chemical reaction γ_1 and the permeability K' parameters on the coefficients of skin friction, couple stress, heat transfer, and mass transfer. It is clear that as permeability parameter K' increases, both skin friction coefficient C_f and couple stress coefficient C'_w increase, while they and Sherwood number Sh decrease as the chemical reaction γ_1 parameter increases. Table 2 illustrates the variation of the coefficients of skin friction, couple stress, heat transfer, and mass transfer with various values of t. It can be concluded that skin friction, couple stress, heat transfer, and mass transfer increase with time.

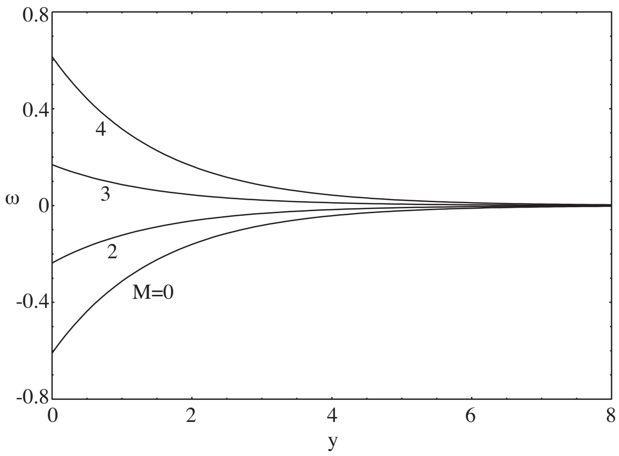

Figure 9. Microrotational velocity distribution for various values of magnetic field parameter $M$for t= 1, $\varepsilon = 0.01$, $n_1 = 0.5$, n= 0.1, $\beta = 1$, $Gr_T = 2$, $Gr_C = 1$, Pr = 1, Sc = 2, $\gamma_1 = 0.1$, $K' = 5$, and $U_p = 0.5$.

Table 1. Effects of variations of chemical reaction and permeability parameters on the coefficients of skin friction, couple stress, heat transfer, and mass transfer.

γ_1	K'	C_f	C'_w	$NuRe_x^{-1}$	$ShRe_x^{-1}$
0	1	-1.70559	-0.19004	1.00981	2.02094
	2	0.04323	0.00392		
	3	0.79096	0.08687		
	5	1.54237	0.17024		
	∞	3.19083	0.35317		
0.1	1	-2.35736	-0.26203		1.91404
	2	-0.16482	-0.01911		
	3	0.63923	0.07007		
	5	1.42384	0.15711		
	∞	3.10976	0.34417		
0.2	1	-4.81429	-0.53826		1.79264
	2	-0.62734	-0.07022		
	3	0.32876	0.03573		
	5	1.19198	0.13146		
	∞	2.95726	0.32729		

The referenced case is t= 1, $\varepsilon = 0.01$, $n_1 = 0.5$, $n = 0.1$, $\beta = 1$, M= 2, $Gr_T = 2$, $Gr_C = 1$, Pr = 1, Sc =2, and $U_p = 0.5$.

Finally, the effects of plate velocity Up and magnetic parameter M on skin friction and couple stress across the boundary layer are presented in Table 3. It can be seen that the skin friction and couple stress decrease as the plate velocity and magnetic parameter increase. We can also note from this table that the increasing of Pr, Gr_T, Gr_C, n_1, β, and ε cause skin friction coefficient C_f and couple stress coefficient C'_w to increase, while the increasing of γ_1, Sc, and n produces lower values of skin friction coefficient C_f and couple stress coefficient C'_w. On the other hand, the Nusselt number increases as Pr and ε increase; it decreases as n increases. The Sherwood number increases as n, Sc, and ε increase, but it decreases as γ_1 increases.

Table 2. Unsteady behaviors of the coefficients of skin friction, couple stress, heat transfer, and mass transfer with various values of t.

t	C_f	C'_w	$NuRe_x^{-1}$	$ShRe_x^{-1}$
0	1.41876	0.15665	1.00887	1.91217
1	1.42384	0.15711	1.00981	1.91404
3	1.43567	0.15819	1.01198	1.91838
5	1.45011	0.15950	1.01463	1.92369
10	1.50181	0.16418	1.02412	1.94267
20	1.72757	0.18466	1.06556	2.02555
30	2.34126	0.24032	1.17822	2.25086
40	4.00945	0.39161	1.48445	2.86332
50	8.54403	0.80286	2.31687	4.52816

The referenced case is $\varepsilon = 0.01$, $n_1 = 0.5$, $n = 0.1$, $\beta = 1$, $M = 2$, $Gr_T = 2$, $Gr_C = 1$, $\Pr = 1$, $Sc = 2$, $K' = 5$, $\gamma_1 = 0.1$, and $U_p = 0.5$.

Table 3. Effects of variations of flow conditions and fluid properties on the coefficients of skin friction, couple stress, heat transfer, and mass transfer.

		C_f	C'_w	$NuRe_x^{-1}$	$ShRe_x^{-1}$
Pr	0.7	0.86122	0.09463	0.70640	1.91404
	1	1.42384	0.15711	1.00981	
Sc	1	3.91898	0.43381	1.00981	0.89530
	2	1.42384	0.15711		1.91404
Gr_T	0	-4.16847	-0.46315		1.91404
	1	-1.37232	-0.15302		
	2	1.42384	0.15711		
Gr_C	0	1.32262	0.14586		
	1	1.42384	0.15711		
	2	1.52505	0.16837		
n_1	0	1.22527	0		
	0.5	1.42384	0.15711		
	1	2.11250	0.70030		
n	0	1.42682	0.15854	1.01000	1.91337
	0.05	1.42552	0.15789	1.00996	1.91374
	0.1	1.42384	0.15711	1.00981	1.91404
	0.15	1.42091	0.15596	1.00948	1.91426
β	0	0.57821	0.14368	1.00981	1.91404
	1	1.42384	0.15711		
	1.5	4.5806	0.33802		
ε	0	1.37042	0.15227	1	1.89443
	0.01	1.42384	0.15711	1.00981	1.91404
	0.1	1.90461	0.20072	1.09806	2.09055
U_p	0	5.69353	0.63152	1.00981	1.91404
	0.5	1.42384	0.15711		
	1	-2.84585	-0.31730		
M	0	3.65837	0.40731		
	2	1.42384	0.15711		
	3	-1.01470	-0.11335		
	4	-3.69145	-0.40873		

The referenced case is $t = 1$, $\varepsilon = 0.01$, $n_1 = 0.5$, $n = 0.1$, $\beta = 1$, $M = 2$, $Gr_T = 2$, $Gr_C = 1$, $\Pr = 1$, $Sc = 2$, $K' = 5$, $\gamma_1 = 0.1$, and $U_p = 0.5$.

Concluding Remarks

An analytical study of the oscillatory MHD heat and mass transfer of the laminar flow of a viscous, incompressible, electrically conducting micropolar fluid over an infinite vertical moving plate in a saturated porous medium was conducted. The governing boundary layer equations for the velocity, microrotation, temperature, and concentration fields were solved using the method of small perturbation approximation. In the presence of a uniform magnetic field, increases in the strength of the applied magnetic field decelerated the fluid motion along the wall of the plate inside the boundary layer, whereas the microrotational velocity of the fluid along the wall of the plate increased. The fluid flow along the wall of the plate accelerated as the chemical reaction parameter increased. On the other hand, the concentration and the microrotational velocity of the fluid increased while the linear velocity decreased with increases in the chemical reaction parameter. Moreover, the results indicated that as the chemical reaction parameter increased, the skin-friction coefficient and the couple stress coefficient at the wall decreased; they had the opposite behavior when the permeability parameter increased. The Nusselt number increased as the Prandtl number increased. The Sherwood number increased as the Schmidt number increased, while it decreased as the chemical reaction parameter increased.

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