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The Effect of Injection/Suction on Flow past Parallel Plates with Transpiration Cooling

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Abstract

This study investigated the effect of injection/suction between 2 horizontal parallel porous flat plates, with transverse sinusoidal injection of fluid at the stationary plate and its corresponding removal by periodic suction through the plate in motion, assuming the sinusoidal injection at the lower plate and its corresponding removal by the upper plate in motion. The approximate solutions were obtained for the flow field, pressure, skin-friction, temperature field, and rate of heat transfer, and are discussed with the help of graphs and tables.

Key Words: Incompressible Fluid, Injection/Suction, Skin-Friction, Heat Transfer.

Introduction

Transpiration cooling methods have been developed in an attempt to protect structural elements in turbojet and rocket engines from the influence of hot gases, such as combustion chamber walls, exhaust nozzles, and gas turbine blades. They have many engineering applications in the development of missiles, satellites, and spacecraft. In view of this Eckert (1958) obtained an exact solution of the plane Couette flow with transpiration cooling. The problem remained 2 dimensional due to uniform injection and suction applied at the porous plates. Flow and heat transfer along a plane wall with periodic suction velocity has been studied by Gersten and Gross (1974). Effects of such a suction velocity on various flow and heat transfer problems along flat and vertical porous plates have been studied by Singh et al. (1978a, 1978b) and Singh (1993). Recently, the problem of transpiration cooling with the application of the transverse sinusoidal injection/suction velocity has been studied by Singh (1999). Chaudhary and Jain (2007) studied exact solutions of incompressible Couette flow with constant temperature and constant heat flux on walls in the presence of radiation. Recently, Sharma et al. (2007) discussed the unsteady free convection oscillatory Couette flow through a porous medium with periodic wall temperature. Hence, the aim of the present study was to examine the effects of injection/suction on the Couette flow between 2 horizontal parallel porous plates, with transverse sinusoidal injection of fluid at the stationary plate and its corresponding removal by periodic suction through the plate in motion. The governing equations were solved for small amplitude oscillation (ε) of injection/suction velocity. The solution was obtained by regular perturbation, in terms of ε. The effect of various parameters on flow characteristics were examined and are discussed with the help of graphs and tables.

Formulation of the Problem

We considered the Couette flow of a viscous incompressible fluid between 2 parallel flat porous plates, with transverse sinusoidal injection of the fluid at the stationary plate and its corresponding removal by periodic suction through the plate in average motion (U). Let x*-z* plane lie along the plates and y*-axis be taken normal to the free stream velocity. The distance (d) is taken between the plates. The lower and upper plates are assumed to be at constant temperature T_0 and T_1, respectively, with $T_1 > T_0$. The plates are considered infinite in the x^* direction. Hence, all physical quantities will be independent of x^*because an asymptotic flow was selected, assuming sinusoidal injection/suction velocity [$\mathbf{V^*(z^*) = V\ (1 + \varepsilon \cos \pi\ z^*/d)}$] at the lower and upper plates. The physical configuration of the problem is visualized in Figure 1 (Singh, 1999). Denoting the velocity components u, v, and w in the x, y, and z directions, respectively, and the temperature by θ, the problem is governed by the following non-dimensional equations:

$$v_y + w_z = 0 \tag{1}$$

$$vu_y + wu_z = [u_{yy} + u_{zz}]/\lambda \tag{2}$$

$$vv_y + wv_z = -p_y + [v_{yy} + v_{zz}]/\lambda \tag{3}$$

$$vw_y + ww_z = -p_z + [w_{yy} + w_{zz}]/\lambda \tag{4}$$

$$v\theta_y + w\theta_z = (\theta_{yy} + \theta_{zz})/\lambda Pr \tag{5}$$

where y = y*/d , z = z*/d, u = u*/U, v = v*/V, w = w*/V, p = p*/ρ V^2, Pr (Prandtl number) = ν/α, λ(injection/suction parameter) Vd/ν, θ = ($T^*- T_0$) / ($T_1- T_0$).

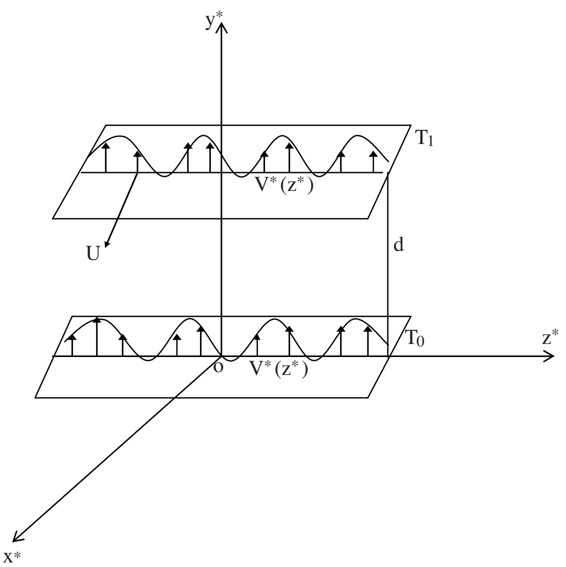

Figure 1. The physical configuration of the problem.

All physical variables are defined in the Nomenclature. The (*) stands for dimensional quantities. The corresponding boundary conditions in the dimensionless form are:

$$y = 0 : u = 0, v(z) = 1 + \varepsilon \cos \pi z, w = 0, \theta = 0,$$

$$y = 1 : u = 1, v(z) = 1 + \varepsilon \cos \pi z, w = 0, \theta = 1 \tag{6}$$

Solution of the Problem

Since the amplitude of the injection/suction velocity ε (<<1) is very small, we now assume the solution of the following form:

$$f(y, z) = f_o(y) + \varepsilon f_1(y, z) + \varepsilon^2 f_2(y, z) + \ldots . \tag{7}$$

where f stands for any of u, v, w, p, and θ . When $\varepsilon = 0$, the problem is reduced to the well-known 2-dimensional flows with constant injection and suction at both plates (Eckert, 1958). The solutions of this 2-dimensional problem are

$$u_0(y) = (e^{\lambda y} - 1)/(e^{\lambda} - 1), (8) \tag{8}$$

$\theta_o(y) = (e^{\lambda Pry} - 1)/(e^{\lambda Pr} - 1), v_0 = 1, w_0 = 0, p_0 =$ constant.

When $\varepsilon \neq 0$, substituting (7) in Eq. (1) to (5) and comparing the coefficient of ε, neglecting those of ε^2, ε^3, etc., the following equations are obtained with the help of solution (8):

$$v_{1y} + w_{1z} = 0 \tag{9}$$

$$v_1 u_{oy} + u_{1y} = (u_{1yy} + u_{1zz})/\lambda \tag{10}$$

$$v_{1y} = -p_{1y} + (v_{1yy} + v_{1zz})/\lambda \tag{11}$$

$$w_{1y} = -p_{1z} + (w_{1yy} + w_{1zz})/\lambda \tag{12}$$

$$v_1\theta_{oy} + \theta_{1y} = (\theta_{1yy} + \theta_{1zz})/\lambda Pr \tag{13}$$

The corresponding boundary conditions reduce to

$$y = 0 : u_1 = 0, v_1 = \cos \pi z, w_1 = 0, \theta_1 = 0$$

$$y = 1 : u_1 = 0, v_1 = \cos \pi z, w_1 = 0, \theta_1 = 0 \tag{14}$$

This is the set of linear partial differential equations that describe the flow. To solve these equations we assume v_1, w_1, p_1, u_1, and θ_1 of the following form:

$$\left.\begin{aligned} u_1(y, z) &= u_{11}(y) \cos \pi z \\ v_1(y, z) &= v_{11}(y) \cos \pi z \\ w_1(y, z) &= -\{v_{11}^{/}(y) sin\pi z\}/\pi \\ p_1(y, z) &= p_{11}(y) \cos \pi z \\ \theta_1(y, z) &= \theta_{11}(y) \cos \pi z \end{aligned}\right\} \tag{15}$$

where primes denote differentiation with respect to y. The expressions for v_1(y,z) and w_1(y,z) were chosen so that the equation of continuity (9) is satisfied. Substituting (15) in (10) to (13) we obtain the following equations

$$u_{11yy} - \lambda u_{11y} - \pi^2 u_{11} = \lambda v_{11} u_{0y} \tag{16}$$

$$v_{11yy} - \lambda v_{11y} - \pi^2 v_{11} = \lambda p_{11y} \tag{17}$$

$$v_{11yyy} - \lambda v_{11yy} - \pi^2 v_{11y} = \lambda p_{11} \pi^2 \tag{18}$$

$$\theta_{11yy} - \lambda Pr \theta_{11y} - \pi^2 \theta_{11} = \lambda Pr v_{11} \theta_{0y} \tag{19}$$

Corresponding boundary conditions are:

$$\left.\begin{aligned} y = 0 : u_{11} = 0, v_{11} = 1, v_{11y} = 0, \theta_{11} = 0, \\ y = 1 : u_{11} = 0, v_{11} = 1, v_{11y} = 0, \theta_{11} = 0 \end{aligned}\right\} \tag{20}$$

Solving Eqs. (16) to (19) under the boundary conditions (20) and using Eqs. (15), we get the solutions for v_1, w_1, p_1, u_1, and θ_1 as

$$v_1(y,z) = A^{-1}[A_1 e^{\alpha y} + A_2 e^{\beta y} - A_3 e^{\pi y} - A_4 e^{-\pi y}] \cos \pi z \ldots .(21) \tag{21}$$

$$w_1(y,z) = -(\pi A)^{-1}[A_1 \alpha e^{\alpha y} + A_2 \beta e^{\beta y} - A_3 \pi e^{\pi y} + A_4 \pi e^{-\pi y}] sin\pi z \tag{22}$$

$$p_1(y,z) = A^{-1}[A_3 e^{\pi y} + A_4 e^{-\pi y}] \cos \pi z \tag{23}$$

$$u_1(y,z) = [E e^{\alpha y} + F e^{\beta y} + C_1\{A_1 e^{(\alpha+\lambda)y}/2\alpha + A_2 e^{(\beta+\lambda)y}/2\beta - A_3 e^{(\pi+\lambda)y}/\pi + A_4 e^{(\lambda-\pi)y}/\pi\}] \cos \pi z \tag{24}$$

$$\begin{aligned} \theta_1(y,z) = [M e^{sy} + N e^{ty} + C_2\{A_1(\alpha\lambda + \alpha\lambda Pr)^{-1}.e^{(\alpha+\lambda Pr)y} \\ + A_2(\beta\lambda + \beta\lambda Pr)^{-1}.e^{(\beta+\lambda Pr)y} - A_3(\pi\lambda Pr)^{-1}.e^{(\pi+\lambda Pr)y} + A_4(\pi\lambda Pr)^{-1}.e^{(\lambda Pr-\pi)y}\}] \cos \pi z \end{aligned} \tag{25}$$

where

$$A = 2\pi(\beta-\alpha)(1+e^{\lambda}) + (\alpha\pi - \beta\pi + \alpha\beta - \pi^2)(e^{\alpha+\beta} + e^{\beta-\alpha})$$
$$+(\alpha\pi - \beta\pi - \alpha\beta + \pi^2)(e^{\alpha-\pi} + e^{\beta+\pi}),$$
$$A_1 = 2\beta\pi(1+e^{\beta}) - (\pi^2 + \beta\pi)(e^{\beta-\pi} + e^{\pi}) + (\pi^2 - \beta\pi)(e^{\beta+\pi} + e^{-\pi}),$$
$$A_2 = -2\alpha\pi(1+e^{\alpha}) + (\pi^2 + \alpha\pi)(e^{\alpha-\pi} + e^{\pi}) - (\pi^2 - \alpha\pi)(e^{\alpha+\pi} + e^{-\pi}),$$
$$A_3 = \pi(\alpha-\beta)(e^{\alpha+\beta} + e^{-\pi}) - (\alpha\pi + \alpha\beta)(e^{\beta-\pi} + e^{\alpha}) + (\beta\pi + \alpha\beta)(e^{\alpha-\pi} + e^{\beta}),$$
$$A_4 = \pi(\alpha-\beta)(e^{\alpha+\beta} + e^{\pi}) - (\alpha\pi - \alpha\beta)(e^{\beta+\pi} + e^{\alpha}) + (\beta\pi - \alpha\beta)(e^{\alpha+\pi} + e^{\beta}),$$
$$\alpha = [\lambda + (\lambda^2 + 4\pi^2)^{1/2}]/2, \beta = [\lambda - (\lambda^2 + 4\pi^2)^{1/2}]/2,$$
$$s = [\lambda Pr + (\lambda^2 Pr^2 + 4\pi^2)^{1/2}]/2, t = [\lambda Pr - (\lambda^2 Pr^2 + 4\pi^2)^{1/2}]/2,$$
$$C_1 = \lambda[A(e^{\lambda} - 1)]^{-1}, C_2 = \lambda^2 Pr^2[A(e^{\lambda Pr} - 1)]^{-1},$$
$$C_3 = \lambda[A(e^{\lambda} - 1).(e^{\alpha} - e^{\beta})]^{-1}, C_4 = \lambda^2 Pr^2[A(e^{\lambda Pr} - 1)(e^{s} - e^{t})]^{-1},$$
$$E = C_3[A_1(e^{\beta} - e^{\lambda+\alpha})/2\alpha + A_2(e^{\beta} - e^{\lambda+\beta})/2\beta - A_3(e^{\beta} - e^{\lambda+\pi})/\pi + A_4(e^{\beta} - e^{\lambda-\pi})/\pi],$$
$$F = C_3[A_1(e^{\lambda+\alpha} - e^{\alpha})/2\alpha + A_2(e^{\lambda+\beta} - e^{\alpha})/2\beta - A_3(e^{\lambda+\pi} - e^{\alpha})/\pi + A_4(e^{\lambda-\pi} - e^{\alpha})/\pi],$$
$$M = C_4[A_1(\alpha\lambda + \alpha\lambda Pr)^{-1}(e^{t} - e^{\alpha+\lambda Pr}) + A_2(\beta\lambda$$
$$+\beta\lambda Pr)^{-1}(e^{t} - e^{\beta+\lambda Pr}) - A_3(\pi\lambda Pr)^{-1}(e^{t} - e^{\pi+\lambda Pr}) + A_4(\pi\lambda Pr)^{-1}(e^{t} - e^{\lambda Pr-\pi})],$$
$$N = C_4[A_1(\alpha\lambda + \alpha\lambda Pr)^{-1}(e^{\alpha+\lambda Pr} - e^{s}) + A_2(\beta\lambda$$
$$+\beta\lambda Pr)^{-1}(e^{\beta+\lambda Pr} - e^{s}) - A_3(\pi\lambda Pr)^{-1}(e^{\pi+\lambda Pr} - e^{s}) + A_4(\pi\lambda Pr)^{-1}(e^{\lambda Pr-\pi} - e^{s})].$$

Now, after knowing the velocity field, we can calculate skin-friction components τ_{xx} and τ_{zz} in the main and transverse directions, respectively, as

$$\tau_{xx} = d\tau^*_{xx}/\mu U = (du_0/dy)_{y=0} + \varepsilon(du_{11}/dy)_{y=0} \cos \pi z \tag{26}$$

$$\tau_{xx} = \lambda(e^{\lambda} - 1)^{-1} + \varepsilon[E\alpha + F\beta + C_1\{A_1(\alpha+\lambda)/2\alpha + A_2(\beta+\lambda)/2\beta - A_3(\pi+\lambda)/\pi + A_4(\lambda-\pi)/\pi\}] \cos \pi z \tag{27}$$

$$\tau_{zz} = d\tau^*_{zz}/\mu V = \varepsilon(\partial w_1/\partial y)_{y=0} \tag{28}$$

$$\tau_{zz} = -\varepsilon(\pi A)^{-1}[A_1\alpha^2 + A_2\beta^2 - A_3\pi^2 - A_4\pi^2]sin\pi z \tag{29}$$

From the temperature field we can obtain the heat transfer coefficient, in terms of the Nusselt number as

$$Nu = dq^*_w/[\kappa(T_0 - T_1)] = (d\theta_0/dy)_{y=0} + \varepsilon(d\theta_{11}/dy)_{y=0}\cos\pi z \tag{30}$$

$$Nu = \lambda Pr(\lambda Pr - 1)^{-1} + \varepsilon[Ms + Nt + C_2\{A_1(\alpha + \lambda Pr).(\alpha\lambda + \alpha\lambda Pr)^{-1}$$
$$+A_2(\beta + \lambda Pr).(\beta\lambda + \beta\lambda Pr)^{-1} - A_3(\pi + \lambda Pr).(\pi\lambda Pr)^{-1} + A_4(\lambda Pr - \pi).(\pi\lambda Pr)^{-1}\}]\cos\pi z \tag{31}$$

Results and Discussion

Main flow velocity profiles are presented in Figure 2. This graph indicates that the main flow velocity decreased with the injection/suction parameter (λ). The transverse velocity component is presented in Figure 3. It was observed that forward flow developed from y = 0 to about y = 0.5, and then onwards there was backward flow. This was due to the fact that the dragging action of the faster layer exerted on the fluid particles in the neighborhood of the stationary plate was sufficient to overcome the adverse pressure gradient and, hence, there was forward flow. The dragging action of the faster layer exerted on the fluid particles reduced due to the periodic suction at the upper plate and, hence, this dragging action was insufficient to overcome the adverse pressure gradient; therefore, there was backward flow. Furthermore, it is evident from Figure 3 that velocity w_1 decreased with increasing λ in the forward and back flow. Pressure values are reported in Table 1; pressure decreased with the injection/suction parameter (λ). The decrease in pressure was sufficiently large for small fluid injection/suction.

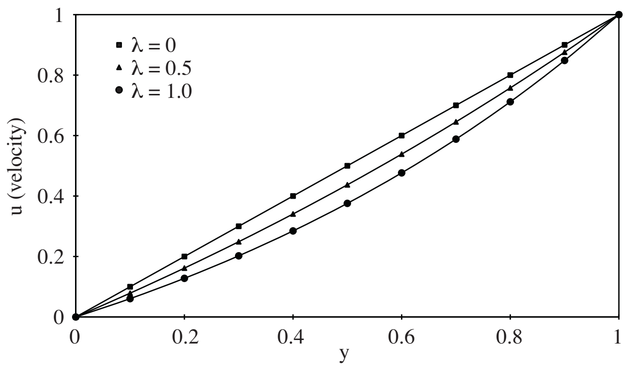

Figure 2. The velocity profiles for ε = 0.2 and z = 0.

The values of skin-friction τ_{xx} and τ_{zz} in the main and transverse flow directions are given in Table 2. It was observed that τ_{xx} and τ_{zz} decreased with increasing λ. It is also clear from Table 2 that the values of τ_{zz} were much lower than those of τ_{xx}.

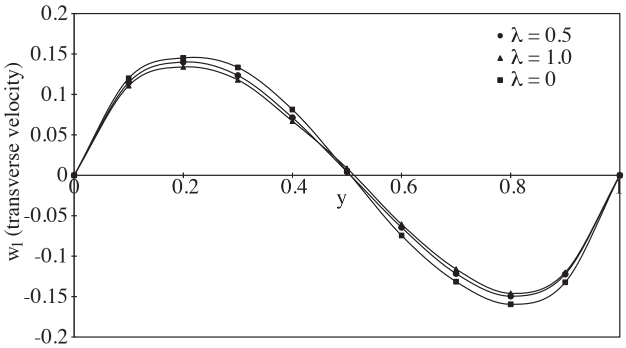

Figure 3. The transverse velocity for z = 0.5.

Table 1. Pressure values of (p_1) for z = 0.

y	$p_1(\lambda = 0.2)$	$p_1(\lambda = 0.5)$
0	22.468	8.8780
0.1	15.752	6.2182
0.2	10.604	4.1773
0.3	6.5114	2.5520
0.4	3.0665	1.1807
0.5	–0.0732	–0.0773
0.6	–3.2202	–1.3342
0.7	–6.6877	–2.7281
0.8	–10.820	–4.3934
0.9	–16.030	–6.4960
1.0	–22.835	–9.2450

Nusselt number (Nu) values are shown in Figure 4. It was observed that Nu decreased with increasing λ in both situations [Pr = 0.71 (air) and Pr = 7 (water)]. It is also clear from Figure 4 that Nu was much lower in the case of water (Pr = 7) than air (Pr = 0.71).

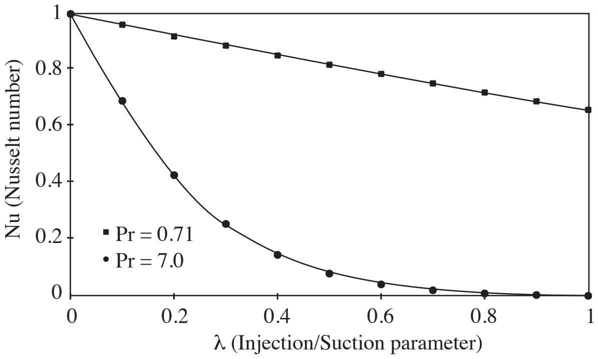

Figure 4. The Nusselt number for $\varepsilon = 0.2$ and z = 0.

Table 2. Skin-friction component (τ_{xx} and τ_{zz}) values for $\varepsilon = 0.2$.

λ	τ_{xx} (z = 0)	τ_{zz} (z = 0.5)
0.1	0.9508	0.3562
0.2	0.9033	0.3529
0.3	0.8575	0.3495
0.4	0.8133	0.3462
0.5	0.7707	0.3429
0.6	0.7298	0.3395
0.7	0.6905	0.3363
0.8	0.6528	0.3330
0.9	0.6166	0.3297
1.0	0.5820	0.3265

Conclusions

On the basis of the above discussion we conclude that the main flow velocity and skin friction components in the main and transverse flow directions decreased with increases in the injection/suction parameter. Additionally, the dimensionless coefficient of heat transfer (Nusselt number) decreased with the injection/suction parameter and the transverse velocity component increased with increasing from small values ($\lambda < 0.5$) of the injection/suction parameter, while the reverse effect was observed for $\lambda > 0.5$. The present analysis gave a better result, as we considered the injection/suction velocity variable at both plates, because in actual practice injection/suction cannot be uniform in all cases.

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Nomenclature

d	distance between plates
Nu	Nusselt number
p	dimensionless pressure
p^*	pressure
Pr	Prandtl number
q_w^*	constant heat flux per unit area
T^*	temperature of fluid
T_∞^*	temperature of free stream
U	average velocity
V	injection/suction velocity
u^*,v^*,w^*	components of velocity
u,v,w	dimensionless velocity components
x^*,y^*,z^*	Cartesian coordinates
x,y,z	dimensionless Cartesian coordinates
α	thermal diffusivity
ε	amplitude of injection/suction velocity (<< 1)
κ	thermal conductivity
λ	injection/suction parameter
μ	viscosity
ν	kinematics viscosity
θ	dimensionless temperature
ρ	density

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