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# Interaction of Couple Stresses and Slip Flow on Peristaltic Transport in Uniform and Nonuniform Channels

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**Abstract**

The effect of slip velocity on peristaltic flow of a couple stress fluid in uniform and nonuniform symmetric channels is studied. This problem has numerous applications. It serves as a model for the blood flow in living creatures. Using long wavelength approximation and neglecting inertial forces, a closed form solution for the axial velocity and the pressure gradient was obtained. Numerical computations were carried out to investigate the effect of couple stress parameter $\alpha$ and Knudsen number $K_n$ on pressure rise, maximum pressure rise, and friction force for uniform and nonuniform channels. It is noted that the pressure rise decreases with increasing $\alpha$ and increasing $K_n$. The friction force has an opposite behavior compared with pressure rise.

**Key words:** Peristalsis, Couple stresses, Slip flow

## Introduction

The study of fluid transport by means of peristaltic waves in both mechanical and physiological situations has been a subject of scientific research since the first investigation by Latham (1966). By peristaltic pumping, we mean a device for pumping fluids, generally from a region of lower pressure to one of higher pressure, by means of a contraction wave traveling along the tube. This mechanism is found in many physiological situations like urine transport from the kidney to the bladder through the ureter, movement of chyme in the gastrointestinal tract, the movements of spermatozoa in the ductus efferentes in the male reproductive tract, and ova in the female fallopian tube. Moreover, a peristaltic mechanism is involved in transporting the lump in lymphatic vessels, movement of bile in the bile duct, and the circulation of blood in small blood vessels such as arterioles, venules, and capillaries. In addition, peristaltic pumping occurs in many practical applications involving biomechanical systems. Furthermore, finger and roller pumps are frequently used for pumping corrosive or very pure materials so as to prevent direct contact of the fluid with the pump's internal surfaces.

The initial mathematical models of peristalsis obtained by a train of sinusoidal waves in an infinitely long symmetric channel or tube were introduced by Fung and Yih (1968), and Shapiro et al. (1969). After these studies, several analytical, numerical, and experimental attempts have been made to understand peristaltic action in different situations for Newtonian and non-Newtonian fluids. Some of these studies have been done by Brown and Hung (1977), Takabatake and Ayukawa (1982, 1988), Srivastava and Srivastava (1983, 1984, 1988), Siddiqui and Schwarz (1994), Ramachandra and Usha (1995), Elshehawey et al. (2000), Elshehawey and Sobh (2001), Sobh (2003), Abd El Naby et al. (2004), Hayat et al. (2006, 2007), and Sobh and Mady (2008).

The study of a couple stress fluid is very useful in understanding various physical problems because it possesses the mechanism to describe rheological complex fluids such as liquid crystals and human blood. By couple stress fluid, we mean a fluid whose particles sizes are taken into account, a special case of non-Newtonian fluids. Some of the recent studies on peristaltic transport of couple stress fluid have been done by Srivastava (1986), Elshehawey and Mekheimer (1994), Elsoud et al. (1998), Mekheimer (2002, 2004), Elshehawey and El-Sebaei (2001), and Ali et al. (2007).

Some of the studies on couple stress fluid just mentioned considered the blood as a couple stress fluid and they were carried out using no slip conditions, although in real systems there is always a certain amount of slip. There are 2 extremely different types of fluids that appear to slip. One class contains the rarefied gases (Kwang and Fang, 2000), while the other fluids have a much more elastic character. In such fluids, some slippage occurs under a large tangential traction. It has been claimed that slippage can occur in non-Newtonian fluids, concentrated polymer solution, and molten polymer. Furthermore, in the flow of dilute suspensions of particles, a clear layer is sometimes observed next to the wall. Poiseuille, in a work that won a prize in experimental physiology, observed such a layer with a microscope in the flow of blood through capillary vessels (Coleman et al., 1966).

With the above discussion in mind, it is convenient to study the interaction of slip conditions with peristaltic flow of couple stress fluid, where the fluid particles' size is taken into account. This mathematical model can be considered a good application for blood transport in blood small vessels.

Because of the complexity of the governing equations, we shall study the problem under long wavelength (the ratio between channel width and wave length is very small) and zero Reynolds number assumptions. The resulting system is solved analytically and the velocity field and pressure gradient are obtained in explicit forms for uniform and nonuniform cases. Furthermore, the pressure rise, the average pressure rise, the maximum average pressure rise, and the friction force per unit wavelength are computed numerically and are plotted and discussed with different parameters of the problem.

## Formulation and Analytic Solution

Consider the flow of a couple stress fluid through a 2-dimensional channel of nonuniform thickness with a sinusoidal wave traveling down its wall. Taking $(\overline{x}, \overline{y})$as rectangular coordinates, the equation of the wall surface is

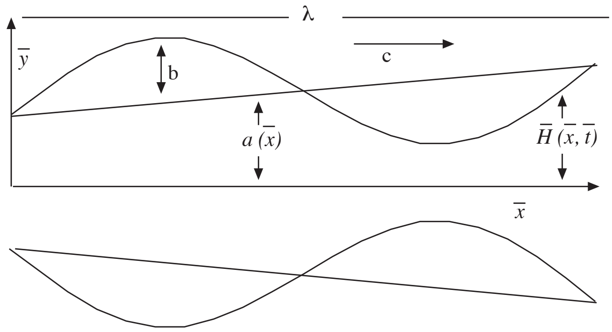


**Figure 1.** Geometry of the problem.

$$\overline{H}(\overline{x}, \overline{t}) = a(\overline{x}) + b \sin \frac{2\pi}{\lambda}(\overline{x} - c\overline{t}), \tag{1}$$

with

$$a(\overline{x}) = a_0 + a_1 \overline{x}. \tag{2}$$

Here $a(\overline{x})$is the half-width of the channel at any axial distance $\overline{x}$ from the inlet, $a_0$ is the half-width at the inlet, ($a_1 <<1$) is a constant whose magnitude depends on the length of the channel and exit and inlet dimensions,$b$ is the wave amplitude, $\lambda$ is the wavelength, $c$ is the propagation velocity of the wave, and $t$ is the time.

The constitutive equations and equations of motion for a couple stress fluid are (Stocks, 1966)

$$T_{ji,j} + \rho f_i = \rho \frac{dv_i}{dt}, \tag{3}$$

$$e_{ijk} T^A_{jk} + M_{ji,j} + \rho C_i = 0, \tag{4}$$

$$\tau_{ij} = -\overline{p}\delta_{ij} + 2\mu\, d_{ij}, \tag{5}$$

$$\mu_{ij} = 4\eta\, w_{j,i} + 4\eta'\, w_{i,j}, \tag{6}$$

where $f_i$ is the body force vector per unit mass,$C_i$ is the body moment per unit mass, $v_i$ is the velocity vector, $\tau_{ij}$and $T^A_{jk}$ are the symmetric and antisymmetric parts of the stress tensor $T_{ji}$, respectively, $M_{ij}$ is the couple stress tensor, $\mu_{ij}$is the deviatoric part of $M_{ij}$, $w_i$ is the vorticity vector, $d_{ij}$ is the symmetric part of the velocity gradient, $\mu$ is the viscosity of the fluid, $\eta$ and $\eta'$ are constants associated with the couple stress, $\overline{p}$ is the pressure, and $\delta_{ij}$is the Kroneker delta.

Neglecting the body force and the body couples, the continuity equations and equations of motion are (Mekheimer, 2002)

$$\frac{\partial \overline{u}}{\partial \overline{x}} + \frac{\partial \overline{v}}{\partial \overline{y}} = 0, \tag{7}$$

$$\rho\left(\frac{\partial\overline{u}}{\partial\overline{t}}+\overline{u}\frac{\partial\overline{u}}{\partial\overline{x}}+\overline{v}\frac{\partial\overline{u}}{\partial\overline{y}}\right)=-\frac{\partial\overline{p}}{\partial\overline{x}}+\mu\nabla^2\overline{u}-\eta\nabla^4\overline{u}, \quad (8)$$

$$\rho\left(\frac{\partial\overline{v}}{\partial\overline{t}}+\overline{u}\frac{\partial\overline{v}}{\partial\overline{x}}+\overline{v}\frac{\partial\overline{v}}{\partial\overline{y}}\right)=-\frac{\partial\overline{p}}{\partial\overline{y}}+\mu\nabla^2\overline{v}-\eta\nabla^4\overline{v}, \quad (9)$$

where

$$\nabla^2=\frac{\partial^2}{\partial\overline{x}^2}+\frac{\partial^2}{\partial\overline{y}^2}, \quad \nabla^4=\nabla^2\nabla^2 \qquad (10)$$

Following Valanis and Sun (1969), the couple stress tensor at the channel wall vanishes; then using the slip condition used by Kwang and Fang (2000), the boundary conditions of the problem take the form

$$\frac{\partial\overline{u}}{\partial\overline{y}}=0, \qquad \frac{\partial^3\overline{u}}{\partial\overline{y}^3}=0 \qquad at \quad \overline{y}=0, \qquad (11a)$$

$$\overline{u}=-\overline{K_n}\frac{\partial\overline{u}}{\partial\overline{y}}, \qquad \overline{v}=c\frac{\partial\overline{H}}{\partial\overline{x}} \qquad at \quad \overline{y}=\overline{H}(\overline{x},\overline{t}). \qquad (11b)$$

$$-\left(\frac{\partial^2\overline{v}}{\partial\overline{x}^2}-\frac{\partial^2\overline{u}}{\partial\overline{y}\partial\overline{x}}\right)\frac{\partial\overline{H}}{\partial\overline{x}}+\frac{\partial^2\overline{v}}{\partial\overline{x}\partial\overline{y}}-\frac{\partial^2\overline{u}}{\partial\overline{y}^2}=0, \quad at \quad \overline{y}=\overline{H}(\overline{x},\overline{t}) \qquad (11c)$$

Using the following nondimensional parameters

$$x=\frac{\overline{x}}{\lambda},\ y=\frac{\overline{y}}{a_0},\ t=\frac{c\overline{t}}{\lambda},\ p=\frac{a_0^2\overline{p}}{c\lambda\mu},\ u=\frac{\overline{u}}{c},\ v=\frac{\lambda\overline{v}}{a_0c},\ Re=\frac{\rho c a_0}{\mu},\ \delta=\frac{a_0}{\lambda},\ \varphi=\frac{b}{a_0},$$

$$\alpha^2=\frac{\eta\,a_0^2}{\mu},\ K_n=\frac{\overline{K_n}}{a_0},\ \ H=\frac{\overline{H}}{a_0}=1+\frac{\lambda a_1}{a_0}x+\varphi\sin 2\pi(x-t), \qquad (12)$$

where $\delta$ is the wave number, $Re$ is the Reynolds number, $\varphi<1$ is the amplitude ratio, and $\alpha$ is the couple stress parameter, the nondimensional equations of motion and the boundary conditions become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (13)$$

$$Re\,\delta\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$
$$-\frac{1}{\alpha^2}\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \qquad (14)$$

$$Re\,\delta^3\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$
$$-\frac{\delta^2}{\alpha^2}\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \qquad (15)$$

$$\frac{\partial u}{\partial y}=0, \qquad \frac{\partial^3 u}{\partial y^3}=0 \qquad at \quad y=0, \qquad (16a)$$

$$u=-K_n\frac{\partial u}{\partial y}, \qquad v=\frac{\partial H}{\partial x} \qquad at \quad y=H(x,t). \qquad (16b)$$

$$-\left(\delta^4\frac{\partial^2 v}{\partial x^2}-\delta^2\frac{\partial^2 u}{\partial y\partial x}\right)\frac{\partial H}{\partial x}+\delta\frac{\partial^2 v}{\partial x\partial y}-\frac{\partial^2 u}{\partial y^2}=0, \quad at \quad y=H(x,t) \qquad (16c)$$

Using long wavelength approximation and neglecting the inertial force, equations of motion (13-15) tend to the following system

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (17)$$

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \frac{1}{\alpha^2}\frac{\partial^4 u}{\partial y^4}, \tag{18}$$

$$\frac{\partial p}{\partial y} = 0, \tag{19}$$

with the boundary conditions

$$\frac{\partial u}{\partial y} = 0, \qquad \frac{\partial^3 u}{\partial y^3} = 0 \quad at \quad y = 0, \tag{20a}$$

$$u = -K_n \frac{\partial u}{\partial y}, \qquad v = \frac{\partial H}{\partial x} \quad at \quad y = H(x,t). \tag{20b}$$

$$\frac{\partial^2 u}{\partial y^2} = 0, \qquad at \quad y = H(x,t) \tag{20c}$$

The solution of Eqs. (17-19), subject to the boundary conditions (20), is

$$u(x,y,t) = \frac{1}{2}\left(\frac{dp}{dx}\right)\left[y^2 - H^2 + \frac{2}{\alpha^2}\left(1 - \frac{\cosh \alpha y}{\cosh \alpha H}\right) - 2K_n\left(H - \frac{Tanh\alpha H}{\alpha}\right)\right]. \tag{21}$$

The instantaneous volume flow rate $Q(x,t)$ is given by

$$Q(x,t) = \int_0^H u\, dy = \left(\frac{dp}{dx}\right)\left[-\frac{H^3}{3} + \left(\frac{1}{\alpha^2} - K_n H\right)\left(H - \frac{1}{\alpha}Tanh\alpha H\right)\right], \tag{22}$$

which implies that

$$\frac{dp}{dx} = \frac{-3Q(x,t)}{\left[H^3 - 3\left(\frac{1}{\alpha^2} - K_n H\right)\left(H - \frac{1}{\alpha}Tanh\alpha H\right)\right]}. \tag{23}$$

The nondimensional pressure rise and nondimensional friction force per wavelength are defined, respectively, as

$$\Delta p(t) = \int_0^1 \frac{dp}{dx}\, dx, \tag{24}$$

$$F(t) = \int_0^1 H(x,t)\left(-\frac{dp}{dx}\right) dx. \tag{25}$$

**Numerical Results and Discussion**

We obtained the solution of the problem theoretically for nonuniform and uniform channels (when $a_1 = 0$). It is clear that as $\alpha \to \infty$and $K_n \to 0$ we obtain the same results as Shapiro et al. (1969) and Sirvastava and Srivastava (1988) when the power-law index n = 1. Moreover, when $K_n \to 0$ our results are in agreement with those obtained by Mekhiemer (2002).

To discuss the results obtained above quantitatively, we use the form of the instantaneous volume flow rate $Q(x,t)$ obtained by Srivastava et al. (1983) as

$$Q(x,t) = \overline{Q} + \varphi \sin 2\pi(x - t), \tag{26}$$

where $\overline{Q}$ is the dimensionless time–mean flow rate, and then evaluate the integrals appearing in Eqs. (24) and (25) numerically using the MATHEMATICA package. Following Srivastava and Srivastava (1984), we take $a_0 = 0.01$ cm, $L = \lambda = 10$ cm, $a_1 = \frac{a_0}{2L}$, and then plot Eqs. (24) and (25) for various values of parameters.

It is important to note that the theory of long wavelength in the present investigation remains applicable here as the radius of the channel at the inlet $a_0 = 0.01$ cm is small compared to the wavelength $\lambda = 10$ cm. This means that$\delta = \frac{a_0}{\lambda} << 1$.

Figure 2 represents the variation in the pressure rise versus the time at $\overline{Q} = 0$, $\varphi = 0.7$, $K_n = 0$, and ($\alpha$ = 1.5, 2, 3). It is noted that the pressure rise decreases as the couple stress fluid parameter $\alpha$ increases. In other words, the pressure rise decreases as the size of the suspended particles decreases. The effect of the slip boundary conditions on the pressure rise appears in Figure 3 for the nonuniform channel at $\overline{Q} = 0.2$, $\varphi = 0.7$, $\alpha = 1.5$,and ($K_n = 0, 0.05, 0.1$). As shown, the pressure rise decreases with increasing Knudsen number $K_n$. Furthermore, we observe from Figures 2 and 3 that the pressure rise decreases as the flow rate increase.

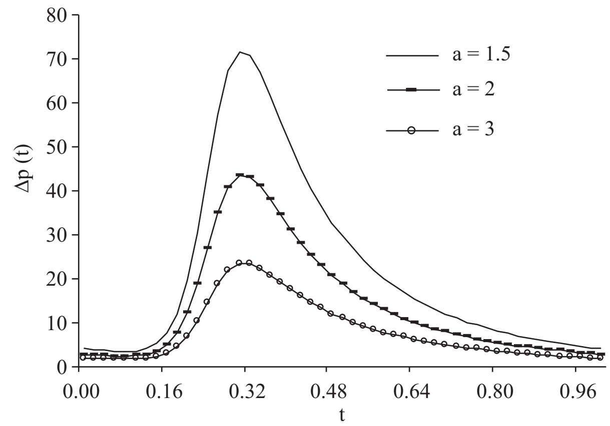


**Figure 2.** Pressure rise versus time for $\overline{Q} = 0$, $\varphi = 0.7$, and $K_n = 0$.

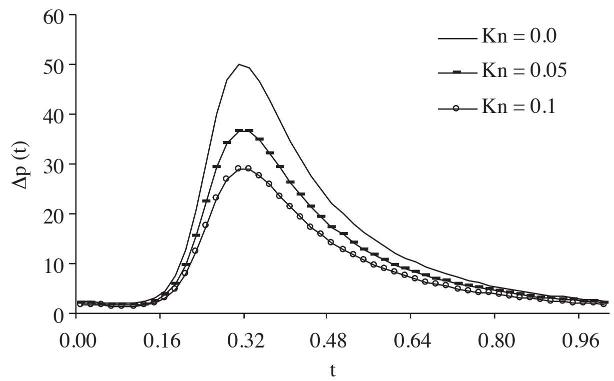


**Figure 3.** Pressure rise versus time for $\overline{Q} = 0.2$, $\varphi = 0.7$, and $\alpha = 1.5$.

The average pressure rise $\overline{\Delta p}$ versus flow rate $\overline{Q}$is plotted in Figures 4 and 5 for uniform and nonuniform channels at $\varphi = 0.7$, $K_n = 0.05$, ($\alpha = 2, 3, 4$)and $\varphi = 0.7$, $\alpha = 2$, ($K_n = 0, 0.05, 0.1$), respectively. As expected, the average pressure rise decreases as the flow rate increases and it achieves its maximum value at zero flow rate. Furthermore, the average pressure rise decreases as the couple stress fluid parameter increases. This means that the peristaltic pumping for the couple stress fluid is greater than for Newtonian fluid. Furthermore, it is clear from Figure 5 that the average pressure rise decreases as the slip parameter (Knudsen number $K_n$) increases. Moreover, the results reveal that the values of the average pressure rise for a uniform channel are greater than those for a nonuniform channel at the same values of physical parameters.

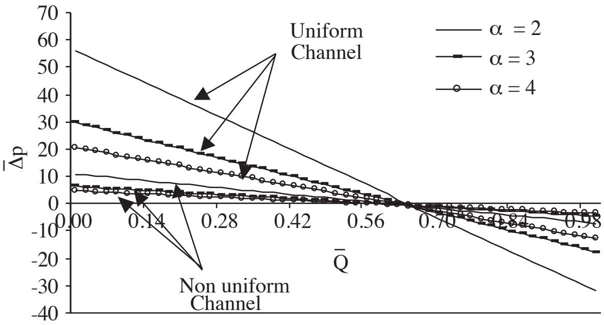


**Figure 4.** Average pressure rise versus flow rate for $\varphi = 0.7$ and $K_n = 0.05$.

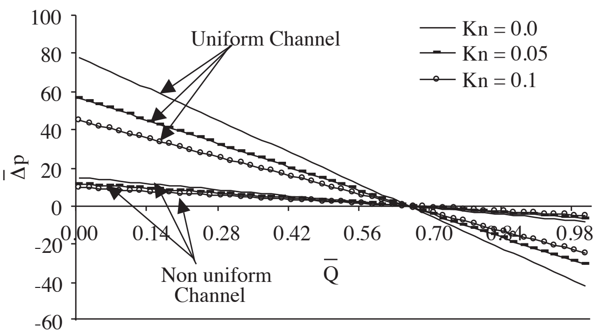


**Figure 5.** Average pressure rise versus flow rate for $\varphi = 0.7$ and $\alpha = 0.2$.

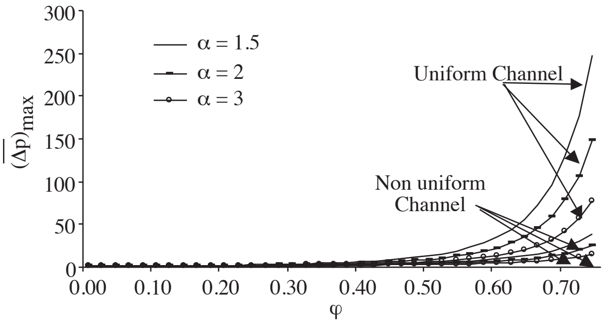


**Figure 6.** Maximum pressure rise versus amplitude ratio $\varphi$ for $K_n = 0$.

In Figures 6 and 7, the maximum pressure rise $(\overline{\Delta p})_{\max}$, which is obtained by putting $\overline{Q} = 0$, is plotted versus the amplitude ratio $\varphi$ for uniform and nonuniform cases for $K_n = 0$, ($\alpha = 1.5, 2, 3$) and $\alpha = 2$, ($K_n = 0, 0.05, 0.1$), respectively. It is shown that the maximum pressure rise increases as the amplitude ratio increases. Moreover, it is seen that the behavior of the maximum pressure rise with variation in the couple stress fluid parameter and the Knudsen number is similar to the behavior of the average pressure rise.

Finally, the friction force is plotted versus time in Figures 8 and 9 for $\overline{Q} = 0$, $\varphi = 0.7$, $K_n = 0$, ($\alpha$ = 1.5, 2, 3) and $\overline{Q} = 0.2$, $\varphi = 0.7$, $\alpha = 1.5$, ($K_n$ = 0, 0.05, 0.1), respectively. It is clear that the friction force has opposite behavior compared with the pressure rise.

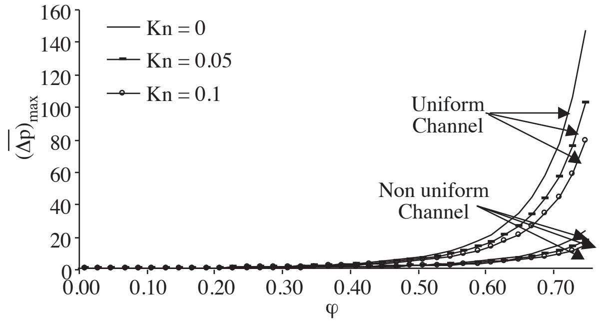


**Figure 7.** Maximum pressure rise versus amplitude ratio $\varphi$ for $\alpha = 2$.

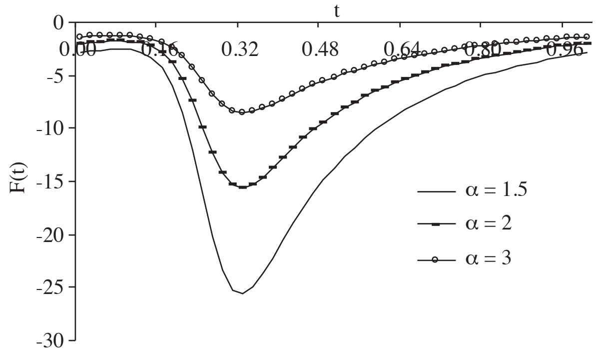


**Figure 8.** Friction force versus time for $\overline{Q} = 0$, $\varphi = 0.7$, and $K_n = 0$.

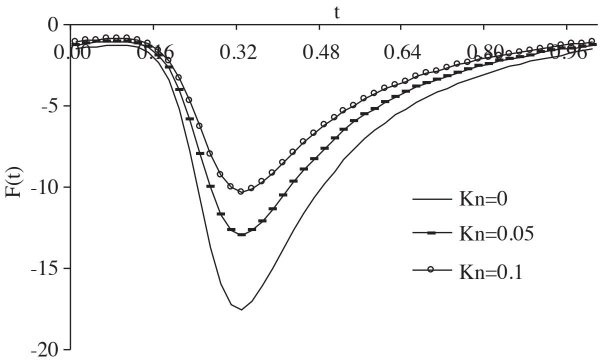


**Figure 9.** Friction force versus time for $\overline{Q} = 0.2$, $\varphi = 0.7$, and $\alpha = 1.5$.

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