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Exact Solutions of Incompressible Couette Flow with Constant Temperature and Constant Heat Flux on Walls in the Presence of Radiation

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Abstract

The present paper investigates a closed form solution for the transient free convection flow of a viscous fluid between 2 infinite vertical parallel plates in the presence of radiation. The flow is set up due to free convective currents occurring as a result of application of constant heat flux (CHF) at one wall and constant temperature on the other wall. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The governing partial differential equations have been solved exactly using the Laplace-transform technique. The numerical values obtained from analytical expressions of velocity, temperature, skin-friction, and Nusselt number have been presented graphically to study the behaviour of flow on momentum and thermal boundary layer.

Key words: Transient, Natural convection, Channel flow, Heat flux, Exact solution, Radiation.

Introduction

The importance of studying natural convection in vertical channels arises from many engineering applications such as cooling of electric and electronic equipment, nuclear reactor fuel elements, home ventilation and many others. Some of the industrial applications in which natural convection in vertical channels formed by parallel plates receives significant attention include solar collectors, fire research, aeronautics, chemical apparatus, building construction, etc. Natural convection occurs in vertical parallel plate channels when at least one of the 2 plates is heated or cooled. The resulting buoyancy driven flow can be laminar or turbulent depending on the channel geometry, fluid properties and temperature difference between the plate and the ambient (Incropera and Dewitt, 1966). Ostrach (1952) extensively presented the steady laminar free-convective flow of a viscous incompressible fluid between 2 vertical walls. Ostrach (1954) and Sparrow et al. (1959) studied the combined effects of a steady free and forced convective laminar flow and heat transfer between vertical walls. The first numerical solution for developing natural convection flow in an isothermal channel was carried out by Bodia and Osterle (1962) using boundary-layer approximation. Aung (1972), Aung et al. (1972), Miyatake and Fuzii (1972), and Miyatake et al. (1973) presented their results for a steady free convective flow between vertical walls by applying different physical treatments for transport processes. Transient convection is of fundamental interest in many industrial and environmental applications such as air conditioning systems, human comfort in buildings, atmospheric flows, motors, thermal regulation processes, and cooling of electronic devices. In view of these applications, Mohanty (1972) investigated transient free convection flow between 2 horizontal parallel plates. The results of a numerical study of the transient natural convection flow between 2 vertical parallel plates were presented by Joshi (1988). In this study, Joshi (1988) applied uniform temperature and a uniform heat flux on the walls. Singh (1988) and Singh et al. (1996) studied the flow behavior of a transient free convective flow of a viscous incompressible fluid between 2 vertical parallel plates in relative motion using the Laplace transform technique. Paul et al. (1996) studied the transient free convective flow in a vertical channel having a constant temperature and constant heat flux on the channel walls. Jha et al. (2003) and Singh et al. (2006) have recently presented an analytical solution for the transient free convection flow in a vertical channel as a result of symmetric/asymmetric heating. The very recent work by Brereton et al. (2006) takes into consideration the convective heat transfer in unsteady laminar parallel flows. In many practical and experimental circumstances, free convection flows are generated adjacent to surfaces dissipating heat at a prescribed heat flux rate. Sacheti et al. (1994) and Pallath et al. (1998, 2002) carried out analytical studies on unsteady free convection flow near an infinite vertical flat plate subjected to uniform heat flux.

All the studies mentioned above are limited only to applications where radiative heat transfer is negligible. Actually, radiative convective flows are encountered in countless industrial and environmental processes, such as heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, and solar power technology. Radiative heat transfer also plays an important role in manufacturing industries in the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites, and space vehicles are examples of such engineering applications. The effects of radiation on heat transfer problems near plates of infinite extent have been discussed by several authors (e.g., Raptis et al. (1998, 2003, 2004, 1999, 2003) under different physical conditions).

However, not too many studies have been published on the closed form solution, which incorporates the effect of radiative heat transfer on the transient free convection flow in a vertical channel when one of the channel walls is at a uniform temperature and heat is supplied at a constant rate at the other wall. Thus the purpose of the present investigation is to obtain an analytical solution (using the Laplace transform technique) for this physical phenomenon. Analytical solutions are important partly because of their wider applicability in understanding the basic physics of any problem and partly because of their possible applications in industrial and technological fields. For flows near the flat plate, the exact solutions of non-linear partial differential equations can be simplified. However, in the case of Couette flow when the flow is generated due to the application of constant heat flux (CHF) at one wall, the degree of complexicity of the solution procedure increases because the heat flux effects entail consideration of derivative boundary conditions. Therefore, exact solutions of such problems are more difficult to obtain when the velocity field gets coupled with the temperature field. Hence, based on the above discussion, the governing equations are solved using Laplace transform technique and numerical results are illustrated graphically in the form of velocity and temperature profiles as well as the skin friction and Nusselt number for the 2 most important fluids – atmospheric air and water.

Mathematical Analysis

Consider the transient free convective flow of a viscous fluid due to temperature gradient in a vertical channel with the walls at a constant distance d apart. The x*-axis is taken along one of the wall of the channel and y*-axis is normal to it. It is also considered that there is radiation only from the fluid. The fluid is a gray, emitting, and absorbing radiation, but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the x* direction is assumed negligible in comparison to that in the y*-direction. Initially, the fluid and the channel walls are at the same temperature and there is no fluid motion. At time $t^* > 0$, the heat is supplied at a constant rate at one of the channel walls ($y^* = 0$) while the other wall at $y^* = d$ is maintained at a constant temperature T_d, which causes free convection currents in the channel. Under usual Boussinesq's approximation, the mathematical model for the above free convection flow in the channel is stated as:

$$\frac{\partial u^*}{\partial t^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta \left(T^* - T_d^*\right) \tag{1}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y^*} \tag{2}$$

The initial and boundary conditions relevant to the fluid flow are:

$$\left.\begin{array}{ll} \text{For } t^* \leq 0 : u^* = 0, T^* = T_d^* & \text{for } 0 \leq y^* \leq d \\ \text{For } t^* > 0 : u^* = 0, \frac{\partial T^*}{\partial y^*} = -\frac{q}{k} & \text{at } y^* = 0 \\ u^* = 0, T^* = T_d^* & \text{at } y^* = d \end{array}\right\} \quad (3)$$

The term $\frac{\partial q_r}{\partial y}$ represents the change in the radiative heat flux with distance. The radiative heat flux term, by using the Rosseland approximation, is given by

$$q_r = -\frac{4\sigma^*}{3a_R}\frac{\partial T^{*4}}{\partial y^*} \quad (4)$$

We assume that the temperature differences within the flow are such that T^{*4} may be expressed as a linear function of the temperature T^*. This is accomplished by expanding T^{*4} in a Taylor series about T_d^* and neglecting higher order terms.

$$T^{*4} \simeq 4T_d^{*3}T^* - 3T_d^{*4} \quad (5)$$

By using Eqs. (4) and (5), Eq. (2) gives

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{16\sigma^* T_d^{*3}}{3a_R C_p}\frac{\partial^2 T^*}{\partial y^{*2}} \quad (6)$$

In order to solve the governing equations in dimensionless form, we introduce the following non-dimensional quantities:

$$y = \frac{y^*}{d}, t = \frac{t^*\nu}{d^2}, \theta = \frac{T^* - T_d^*}{dq/k}, u = \frac{u^* k\nu}{g\beta q d^3}, R = \frac{k a_R}{4\sigma^* T_d^3}, Pr = \frac{\mu C_p}{k} \quad (7)$$

Equations (1) and (6) in dimensionless form are now expressed as:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \theta \quad (8)$$

$$\frac{\partial \theta}{\partial t} = \left(\frac{3R+4}{3RPr}\right)\frac{\partial^2 \theta}{\partial y^2} \quad (9)$$

The initial and boundary conditions in the dimensionless form for the physical system considered in the present study are obtained as:

$$\left.\begin{array}{ll} \text{For } t \leq 0: u = \theta = 0 & \text{for } 0 \leq y \leq 1 \\ \text{For } t > 0 : u = 0, \frac{\partial \theta}{\partial y} = -1 & \text{at } y = 0 \\ u = 0, \theta = 0 & \text{at } y = 1 \end{array}\right\} \quad (10)$$

The physical quantities used in the above equations are defined in the Nomenclature. Under the assumptions of the flow problem, we observe that the energy Eq. (9) is uncoupled from the momentum Eq. (8). We can, therefore, solve for the temperature θ(y,t) whereupon u(y,t) can be expressed in terms of θ(y,t). As stated before, the governing partial differential Eqs. (8) and (9) with the initial and boundary conditions (10) are amenable to an exact analytical treatment using Laplace transforms. Taking Laplace transforms of Eqs. (8) and (9) will result in a set of ordinary differential equations for the transformed functions in the independent variable y. On solving, the transformed temperature variable $\bar{\theta}(y,p)$ can be obtained as

$$\bar{\theta}(y,p) = \frac{1}{\sqrt{s}}\sum_{n=0}^{\infty}(-1)^n \frac{1}{p^{3/2}}\{exp\,(-a\sqrt{p}) - exp(-b\sqrt{p})\} \quad (11)$$

where

$$s = \frac{3RPr}{3R+4}, a = (2n+y)\sqrt{s}, b = (2n+2-y)\sqrt{s}$$

which on inversion (Abramowitz et al., 1972), yields

$$\theta(y,t) = \frac{1}{\sqrt{s}}\sum_{n=0}^{\infty}(-1)^n\{F_1(a,t) - F_1(b,t)\}, R \neq 0 \quad (12)$$

To obtain the solution for the velocity variable, we solve the transformed momentum equation and write the velocity in the (y,p) – plane in the form:

$$\bar{u}(y,p) = \frac{1}{(s-1)\sqrt{s}}\sum_{n=0}^{\infty}(-1)^n\left[\frac{1}{p^{5/2}}(exp(-b\sqrt{p}) - exp(-a\sqrt{p}) + \sum_{m=0}^{\infty}\frac{1}{p^{5/2}}(exp(-d\sqrt{p}) - exp(-c\sqrt{p}) + exp(-e\sqrt{p}) - exp(-f\sqrt{p})\right] \quad (13)$$

where

$$c = (2n+2)\sqrt{s} + 2m + y,$$

$$d = (2n+2)\sqrt{s} + 2m + 2 - y$$

$$e = 2n\sqrt{s} + 2m + y, \quad f = 2n\sqrt{s} + 2m + 2 - y.$$

The solution for the velocity variable in the physical (y,t) – plane is obtained by applying the inverse Laplace transform to Eq. (13).

$$\begin{aligned} u(y,t) = \; & \tfrac{1}{(s-1)\sqrt{s}} \sum_{n=0}^{\infty} (-1)^n \\ & \Big[\{F_2(b,t) - F_2(a,t)\} + \sum_{m=0}^{\infty} \{F_2(d,t) \\ & -F_2(c,t) + F_2(e,t) - F_2(f,t)\}\Big], R \neq 0 \end{aligned} \tag{14}$$

The functionals F_1 and F_2 used in the above equations are defined as:

$$F_1(\ell,t) = 2\sqrt{\frac{t}{\pi}} exp(-\ell^2/4t) - \ell erfc\left(\frac{\ell}{2\sqrt{t}}\right) \tag{15}$$

$$\begin{aligned} F_2(\ell,t) = & \tfrac{1}{3}\sqrt{\tfrac{t}{\pi}}(4t + \ell^2) exp\left(-\tfrac{\ell^2}{4t}\right) \\ & -\ell\left(t + \tfrac{\ell^2}{6}\right) erfc\left(\tfrac{\ell}{2\sqrt{t}}\right) \end{aligned} \tag{16}$$

where erfc is a complementary error function defined as

$$erfc(\ell) = \frac{2}{\sqrt{\pi}} \int_{\ell}^{\infty} exp^{-\eta^2} d\eta$$

Solution when R → 0

We observe that the solutions for the temperature and velocity variable given by Eqs. (12) and (14), respectively, are not applicable when R → 0. The exact solution in this case has to be re-derived starting from the following equation:

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \tag{17}$$

Solving the above equation with boundary conditions (10) yields the following expression for temperature field:

$$\theta(y,t) = \frac{1}{\sqrt{\Pr}} \sum_{n=0}^{\infty} (-1)^n \{F_1(g,t) - F_1(h,t)\} \tag{18}$$

where

$$g = (2n + y)\sqrt{\Pr}, h = (2n + 2 - y)\sqrt{\Pr}$$

Using Eq. (18) in Eq. (8), we obtain the solution for velocity variable as:

$$\begin{aligned} u(y,t) = \; & \tfrac{1}{(Pr-1)\sqrt{Pr}} \sum_{n=0}^{\infty} (-1)^n \\ & \Big[\{F_2(h,t) - F_2(g,t)\} + \sum_{m=0}^{\infty} \{F_2(k,t) \\ & -F_2(j,t) + F_2(o,t) - F_2(r,t)\} \Big] \end{aligned} \tag{19}$$

where

$$j = (2n + 2)\sqrt{\Pr} + 2m + y,$$

$$k = (2n + 2)\sqrt{\Pr} + 2m + 2 - y$$

$$o = 2n\sqrt{\Pr} + 2m + y, \quad r = 2n\sqrt{\Pr} + 2m + 2 - y.$$

The functionals F_1 and F_2 used in Eqs. (18) and (19) are the same as those defined in Eqs. (15) and (16).

For practical applications, the major physical quantities of interest include the shear stress and the rate of heat transfer at the wall. They are expressed as follows:

The skin-friction in non-dimensional form is given by:

$$\tau_0 = \frac{k\tau_0^*}{\rho g \beta q d^2} = \left.\frac{\partial u}{\partial y}\right|_{y=0} \tag{20}$$

$$\begin{aligned} \tau_0 = \; & \frac{1}{(s-1)\sqrt{s}} \sum_{n=0}^{\infty} (-1)^n \left\{ \sqrt{s} \left[\left(t + \frac{b_1^2}{2}\right) erfc\left(\frac{b_1}{2\sqrt{t}}\right) \right.\right. \\ & -b_1\sqrt{\frac{t}{\pi}} exp\left(-\frac{b_1^2}{4t}\right) + \left(t + \frac{a_1^2}{2}\right) erfc\left(\frac{a_1}{2\sqrt{t}}\right) \\ & \left. -a_1\sqrt{\frac{t}{\pi}} exp\left(-\frac{a_1^2}{4t}\right)\right] + \sum_{m=0}^{\infty} \left[\left(t + \frac{d_1}{2}\right) erfc\left(\frac{d_1}{2\sqrt{t}}\right) \right. \\ & -d_1\sqrt{\frac{t}{\pi}} exp\left(-\frac{d_1^2}{4t}\right) + \left(t + \frac{c_1^2}{2}\right) erfc\left(\frac{c_1}{2\sqrt{t}}\right) \\ & -c_1\sqrt{\frac{t}{\pi}} exp\left(\frac{c_1^2}{4t}\right) - \left(t + \frac{e_1^2}{2}\right) erfc\left(\frac{e_1}{2\sqrt{t}}\right) \\ & +e_1\sqrt{\frac{t}{\pi}} exp\left(-\frac{e_1^2}{4t}\right) - \left(t + \frac{f_1^2}{2}\right) erfc\left(\frac{f_1}{2\sqrt{t}}\right) \end{aligned}$$

$$+f_1\sqrt{\frac{t}{\pi}}exp\left(-\frac{f_1^2}{4t}\right)\Bigg]\Bigg]\Bigg\}, R \neq 0$$

where

$$a_1 = 2n\sqrt{s}, b_1 = (2n+2)\sqrt{s},$$

$$c_1 = (2n+2)\sqrt{s} + 2m,$$

$$d_1 = (2n+2)\sqrt{s} + 2m + 2, \quad e_1 = 2n\sqrt{s} + 2m,$$

$$f_1 = 2n\sqrt{s} + 2m + 2$$

The rate of heat transfer is given by

$$q = -k\left(\frac{\partial T^*}{\partial y^*}\right)_{y^*=0} \tag{21}$$

The coefficient of heat transfer, which is generally known as the Nusselt number Nu, is given by

$$Nu_0 = \frac{qd}{k(T^* - T_d^*)} = -\frac{1}{\theta(0)}\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = \frac{1}{\theta(0)} \tag{22}$$

Discussion and Conclusion

The aim of this study was to obtain an analytical solution for the transient free convective flow in a vertical channel in the presence of radiation. The thermal radiation heat transfer effects are included in the energy equation and the surface heat flux is assumed to be constant. The exact solution thus obtained has been used in this section to investigate numerically the influence of the external forces. The present analysis is carried out for 2 sample liquids: air and water, whose Prandtl numbers at 20 °C and 1 atmospheric pressure were measured as 0.71 and 7.0, respectively.

Figure 1 shows the dimensionless temperature profiles θ(y,t) inside the boundary layer for different values of the radiation parameter (R), temporal variable (t), and Prandtl number (Pr). This figure illustrates that the temperature profiles attain their maximum value on the wall (y = 0) and descend smoothly to zero at the other wall (y = 1) of the channel. It is interpreted from Eqs. (12) and (15) that as the time (t) progresses the fluid temperature also increases. This trend is clearly visible in Figure 1, in which the thickness of the thermal boundary layer, for both of the fluids, increases with an increase in time because of the continuous heating of both surface and fluid. From this figure, it is also noticed that the temperature falls more rapidly for water (Pr = 7.0) than for air (Pr = 0.71). This is in agreement with the physical fact that the thermal boundary thickness increases with the decrease in the Prandtl number (Pr). The reason for such behavior is that smaller values of Prandtl number $\left(Pr = \frac{\mu C_p}{k}\right)$ yield increases in thermal conductivities and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of Pr. Furthermore, for both air and water, the fluid temperature is found to decrease due to an increase in the radiation parameter (R). The limit R $\rightarrow \infty$ represents the case of absent radiation heat transfer effects and, in this case, our result is comparable with that reported by Paul et al. (1996).

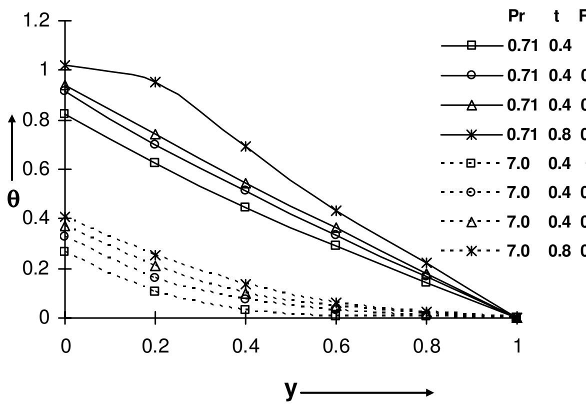

Figure 1. Temperature profiles.

The effects of temporal variable (t) and radiation parameter (R) on the velocity distribution of air and water within the channel are depicted in Figures 2 and 3, respectively. It can be observed that the nature of the flow is of parabolic type with maximum magnitude along the axis of the channel and minimum at the walls.

It is evident in both figures that the fluid velocity is enhanced by increasing the values of the temporal variable due to the important role that viscosity plays in transferring momentum between fluid layers. Moreover, the figures show that the larger the value of radiation parameter, the thinner the momentum boundary layer size. This result can be explained by the fact that an increase in the radiation parameter $R = ka_R/4\sigma^*T_d^{*3}$ for given k and T_d^* means an increase in the Rosseland radiation absorptivity a_R.

In view of Eqs. (2) and (4), it is concluded that the divergence of radiative heat flux $\frac{\partial q_r}{\partial y}$ decreases as a_R increases. Therefore, the rate of radiative heat transferred to the fluid decreases and consequently the fluid temperature and along with the velocity of its particle also decreases. The curve corresponding to R $\rightarrow \infty$ represents the case of absent radiation heat transfer effect. A comparative study of these figures reveals that the velocity of air (Pr = 0.71) is greater than the velocity of water (Pr = 7), keeping the other parameters fixed. These results are consistent with the physical observation that the fluids with high Prandtl number have greater viscosity, which makes the fluid thick and hence move slowly.

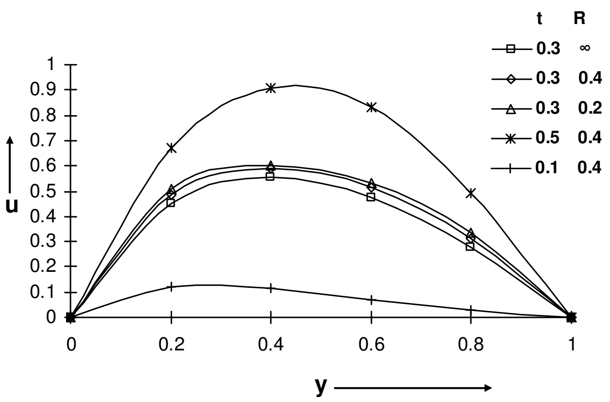

Figure 2. Velocity Profiles of air (Pr = 0.71).

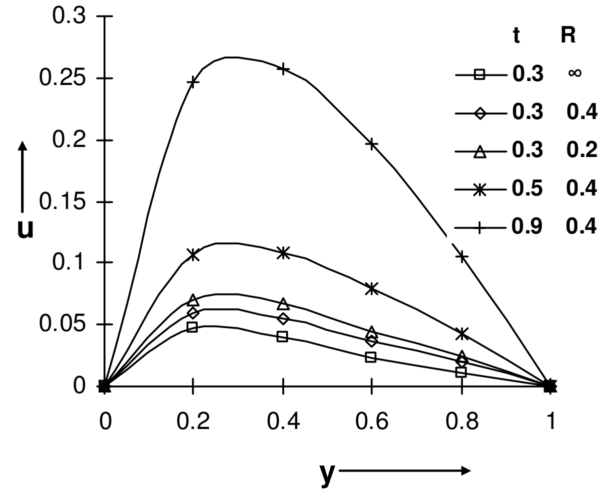

Figure 3. Velocity profiles of water (Pr = 7.0).

Results for the coefficient of skin friction τ_0 against time for different parameters at the wall y = 0 are presented in Figure 4. An inspection of this figure reveals that the shear stress decreases as Pr increases since an increasing value of Pr (which encapsulates the ratio of momentum diffusivity to thermal diffusivity) implies a reduced buoyancy effect and hence there is less friction at the plate. Moreover, it is observed that skin friction profiles at the walls increase with increasing temporal variable but decrease with increasing values of the radiation parameter. For small values of time, skin friction is more affected by Pr while less affected for large values of time.

Similarly, the effects of Pr and R on Nusselt number Nu_0 at the wall y = 0 are demonstrated in Figure 5. It is noticed that the rate of heat transfer is higher

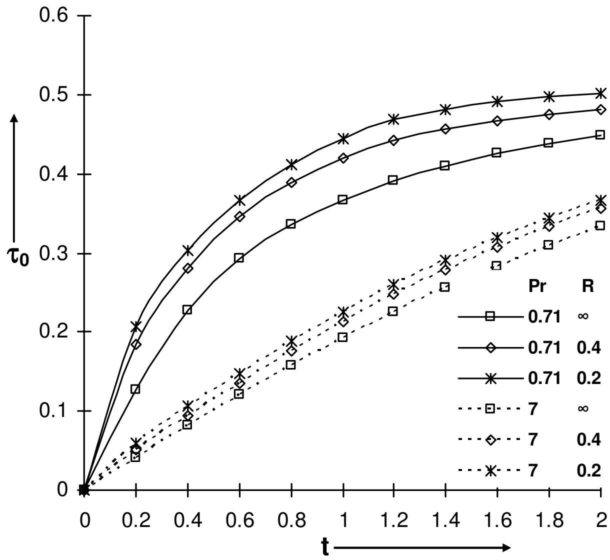

Figure 4. Skin-friction profiles at the plate $y = 0$.

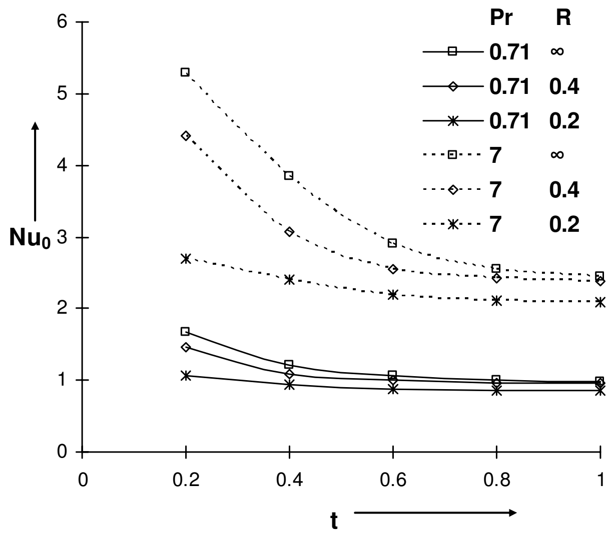

Figure 5. Nusselt number at the plate $y = 0$.

for water than it is for air. This is evident from the fact that, for increasing values of Pr, frictional forces become dominant and hence yield greater heat transfer. Furthermore, as time advances, the value of Nusselt number decreases, while the reverse happens for increasing values of R.

Nomenclature

a_R	absorption coefficient
C_p	specific heat at constant pressure
d	width of the channel
g	acceleration due to gravity
k	thermal conductivity of the fluid
Nu	Nusselt number
p	Laplace parameter
Pr	Prandtl number
q	rate of heat transfer
q_r	radiative heat flux in the y-direction
R	radiation parameter
T^*	temperature of the fluid in non-dimensional form
T_d^*	temperature of one of the walls
t^*	time
t	dimensionless time
u^*	velocity of the fluid in the x^*-direction
u	dimensionless velocity of the fluid in the x-direction
y^*	coordinate axis normal to the plate
y	dimensionless coordinate axis normal to the plate

Greek symbols

β	volumetric coefficient of thermal expansion
μ	coefficient of viscosity
ν	kinematic viscosity
ρ	density
σ^*	Stefan-Boltzmann constant
τ	dimensionless skin-friction
θ	dimensionless temperature

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