Turkish J. Eng. Env. Sci.
30 (2006) , 237 – 249.


# Reliability and Reliability-Based Design Optimization

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Received 18.11.2005

**Abstract**

Reliability is often understood as the probability that a structure will not fail to perform its intended function. In this paper, the general formulation of consistent evaluation of the safety of structures and the associated methods of analysis are reviewed. After the reliability analysis of a cantilever beam is demonstrated, the methodology of reliability-based optimization and the related problem are discussed. A typical 2D roof truss system is optimized by various optimization methods, such as Sequential Quadratic Programming (SQP), Evolution Strategy (EVOL), and Genetic Algorithms (GA). It is concluded that the algorithms work well and an efficient design of the roof truss is achieved under the constraint of the failure probabilities of structural element.

**Key words:** Optimization, Reliability, Reliability-based design optimization, Truss, Evolutionary computation.

## Introduction

During the last decade, structural reliability theory has been examined in a large number of research studies, and now the reliability of a structure could be defined as an ability that fulfills its design purpose for some specified reference period. For a structure, design purposes are safety against torsion, shear, flexure, and so on. However, due to many sources of uncertainty, which are inherent in structural design, there is the risk of unacceptable performance for structures, which is called failure. It could be said that a structure fails if it cannot perform its intended function; however, this is a vague definition because the function of the structure has not been specified. Therefore, the concept of a limit state is used to help define failure in the context of structural reliability analyses. A limit state is a boundary between desired and undesired performance of a structure (Nowak and Collins, 2000). It is usually characterized mathematically by a limit state function and there are 3 types of limit states considered in structural reliability analyses. These are as follows:

1. Ultimate limit states: mostly related to the loss of load-carrying capacity.

2. Serviceability limit states: related to gradual deterioration.

3. Fatigue limit states: related to loss of strength under repeated loads.

These 3 limit states are incorporated into structural reliability analysis, which is concerned with the treatment of uncertainties (random) in structural engineering design. A set of basic variables must be defined for the purpose of quantifying uncertainties in the field of structural engineering and for subsequent reliability. These are defined as the set of basic quantities governing the static or dynamic response of a structure (Thoft-Christensen and Baker, 1982). In the beginning, the random variables were simply represented by 2 basic variables; the resistance ($R$ load-carrying capacity) and the load effect ($Q$). The limit state can be defined corresponding to the ability of a structure to fulfill its design purpose as follows:

$Q > R$, the structure has no ability to fulfill its design purpose, failure.

$R > Q$, the structure has the ability to fulfill its design purpose, no failure or safe.

**Structural reliability and reliability methods**

Having accepted the dichotomy of structural behavior as failure and no failure, one can proceed to consider the methods that can be used to determine the probability of each state. A reliability method, in the narrowest sense, is a method to evaluate the reliability of a system (Madsen et al., 1986). The probability of failure, $P_f$, is equal to the probability that the undesired performance will occur. Mathematically, this can be expressed, depending on the basic design variables mentioned above, as follows:

$$P_f = P(R < Q) = P(R - Q < 0) = P(R/Q < 1) \quad (1)$$

As previously mentioned, the state of the structure can be described using various random parameters (variables) $X_1, X_2, \ldots,$ X$_n$, which are load and resistance parameters, such as dead load, live load, compressive strength, and yield strength. In this case, the limit state function showing the conditions of failure can be expressed as a function of the vector of random variables, $x$. The failure domain is denoted by $F = \{x; g(x) < 0\}$, the safe domain $S = \{x; g(x) > 0\}$, and the limit surface, the boundary of $F$ by $G = \{x; g(x) = 0\}$ (Breitung, 1994). This is illustrated in Figure 1 in the case of a 2-dimensional state space.

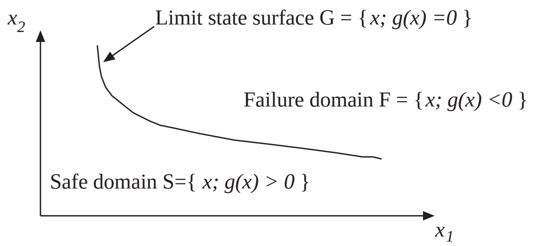



**Figure 1.** Failure domain, limit state surface, and safe domain.

Then the problem is to compute the probability of failure given as Eq. (2),

$$P_f = \int_{g(x) \leq 0} f_x(x) \; dx, \quad (2)$$

where $f_x(x)$ represents the joint probability density function (PDF) of the random vector and $P_f$ is valid for continuous random variables only. Even though Eq. (2) seems simple, evaluating this integral is very difficult in most cases. The integration requires special numerical techniques and the accuracy of these techniques may not be adequate (Nowak and Collins, 2000). Therefore, some other procedures, which will be explained in the following sections, need to be used to evaluate the integral in Eq. (2).

The calculation of the probability of failure was not a simple task until the concept of the reliability index, first proposed by Freudenthal (1956), was introduced. Further proposal for a reliability index is given by Cornell (1969). Later, Hasofer and Lind (1974) introduced a new reliability index definition that is the shortest distance from the origin of reduced variables to the limit state surface ($g(x) = 0$). It has a very important characteristic that is invariant with respect to different choices of the limit state function for a given failure domain. According to this definition, the reliability index, $\beta$, is calculated as follows:

$$\beta = \mu_R - \mu_Q \Big/ \sqrt{\sigma_R^2 + \sigma_Q^2}, \quad (3)$$

where $g(R, Q) = R - Q; R$ and $Q$ are uncorrelated, normally distributed random variables.

As can be seen in Eq. (3), $\beta$ depends only on the means and standard deviations of random variables. Therefore, $\beta$ is called a second-moment measure of structural safety because only the first 2 moments (mean and variance) are required to calculate $\beta$. A one-to-one relation between the reliability index and the probability of failure exists for linear limit state functions, Eq. (4). It is also necessary for random variables to be uncorrelated and normally distributed for this relation.

$$\beta = -\Phi^{-1}(P_f) \quad \text{or} \quad P_f = \Phi(-\beta) \quad (4)$$

The situation is different in the case of domains defined by non-linear functions, and $P_f$ is not computed from the reliability index in this case. Several attempts were made to generalize these relations between the index and probability contents to nonlinear functions (Breitung, 1994). The basic idea in this connection was to replace the nonlinear function, $g$, in the point, $x^*$(design point), on the failure surface with minimal distance to the origin by a linear function, $g_L$, defined by

$$g_L(x) = (\nabla g(x^*))^T \ (x - x^*) \tag{5}$$

i.e. by its first-order Taylor expansion at the point $x^*$. This method described above is known as the First-Order Reliability Method (FORM) since only a first-order Taylor expansion of the limit state function is made. In addition to the FORM, approximations based on linear Taylor expansion of the limit state function at the maximum point of the normal density, the Second-Order Reliability Method (SORM) was studied in the late 70s in which a second-order Taylor expansion of the limit state function is made (Breitung, 1994). That means that the Hessian matrix ($H_g(x)$) of the limit state function is used to fit an approximation. The second-order approximation of the failure surface at the design point is given by

$$g(x) \approx \nabla g(x^*)^T (x - x^*) + \frac{1}{2}(x - x^*)^T H_g(x^*)(x - x^*), \tag{6}$$

where $\nabla g(x^*)$ is the gradient vector at the design point and the symmetric matrix, $H_g(x^*)$, is the matrix of second-order partial derivatives at this point.

If the distributions of the random variables are known, then the procedures used for calculating reliability indexes can be improved. In such cases, the non-normally distributed sets of basic variables are transformed into a standardized Gaussian space. This transformation is defined as:

$$F_{Xi}(x_i) = \Phi(y_i) \tag{7a}$$

$$y_i = \Phi^{-1}(F_{Xi}(x_i)) \tag{7b}$$

$$x_i = F_{Xi}^{-1}(\Phi(y_i)), \tag{7c}$$

where $F_{Xi}$ is the cumulative distribution function (CDF) of $i$th basic variables, $\Phi(.)$ is the CDF of the standard normal distribution, $x_i$ is the $i$th basic variables, and $y_i$ are transformed Gaussian variables that are also standardized to the $i$th basic variables. This transformation yields a nonlinear limit state function in almost all cases. In the original space, as previously mentioned, the limit state function is defined as

$$g(x) = g(x_1, x_2, ..., x_n) = 0. \tag{8}$$

According to Eq. (7) the limit state function may also be defined as

$$g(F_{X1}^{-1}(\Phi(y_1)), F_{X2}^{-1}(\Phi(y_2)), ...., F_{Xn}^{-1}(\Phi(y_n))) = 0 \tag{9}$$

$$h(y) = h(y_1, y_2, ...., y_n) = 0. \tag{10}$$

Equation (10) leads to a hyper surface and it has safe and failure domains in n dimensional standardized space. To simplify these functions, linear and quadratic tangential surfaces on the so-called *design point y** are calculated by FORM and SORM, respectively. This point of the limit state function, $h(y)$, is defined via the shortest distance between $h(y)$ and the coordinate origin of the

$$\delta = \frac{h(y) - \sum\limits_{j=1}^{n} y_j \frac{\partial h}{\partial y_j}}{\left( \sum\limits_{j=1}^{n} \left( \frac{\partial h}{\partial y_j} \right)^2 \right)^{1/2}} \tag{11}$$

standardized Gaussian space. From this distance measure the safety index is derived.

$$\beta = \begin{cases} +\delta \ \ h(0) > 0 \\ -\delta \ \ h(0) < 0 \end{cases} \tag{12}$$

Afterwards, the failure probability is easily calculated for FORM and SORM, respectively, as

$$P_f \approx \Phi(-\beta) \tag{13}$$

$$P_f = \Phi(-\beta) \prod_{i=1}^{n-1} (1 - \beta\kappa_i)^{-1/2}, \tag{14}$$

where $\kappa_i$ are the main curvatures of the failure surface at $y^*$. The most time-consuming part in these methods is to find the design point. Several iterations have to be calculated until the distance measure, $\delta$, shows good convergence (Jelic et al., 2004);

however, there is no guarantee that a good convergence will be reached in all cases (Nowak and Collins, 2000).

In addition to the approximation methods defined above, simulation techniques are presented as one possible way to solve such problems. The basic idea behind simulation is to numerically simulate some phenomenon and then to observe the number of times a specific event of interest occurs. Although it seems relatively straightforward, the procedure can become time consuming. One of the methods to generate some results numerically is the Monte Carlo (MC) method. The distribution information of the important parameters of a problem is used to generate samples of numerical data according to MC. Then the limit state function is checked for each set of samples of numerical data. For this task, an indicator function, $I(g(x))$, is used in order to show the simulation results. The state of structure is failure if it takes the value of 1; otherwise no failure occurs (Eq. 15).

$$I(g(x)) = \begin{cases} 1 & \text{if} \quad g(x) < 0 \\ 0 & \text{if} \quad g(x) \geq 0 \end{cases} \tag{15}$$

If a sufficient number of simulated values are available, it is possible to estimate the probability of failure as

$$P_f = \frac{1}{n} \sum_{i=1}^{p} I(g(x)), \tag{16}$$

where $n$ is the total number of simulations and $p$ is the number of times that $I(g(x)) = 1$; in other words, $p$ is the number state of failure, $g(x) < 0$, in the total number of simulated $g(x)$. The accuracy of the MC simulation approach increases as the number of sample values increases, and it is proportional to $1/\sqrt{n}$. Moreover, in some cases, the problem is extremely complex and the time needed to evaluate the problem for a single trial is extensive. Therefore, to reduce the number of simulations needed to obtain a reasonable result, the methods known as variance reduction have been introduced. Among them, Importance Sampling, Latin Hypercube, and Adaptive Sampling are well-known variance reduction methods.

### Example 1

To illustrate how the reliability analysis on a structural system by any reliability methods is applied, a cantilever beam, Figure 2, is considered. The assumed limit state is the moment capacity of the beam at the support. The random variables in the problem are concentrated load ($P = X_2$) and yield stress ($\sigma_y = X_1$). The mean, standard deviation, and type of distribution of random variables are listed in Table 1.

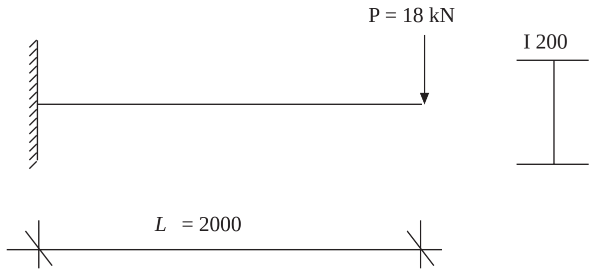



**Figure 2.** Cantilever beam.

This example is also solved by Spaethe (1992) and Jelic et al. (2004). In this work, the failure probability of the cantilever beam is calculated by FORM, SORM, and MC, again in order to show the efficiency of the algorithm coded in MATLAB. Solution algorithms of FORM, SORM, and MC are based on the algorithms given by Spaethe (1992). The comparison of the results obtained by coded algorithm and the results given by other researchers are summarized in Table 2. The limit state function for the cantilever beam is expressed as follows:

$$g(x) = X_1 w - X_2 L = 0, \tag{17}$$

where $w$ is the plastic section modulus and is taken as 214,000 mm$^3$.

**Table 1.** Parameters of random variables.

| $X_i$ | $\mu_{xi}$ | $\sigma_{xi}$ | $X_{0i}$ | Type of distribution |
|---|---|---|---|---|
| 1 | 265 N/mm$^2$ | 25 N/mm$^2$ | 160 N/mm$^2$ | Log. Normal |
| 2 | 18 kN | 2 kN | - | Gumbel, Smallest values |

**Table 2.** Calculation results.

| Methods | $P_f$−Spaethe (1992) | $P_f$−Jelic et al. (2004) | $P_f$−this work |
|---|---|---|---|
| MC (10,000 samples) | 2.130E-03 | | 2.200E-03 |
| MC (100,000 samples) | | 2.46E-3 | 2.210E-03 |
| MC (5,000,000 samples) | | 2.09E-3 | 2.581E-03 |
| FORM | 2.242E-03 | 2.24E-03 | 2.243E-03 |
| SORM | 2.138E-03 | 2.14E-03 | 2.142E-03 |

Comparing the results obtained in the current work and the exact value computed by Spaethe (1992) using the numerical integration method, 2.131E-03, it can be said that the coded algorithm works well, except for the MC method with 5 million samples. It is thought that this is due to the MATLAB random number generator tool. The built-in generators found in many software packages should be used with caution. Some random generation algorithms work better than others (Nowak and Collins, 2000); hence, it may be possible that the same set of uniformly distributed random numbers is generated over and over again. It is also observed that the time needed to reach a reasonable result increases as the size of the samples increases during the process of reliability analysis.

### Reliability-based optimization

The aim of a design is to achieve adequate safety with minimum cost. It has been observed that the structures designed through deterministic optimization do not necessarily have high reliability. Optimization based on the reliability concept will lead to more consistent safety in the structure system (Pu et al., 1996).

Reliability-based optimization formulations can be divided into 2 main categories as the component and the system reliability index-based optimization. In this work, the reliability-based optimization formulation based on the probabilities of failure of the members is adopted.

The formulation of the optimization problem based on member reliability can be expressed as follows:

$$\min_A \left\{ W = \sum_{i=1}^{m} \rho\, l_i\, A_i \quad | \quad P_{fi} < P_{fa} \right\}, \tag{18}$$

where $W$ is the weight of the structural system, $l_i$, $A_i$ are the length and cross-section area of member $i$, respectively, $P_{fi}$ is the failure probability of member $i$, $P_{fa}$ is the specified allowable probability of failure, and $m$ is the number of members of the structural system. The safety margin or limit state function of member $i$ $g_i(\hat{A}, = \{A_1,\, A_2,\, ...,\, A_m\})$ is calculated as

$$g_i(\hat{A}) = R_{ai}\, A_i - \sum_{j=1}^{\ell} b_{ij}(\hat{A})\, L_j, \tag{19}$$

where $R_{ai}$ is the allowable stress of member $i, b_{ij}(\hat{A})$ is the load coefficient of member $i$ with respect to $L_{j,}, L_j$ is the applied load, $\ell$ is the number of the applied loads, and $\hat{A}$ is the vector composed of the cross-sectional areas.

The probability of failure of member $i$ is calculated by using the First-Order Second Moment (FOSM) method:

$$P_{fi} = P(g_i(\hat{A}) \leq 0) = \Phi(-\beta) \tag{20}$$

Although the strength $(R_{ai}A_i)$ is easily determined specifying the material and dimension of the member, evaluation of the internal forces of the member is very complex, and it is derived by applying the matrix method (Murotsu et al., 1980). According to this method, the internal forces, which are the function of random influences, are obtained as follows:

$$Q = \sum_{j=1}^{\ell} b_{ij}(\hat{A})\, L_j, \tag{21}$$

where $b_{ij}(\hat{A})$ is the element of the matrix $[k_i][T_i][K_i]^{-1}$, $[k_i]$ represents the member stiffness matrix, $[T_i]$ is the transformation matrix, $[K_i]^{-1}$ indicates the matrix formed by extracting the rows corresponding to the vector $\{d_i\}$ from the matrix $[K]^{-1}$, and $[K]$ is the total structure stiffness matrix. The coefficient of $b_{ij}$ becomes functions of the cross-section areas of the members. The purpose is to find the areas of cross-section under the constraints that are taken as failure probabilities of the members in terms of design procedures.

Sequential Quadratic Programming (SQP), Evolution Strategy (EVOL), and Genetic Algorithms (GA) are used to determine the cross-sectional areas under the specified failure probabilities of the members.

There are 2 approaches used to gain insight for practical applications of optimization: indirect and direct methods. Minimization techniques seeking solutions to optimality conditions are often called indirect methods. On the other hand, the direct techniques have a different philosophy. An estimate of the optimum design for the problem is required to start the design process. Then the initial design is improved iteratively until some criteria, which are specified in the design, are satisfied (Arora, 1989; Saka, 1990).

The so-called direct methods are based on the idea of finding a search direction by linearizing the objective and constraint functions and then taking a suitable step in this direction. SQP is based on this idea and it is one of the most popular direct methods (Bhatti, 2000).

Since the 1960s, researchers have been interested in imitating living beings to develop powerful algorithms for difficult optimization problems. A term now in common use to refer to such techniques is evolutionary computation. The most widely known types of evolutionary computation methods are GA and EVOL (Gen and Cheng, 2000). Both these methods use the solution principles and mechanisms of biological evolution processes, in which numerous optimization mechanisms are embedded. EVOL imitates, in contrast to GA, the effects of genetic procedures on the phenotype.

An algorithmic procedure to solve the optimization problem outlined above is as follows:

1. Specify the allowable probability of failure, $P_{fa}$, and initial values of the cross-section areas for members.

2. Calculate the load coefficients ($b_{ij}(\hat{A})$) corresponding to the cross-section areas.

3. Calculate the failure probabilities of the members from Eq. (20).

4. Check the convergence. If there is no convergence, change the cross-section areas and go back to step 2.

To perform Reliability-Based Design Optimization (RBDO) of a roof truss with SQP and EVOL, the programs called miniFE and optimization component work interactively and are available in the laboratory of the Institute for Computational Engineering, Faculty of Civil Engineering, Ruhr University, Bochum, Germany. miniFE contains a set of Java classes, which can be used to perform structural analysis. A method to specify the failure probability of an element was coded in JAVA and added the class, which coordinates the data linking between the miniFE and optimization components. On the other hand, the calculations based on GA are performed on a personal computer. This program was coded in MATLAB and it contains several functions to calculate structural response and to optimize the design problem based on GA methodology (Toğan, 2004; Toğan and Daloğlu, 2004).

#### Example 2

The roof truss structure shown in Figure 3 is considered as an example to demonstrate the reliability concept inclusion in the optimization process. It is

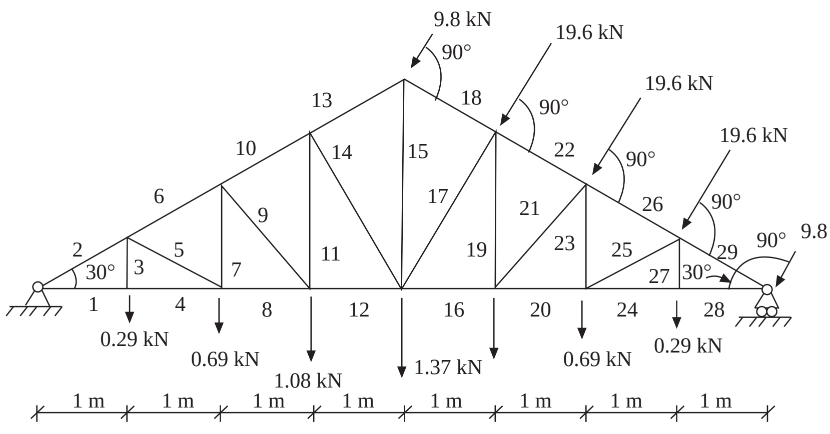



**Figure 3.** 29-member truss structure.

assumed that the allowable stress of the members and the applied loads are statistically independent Gaussian random variables, while the cross-section areas of members and length are deterministic. The The mean value of the loads and the geometrical properties of the truss structure are given in Figure 3. The mean value of the allowable stress and modulus of elasticity are taken as 276 and 68,900 N/mm$^2$, respectively. Failures of the members are assumed to occur under tension or compression.

However, the allowable tension and compression stresses are taken to be the same. The allowable failure probability of the members, ($P_{fa}$), is assumed to be 3.45E-07 in the design process and the values of the coefficients of variation for loads and stress are specified as 0.20 and 0.05, respectively. The model used to assess the system reliability at the state of collapse is based on a series system model. The system reliability can be evaluated as Cornell's upper bound for small values of probability of member failure. All the $n_c$ component failures are assumed to be independent of each other and of equal importance, in that they would try to attain the same target reliability in any optimized structure (Thampan and Krishnamoorthy, 2001). Hence,

$$P_{fa}= P_{fsys}/n_c=1.0\text{E-}05/29=3.45\text{E-}07.$$

The system is optimized by various optimization methods. The results are given in Table 3. It can be

**Table 3.** Comparison between Thoft-Christensen and Murotsu (1986)[1], and this work.

| Member | [1] (mm$^2$) | SQP | EVOL | GA | P$_{fi}$ -SQP | P$_{fi}$ -EVOL | P$_{fi}$ -GA |
|---|---|---|---|---|---|---|---|
| 1 | 56 | 85 | 84.8 | 161 | 3.20E-07 | 3.41E-07 | 2.95E-17 |
| 2 | 298 | 282 | 282.3 | 318 | 3.45E-07 | 3.27E-07 | 7.58E-11 |
| 3 | 2 | 104.9 | 2.2 | 161 | 2.05E-87 | 1.56E-07 | 4.23E-88 |
| 4 | 56 | 87.6 | 84.8 | 161 | 1.51E-07 | 3.40E-07 | 2.95E-17 |
| 5 | 2 | 10.68 | 2.36 | 161 | 1.69E-63 | 3.09E-08 | 4.23E-88 |
| 6 | 297 | 280.7 | 280.8 | 318 | 3.45E-07 | 3.41E-07 | 5.44E-11 |
| 7 | 6 | 6.5 | 5.97 | 161 | 3.18E-09 | 2.78E-07 | 1.01E-85 |
| 8 | 55 | 86.3 | 85.8 | 161 | 2.91E-07 | 3.44E-07 | 1.86E-17 |
| 9 | 5 | 5.62 | 5.3 | 161 | 1.37E-08 | 2.61E-07 | 3.57E-86 |
| 10 | 295 | 278.5 | 278.3 | 279 | 3.27E-07 | 3.42E-07 | 2.44E-07 |
| 11 | 11 | 11.69 | 11 | 161 | 1.28E-08 | 2.90E-07 | 4.04E-82 |
| 12 | 53 | 88 | 87.6 | 161 | 2.94E-07 | 3.41E-07 | 7.52E-18 |
| 13 | 291 | 274.5 | 274.5 | 279 | 3.44E-07 | 3.42E-07 | 1.07E-07 |
| 14 | 9 | 9.5 | 9.5 | 161 | 3.39E-07 | 3.35E-07 | 3.67E-83 |
| 15 | 233 | 221 | 219 | 279 | 1.51E-07 | 2.64E-07 | 6.69E-15 |
| 16 | 161 | 179.9 | 178 | 206 | 2.44E-07 | 3.42E-07 | 1.82E-08 |
| 17 | 256 | 241.8 | 238.5 | 279 | 1.58E-07 | 3.44E-07 | 1.06E-11 |
| 18 | 260 | 250 | 249 | 279 | 3.30E-07 | 3.44E-07 | 2.34E-10 |
| 19 | 161 | 149.9 | 148.6 | 161 | 2.13E-07 | 3.44E-07 | 2.71E-09 |
| 20 | 274 | 282 | 281.8 | 318 | 3.31E-07 | 3.42E-07 | 1.11E-08 |
| 21 | 207 | 191.27 | 190.3 | 206 | 2.63E-07 | 3.43E-07 | 3.00E-09 |
| 22 | 331 | 334 | 334 | 412 | 3.44E-07 | 3.43E-07 | 2.86E-13 |
| 23 | 91 | 84.6 | 82 | 161 | 8.11E-08 | 3.39E-07 | 4.40E-30 |
| 24 | 391 | 389.5 | 388 | 412 | 2.91E-07 | 3.43E-07 | 1.37E-07 |
| 25 | 176 | 160.67 | 158.5 | 161 | 1.81E-07 | 3.38E-07 | 1.37E-07 |
| 26 | 401 | 413 | 413 | 431 | 3.43E-07 | 3.42E-07 | 2.75E-08 |
| 27 | 2 | 8.38 | 2.2 | 161 | 2.10E-55 | 1.01E-07 | 4.23E-88 |
| 28 | 391 | 392 | 388 | 412 | 2.04E-07 | 3.40E-07 | 1.37E-07 |
| 29 | 474 | 491 | 491 | 515 | 3.45E-07 | 3.40E-07 | 2.22E-08 |
| Mass (kN) | 0.1752 | 0.1772 | 0.1743 | 0.2388 | | | |
| $\sum$ P$_{fi}$ (i=1,..29) | 1.00E-05 | | | | 6.26E-06 | 8.88E-06 | 8.48E-07 |

Initial value: A$_i$ = 465 mm$^2$ and density of material, $\rho$ = 2.7E-08 kN/mm$^3$

easily seen from Table 3 that the results given in Thoft-Christensen and Murotsu (1986) and obtained by SQP and EVOL are smaller than the result obtained by GA.

GA takes the design variables as discrete and it is very usual that it obtain more mass than gradient-based methods. The pipe sections listed in AISC are adopted in the GA for cross-sectional areas of members so that the results obtained by GA can be directly used in practice.

The failure probabilities of members and the total failure probability of members are close to their allowable upper limit when SQP and EVOL are used; however, the probability of failure decreases in GA because the minimum cross-section is 161 $mm^2$ and the bigger cross-sections have greater resistance. In this example, each member of the truss structure is taken as a member group. The figure obtained at the end of the EVOL optimization process is shown in Figure 4. It can be said that the construction of this truss is not practical or meaningful. The number of sections in the final set should be as few as possible to make fabrication easy.

Additionally, the upper and lower chords are usually preferred to be designed with the same cross-sections. Therefore, the truss structure shown in Figure 3 is optimized with 2 distinct member group strategies. In the first strategy, the members of the truss are grouped into groups of 15, while in another strategy the member groups are fixed at 9. Corresponding results and configurations are summarized in Table 4, Figure 5, Table 5, and Figure 6, respectively.

The minimum weight of the truss obtained with member groups of 15 is bigger than the truss having 29-member groups. Although the configuration after the optimization process is better than the previous one, it is still not acceptable in practice.

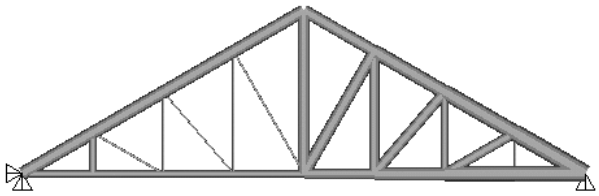


**Figure 4.** Configuration of truss structure after optimizing.

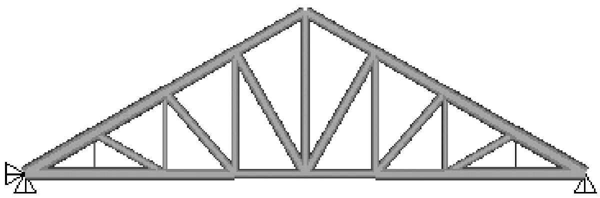


**Figure 5.** Configuration of truss that has 15 member groups after optimizing.

**Table 4.** The optimization results for truss structure that has 15-member groups.

| Member | SQP ($mm^2$) | EVOL | GA | $P_{fi}$-SQP | $P_{fi}$-EVOL | $P_{fi}$-GA |
|---|---|---|---|---|---|---|
| 1, 28 | 388.19 | 388 | 412.2 | 3.45E-07 | 3.45E-07 | 1.37E-07 |
| 2, 29 | 491.31 | 491.3 | 515.5 | 3.42E-07 | 3.45E-07 | 2.21E-08 |
| 3, 27 | 4.77 | 2.25 | 206.5 | 6.79E-32 | 3.19E-07 | 2.09E-88 |
| 4, 24 | 388 | 388 | 412.2 | 3.41E-07 | 3.45E-07 | 1.37E-07 |
| 5, 25 | 158.4 | 158.49 | 161.3 | 3.45E-07 | 3.45E-07 | 1.37E-07 |
| 6, 26 | 413.08 | 413.18 | 431.6 | 3.49E-07 | 3.45E-07 | 2.74E-08 |
| 7, 23 | 82.23 | 82 | 161.3 | 3.42E-07 | 3.45E-07 | 4.39E-30 |
| 8, 20 | 281.99 | 281.79 | 431.6 | 3.38E-07 | 3.45E-07 | 9.19E-18 |
| 9, 21 | 190 | 190 | 206.5 | 3.48E-07 | 3.45E-07 | 3.00E-09 |
| 10, 22 | 334 | 334 | 412.2 | 3.27E-07 | 3.45E-07 | 2.86E-13 |
| 11, 19 | 148.5 | 148.59 | 161.3 | 3.45E-07 | 3.44E-07 | 2.70E-09 |
| 12, 16 | 178.4 | 178 | 206.5 | 3.34E-07 | 3.45E-07 | 1.82E-08 |
| 13, 18 | 274.5 | 274.5 | 279.3 | 3.42E-07 | 3.45E-07 | 2.34E-10 |
| 14, 17 | 238.5 | 238.56 | 297.3 | 3.47E-07 | 3.45E-07 | 1.05E-11 |
| 15 | 218 | 218 | 297.3 | 3.45E-07 | 3.44E-07 | 6.60E-15 |
| Mass (kN) | 0.2453 | 0.2452 | 0.2874 | | | |
| $\sum P_{fi}$ (i=1,..29) | | | | 4.79E-06 | 5.15E-06 | 5.93E-07 |

**Table 5.** The optimization results for truss structure that has 9-member groups.

| ($CV_R$, $CV_L$) | (0.05, 0.20) | | (0.03, 0.40) | | (0.10, 0.40) | |
|---|---|---|---|---|---|---|
| | EVOL | GA | EVOL | GA | EVOL | GA |
| Member | A(mm$^2$) | A(mm$^2$) | A(mm$^2$) | A(mm$^2$) | A(mm$^2$) | A(mm$^2$) |
| 1,4,8,12,16,20,24,28 | 388 | 412.3 | 516 | 568.4 | 648 | 690.3 |
| 2,6,10,13,18,22,26,29 | 491 | 515.5 | 628 | 690.3 | 803.6 | 954.8 |
| 3,27 | 2.2 | 241.8 | 3.2 | 214 | 3.9 | 161.3 |
| 7,23 | 82 | 161.3 | 110.6 | 161.3 | 138 | 161.3 |
| 11,19 | 148 | 161.3 | 188 | 214.8 | 241.8 | 279.3 |
| 15 | 218.1 | 279.3 | 264 | 279.3 | 347.7 | 412.2 |
| 5,25 | 158 | 161.3 | 215.7 | 279.3 | 268 | 279.3 |
| 9,21 | 190 | 206.4 | 243 | 279.3 | 311 | 318.7 |
| 14,17 | 238 | 279.3 | 293 | 318.7 | 382.7 | 412.2 |
| Mass (kN) | 0.2904 | 0.3241 | 0.3745 | 0.4241 | 0.4772 | 0.5424 |
| $\sum P_{fi}$ (i=1, ... ,29) | 3.74E-06 | 1.87E-07 | 3.78E-06 | 4.04E-08 | 3.77E-06 | 6.95E-07 |

Initial value : $A_i = 445$ mm$^2$

The last optimization results are presented in Table 5 and the last configuration shown in Figure 6 is the best in terms of practicality and construction.

Table 5 also shows the effect of the values of the variations coefficient of the random variables on the weight of the truss. The cross-sectional areas increase as the coefficients of variation increase.

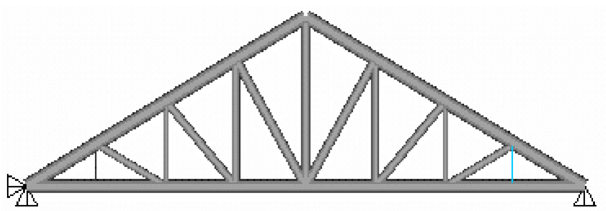


**Figure 6.** Configuration of truss that has 9 member groups after optimizing.

The design optimization problem in which the member groups are fixed at 9 is evaluated only by EVOL and GA. EVOL is not sensitive to any changes in the initial values of cross-section areas that are required to start the optimization process for this problem.

The same truss is optimized, once again, using GA under the continuous design variables to show how the result is affected. Java Genetics Algorithms Package (JGAP), which is free software (freeware), was used for this purpose. JGAP is a GA component written in the form of a Java framework. It provides basic genetic mechanisms that can be easily used to apply evolutionary principles to the solutions of design problems. Since both JGAP and miniFE are based on OOP, it is possible to link them together by writing appropriate software. Hence, the method needed to perform RBDO of the roof truss was coded in JAVA in the Eclipse Platform and added the class, which is responsible for providing the data linking between JGAP and miniFE. The continuous design variables were defined in JGAP as follows:

```
Gene[] sampleGenes = new Gene[9];
for (int i = 0; i < sampleGenes.length; i++) {
    sampleGenes[i] = new DoubleGene(0.02,6.65);
}
```

where sampleGenes represents a chromosome composed of 9 genes, which is the total of the number of the design variables, and DoubleGene(0.02,6.65) specifies the lower and upper limits of design variables (genes). The initiating of the design variables are randomly formed by JGAP. The result obtained by JGAP using continuous design variables under $P_{fa}$ = 3.45E-07 is presented in the second row of Table 6.

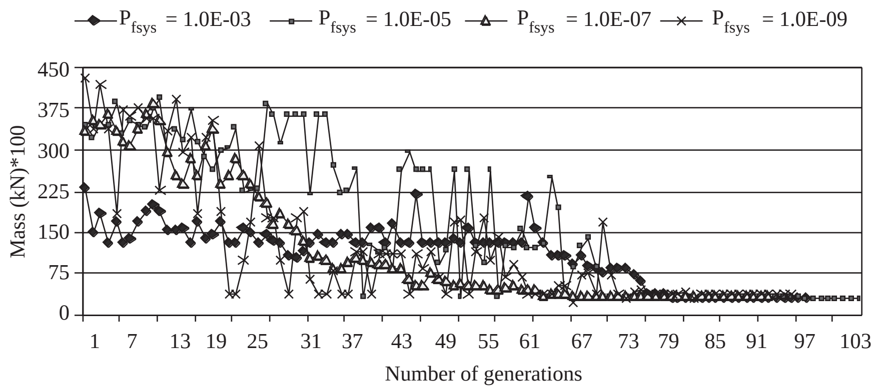
Another investigation performed in the study explored the variation of optimum weight in connection with the imposed level of failure probability for the design problem given. This optimization is implemented for both continuous and discrete design variables in GA. The results are illustrated in Tables 6 and 7, respectively. Studying both the tables, it can be concluded that as the imposed level of failure probability increases, the weight of the roof truss also increases. This is meaningful because the increased level of failure probability causes the optimization method to assign larger cross-section areas than the

**Table 6.** The optimization results with continuous design variables in GA.

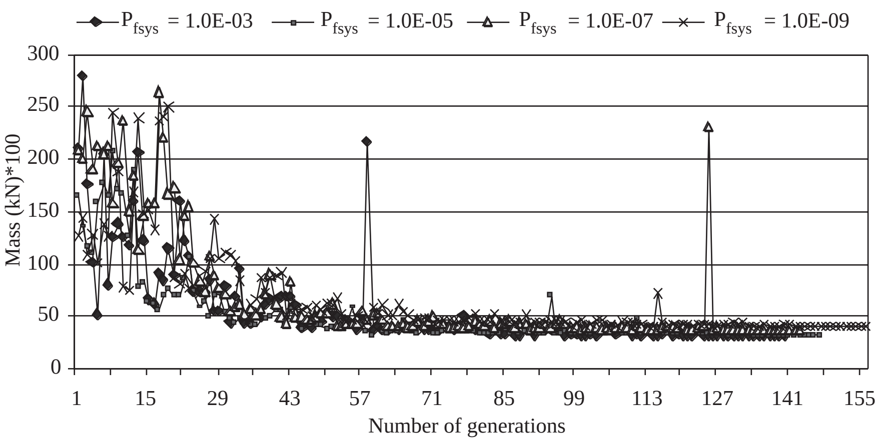
| Allow. failure prob. ($P_{fsys}$) | Mass (kN) | Design variables ($mm^2$) $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ | $A_8$ | $A_9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.00E-03 | 0.2799 | 369 | 465.3 | 9 | 77.5 | 135.6 | 258.9 | 153.9 | 184.5 | 235.2 |
| 1.00E-05 | 0.2936 | 390.1 | 491.3 | 2.4 | 85.3 | 153 | 218.8 | 162.5 | 191.4 | 255.4 |
| 1.00E-07 | 0.3333 | 425 | 541 | 24.5 | 135 | 221.8 | 312.8 | 183.3 | 217.3 | 257.6 |
| 1.00E-09 | 0.3633 | 466.7 | 573 | 28.1 | 188.7 | 200.4 | 384.1 | 198.2 | 266.2 | 277.4 |

**Table 7.** The optimization results with discrete design variables in GA.

| Allow. failure prob. ($P_{fsys}$) | Mass (kN) | Design variables ($mm^2$) $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $A_7$ | $A_8$ | $A_9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.00E-03 | 0.3198 | 412.2 | 412.2 | 161.3 | 161.3 | 206.4 | 206.4 | 161.3 | 161.3 | 206.4 |
| 1.00E-05 | 0.3241 | 412.2 | 515.5 | 241.8 | 161.3 | 161.3 | 279.3 | 161.3 | 206.4 | 279.3 |
| 1.00E-07 | 0.3636 | 515.5 | 568.4 | 161.3 | 206.4 | 161.3 | 279.3 | 206.4 | 206.4 | 279.3 |
| 1.00E-09 | 0.4019 | 515.5 | 690.8 | 161.3 | 161.3 | 206.4 | 279.3 | 214.8 | 279.3 | 279.3 |




a. continuous design variables




b. discrete design variables

**Figure 7.** a, b. Variation of the weight of truss with the imposed level of failure probability.

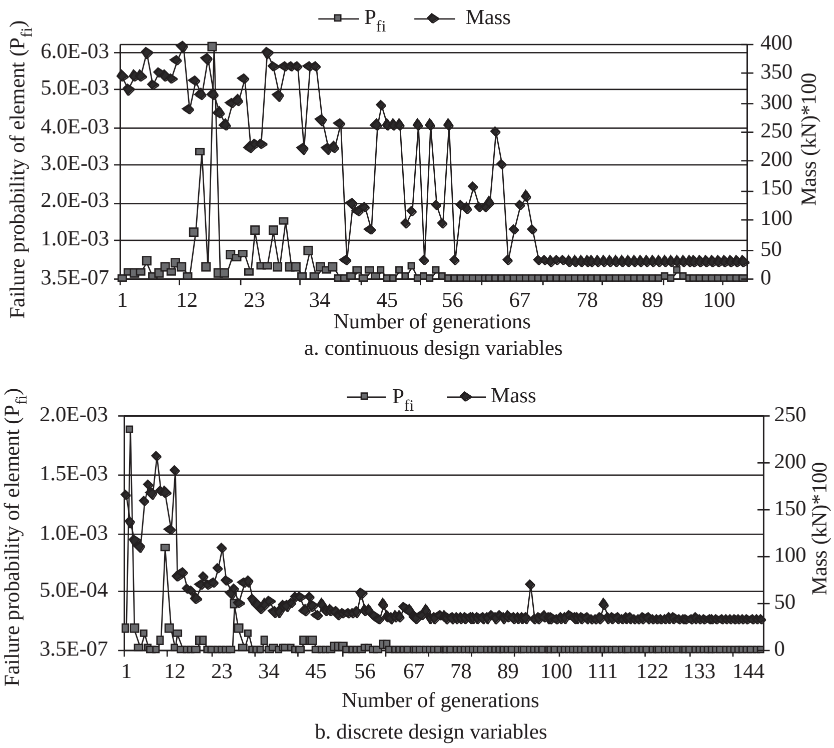



**Figure 8.** a,b. Variation of the weight of truss and failure probability of element with generations.

ones reached under the smaller level of failure probability. Figures 7a and 7b represent graphical illustrations of the results given in Tables 6 and 7.

Additionally, it is very common to observe a reduction of the weight in subsequent iterations of optimization when the value of failure probability of the element, which is taken as a constraint in the design problem, is getting closer to or is under its upper boundary. These variations are presented in Figures 8a and 8b, for both continuous and discrete design variables.

## Observation and Conclusion

Reliability and RBDO are reviewed in this study. The application of these concepts to structural problems is explained through numerical examples and the conclusions reached in the study follow.

Both the algorithms coded for Reliability and RBDO in the study are accurate and efficient. The design obtained by RBDO balances both cost and safety.

If design variables are taken to be discrete, GA finds more mass and the failure probabilities decrease. In the case of continuous design variables in GA, the weight of the roof truss shows similarity between the results obtained by EVOL. Hence, the results obtained by continuous design variables verify the results obtained by discrete design variables, but they are not applicable in practice. Therefore, GA working with discrete design variables submits a one to one practical design in practice

The weight of the structures changes with different member groups strategies and the change tends to increase as the number of member groups used decreases; however, the shape of the truss becomes more applicable from the practical point of view.

The changes in the coefficient of variation of random variables influence the weight of the structure and the changes in the imposed level of failure probability influence the weight. However, this influence is less than the influence of changes in the variation coefficient of random variables.

## Acknowledgment

The authors thank Dr. Baitsch, of the Faculty of Civil Engineering, at Ruhr University Bochum, Germany, for his help, interest, and support.

## Nomenclature

| | |
|---|---|
| $R$ | load carrying capacity |
| $Q$ | load effect |
| g(.) | limit state function |
| $P_f$ | probability of failure |
| $X_1, X_2, \ldots, X_n$ | random variables |
| $x_1, x_2, \ldots, x_n$ | realization of the random variables |
| $y_1, y_2, \ldots, y_n$ | transformed and standardized random variables |
| $f_x(x)$ | probability density function |
| $\beta$ | reliability index |
| $\mu$ | mean value |
| $\sigma$ | standard deviation |
| $x^*$ | design point of random variables |
| $\Phi(.)$ | cumulative distribution function of standard normal distribution |
| $g_L$ | linear function |
| $\nabla g\ (.)$ | gradient of function |
| $H(.)$ | Hessian matrix of function |
| $F_{xi}$ | cumulative distribution function |
| $y^*$ | design point of transformed random variables |
| $h(y)$ | limit state function of transformed random variables |
| $\kappa$ | main curvature of the failure surface |
| $\delta$ | distance measure |
| $I(g(x))$ | indicator function |
| $n$ | total number of simulations |
| $p$ | number of state of failure |
| $\sigma_y$ | yield stress |
| $w$ | elastic section modulus |
| $W$ | weight of structure |
| $l_i$ | length of member $i$ |
| $A_i$ | area of member $i$ |
| $\rho$ | density of material |
| $P_{fi}$ | failure probability of member $i$ |
| $P_{fa}$ | specified allowable probability of failure |
| $m$ | number of members in the structure |
| $R_{ai}$ | allowable stress of member $i$ |
| $b_{ij}(.)$ | load coefficient of member $i$ with respect to $L_j$ |
| $L_j$ | applied load |
| $\ell$ | number of applied load |
| $[k]$ | member stiffness matrix |
| $[K]$ | global stiffness matrix |
| $[T]$ | transformation matrix |
| $P_{fsys}$ | probability of system failure |

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