

On the Effectiveness of Uniform Suction and Injection on Steady Rotating Disk Flow in Porous Medium with Heat Transfer

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Abstract

The steady flow of an incompressible viscous fluid above an infinite rotating porous disk in a porous medium is studied with heat transfer. A uniform injection or suction is applied through the surface of the disk. Numerical solutions of the nonlinear governing equations that govern the hydrodynamics and energy transfer are obtained. The effects of the porosity of the medium and the suction or injection velocity on the velocity and temperature distributions are considered.

Key words: Rotating disk flow, Heat Transfer, Porous medium, Suction or Injection.

Introduction

The pioneering study of fluid flow due to an infinite rotating disk was carried out by von Karman (1921). von Karman gave a formulation of the problem and then introduced his famous transformations, which reduced the governing partial differential equations to ordinary differential equations. Cochran (1934) obtained asymptotic solutions for the steady hydrodynamic problem formulated by von Karman. Benton (1966) improved Cochran's solutions and solved the unsteady problem. The problem of heat transfer from a rotating disk maintained at a constant temperature was first considered by Millsaps and Pohlhausen (1952) for a variety of Prandtl numbers in the steady state. Sparrow and Gregg (1960) studied the steady state heat transfer from a rotating disk maintained at a constant temperature to fluids at any Prandtl number. The influence of an external uniform magnetic field on the flow due to a rotating disk was studied (Attia, 1998, 2002; Attia and Aboul-Hassan, 2001). The effect of uniform suction or injection through a rotating porous disk on the steady hydrodynamic or hydromagnetic flow induced by the disk was investigated (Stuart, 1954; Kuiken,

1971; Ockendon, 1972).

In the present work, the steady laminar flow of a viscous incompressible fluid due to the uniform rotation of a porous disk of infinite extent in a porous medium is studied with heat transfer. A uniform injection or suction is applied through the surface of the disk. In the analysis of the flow in the porous media the differential equation governing the fluid motion is based on Darcy's law, which accounts for the drag exerted by the porous medium (Joseph et al., 1982; Ingham and Pop, 2002; Khaled and Vafai, 2003). The temperature of the disk is maintained at a constant value. The governing nonlinear differential equations are integrated numerically using finite difference approximations. The effect of the porosity of the medium on the steady flow and heat transfer is presented and discussed.

Basic Equations

Let the disk lie in the plane $z = 0$ and the space $z > 0$ is occupied by a viscous incompressible fluid. The motion is due to the rotation of an insulated disk of infinite extent about an axis perpendicular to its plane with constant angular speed ω where

the space above the plane sheet is filled with the porous medium as shown in Figure 1. Otherwise the fluid is at rest under pressure p_∞ . The disk is maintained at a constant temperature T_w . A uniform injection or suction is applied at the surface of the disk for the entire range from large injection velocities to large suction velocities. As was pointed out by Joseph et al. (1982), the self consistent non-linear Navier-Stokes equation that would govern the flow in a surrounding porous medium is given by (Wu et al., 2005)

$$\rho(\vec{u} \cdot \vec{\nabla})\vec{u} + \vec{\nabla}P = \mu \nabla^2 \vec{u} - \frac{\mu}{K} \vec{u} - \frac{c\rho}{\sqrt{K}} |\vec{u}| \vec{u} \quad (1)$$

$$u = 0, v = 0, p = p_\infty$$

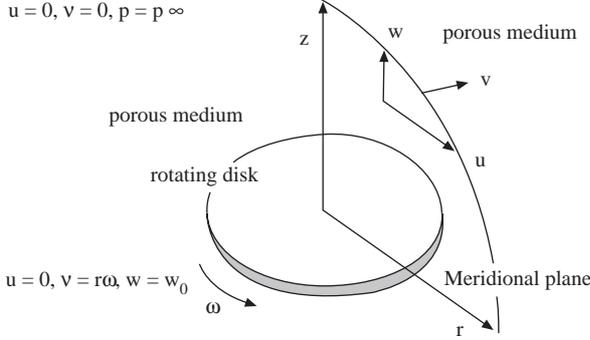


Figure 1. Flow configuration.

where the last 2 terms on the right-hand side of Eq. (1) describe the non-linear Darcy-Forchheimer resistance of the surrounding porous medium. Here μ is the fluid viscosity, K is the Darcy permeability, \vec{u} is the local velocity, ρ is the density of the fluid, and c is the Forchheimer constant, which has been experimentally measured for different porous media. In the present paper, we shall limit our consideration to flows where the non-linear Forchheimer term is neglected but the linear Darcy term describing the distributed body force exerted by the fibers in the porous medium is retained (Wu et al., 2005).

The equations of steady motion are given by

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (2)$$

$$\begin{aligned} \rho(u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r}) + \frac{\partial p}{\partial r} = \\ \mu(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}) - \frac{\mu}{K} u \end{aligned} \quad (3)$$

$$\begin{aligned} \rho(u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{uv}{r}) = \\ \mu(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2}) - \frac{\mu}{K} v \end{aligned} \quad (4)$$

$$\rho(u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z}) + \frac{\partial p}{\partial z} = \mu(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}) - \frac{\mu}{K} w \quad (5)$$

where u , v , and w are velocity components in the directions of increasing r, φ , and z respectively, and p denotes the pressure. We introduce von Karman transformations (von Karman, 1921)

$$\begin{aligned} u = r\omega F, v = r\omega G, w = \sqrt{\omega\nu}H, z = \sqrt{\nu/\omega}\zeta, p - p_\infty \\ = -\rho\nu\omega P \end{aligned}$$

where ζ is a non-dimensional distance measured along the axis of rotation; F , G , H and P are non-dimensional functions of ζ ; and ν is the kinematic viscosity of the fluid, $\nu = \mu/\rho$. With these definitions, Eqs. (2)-(5) take the form

$$\frac{dH}{d\zeta} + 2F = 0 \quad (6)$$

$$\frac{d^2 F}{d\zeta^2} - H \frac{dF}{d\zeta} - F^2 + G^2 - MF = 0 \quad (7)$$

$$\frac{d^2 G}{d\zeta^2} - H \frac{dG}{d\zeta} - 2FG - MG = 0 \quad (8)$$

$$\frac{d^2 H}{d\zeta^2} - H \frac{dH}{d\zeta} + \frac{dP}{d\zeta} - MH = 0 \quad (9)$$

where $M = \nu/K\omega$ is the porosity parameter. The boundary conditions for the velocity problem are given by

$$\zeta = 0, F = 0, G = 1, H = S, \quad (10a)$$

$$\zeta \rightarrow \infty, F \rightarrow 0, G \rightarrow 0, P \rightarrow 0, \quad (10b)$$

where $S = w_o/\sqrt{\omega\nu}$ is the uniform suction or injection parameter, which takes constant negative values

for suction and constant positive values for injection, and w_o is the vertical velocity component at the surface. Equation (10a) indicates the no-slip condition of viscous flow applied at the surface of the disk, but due to the uniform suction or injection the vertical velocity component takes a constant non-zero value at $z = 0$. Far from the surface of the disk, all fluid velocities must vanish aside the induced axial component as indicated in Eq. (10b). The above system of Eqs. (6)-(8) with the prescribed boundary conditions given by Eq. (10) is sufficient to solve for the 3 components of the flow velocity. Equation (9) can be used to solve for the pressure distribution if required.

Due to the difference in temperature between the wall and the ambient fluid, heat transfer takes place. The energy equation, neglecting the dissipation and assuming constant thermal conductivity (Wu et al., 2005), takes the form (Millsaps and Pohlhausen, 1952; Sparrow and Gregg, 1960)

$$\rho c_p \left(u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) - k \left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) = 0 \quad (11)$$

where T is the temperature of the fluid, c_p is the specific heat at constant pressure of the fluid, and k is the thermal conductivity of the fluid. The boundary conditions for the energy problem are that, by continuity considerations, the temperature equals T_w at the surface of the disk. At large distances from the disk, T tends to $T_{\infty 8}$ where $T_{\infty 8}$ is the temperature of the ambient fluid. In terms of the non-dimensional variable $\theta = (T - T_{\infty 8}) / (T_w - T_{\infty 8})$ and using von Karman transformations, Eq. (12) takes the form

$$\frac{1}{\text{Pr}} \frac{d^2 \theta}{d\zeta^2} + H \frac{d\theta}{d\zeta} = 0 \quad (12)$$

where Pr is the Prandtl number, $\text{Pr} = c_p \mu / k$. The boundary conditions in terms of θ are expressed as

$$\theta(0) = 1, \theta(\infty) = 0 \quad (13)$$

The system of non-linear ordinary differential equations (6)-(8) and (12) is solved under the conditions given by Eq. (10) and (13) for the 3 components of the flow velocity and temperature distribution, using the Crank-Nicolson method (Ames, 1977). The resulting system of difference equations has to be solved in the infinite domain $0 < \zeta < \infty$.

A finite domain in the ζ -direction can be used instead with ζ chosen large enough to ensure that the solutions are not affected by imposing the asymptotic conditions at a finite distance. The independence of the results from the length of the finite domain and the grid density was ensured and successfully checked by various trial and error numerical experimentations. Computations are carried out for $\zeta_{\infty} = 12$. It should be mentioned that the results obtained herein reduce to those reported by Attia (2002), when $M = 0$, and the results of the 2 papers are in close agreement, which ensures the validity of the presented solution.

Results and Discussion

Figures 2-4 present the influence of the axial flow at the disk surface on the steady state velocity profiles for the case of suction or injection and for the porosity parameter $M = 0$ and 1. These figures indicate the restraining effect of the porosity of the medium on the flow velocity in the 3 directions. Increasing the porosity parameter M decreases G , F , and H , and the thickness of the boundary layer. Increasing the suction velocity leads to a rapid decrease in the azimuthal and radial velocity components as shown in Figures 2 and 3, while Figure 4 indicates that the axial flow at infinity towards the disk is larger. Increasing the injection velocity leads to an increase in the azimuthal and radial flows as shown in Figures 2 and 3, while Figure 4 shows that the axial flow towards the disk is smaller. With increasing injection velocity, the outflow penetrates to greater distances from the disk surface. Consequently, the crossover point between the positive and negative axial velocity is pushed farther outward in the ζ -direction. In Figure 2, it is seen that the fluid injection gives rise to the familiar inflection-point profiles, especially for high values of the injection parameter S . Hence, high injection velocities are expected to destabilize the laminar flow and lead to transition to turbulence. The influence of the porosity parameter in reducing the axial flow towards the disk is more apparent for the case of injection than suction.

When injection is applied, the porosity effect reduces the azimuthal and radial flows and, consequently, the injection stream sustains its axial motion towards the disk. Figure 4 shows the effect of the porosity in the suppression of the crossover of the axial component of velocity and then reversal of the direction of the axial motion. In Figure 4, it is

clear that the porosity has a marked effect in changing the shape of the inflection-point profiles in the case of high injection velocities. Consequently, the

porosity of the medium works to stabilize the laminar boundary layer and prevents the transition to turbulence.

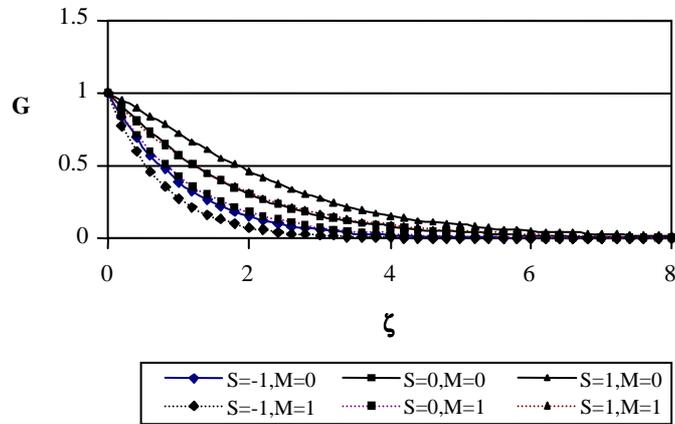


Figure 2. Effect of the porosity parameter M and the suction parameter S on the profile of G .

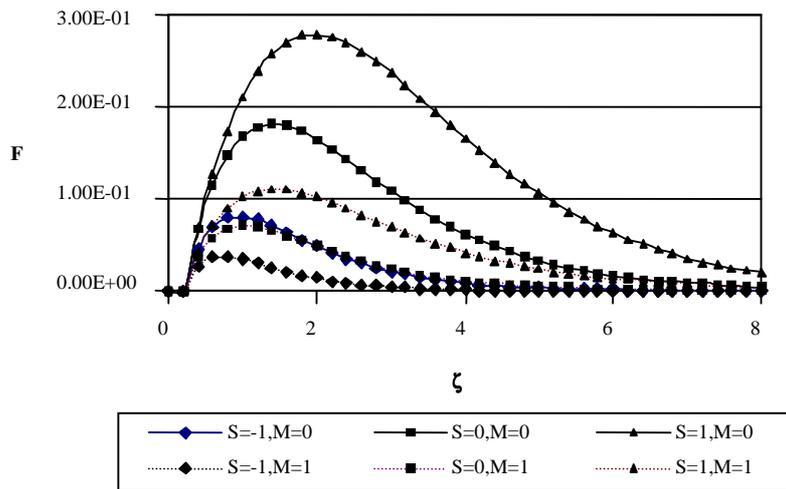


Figure 3. Effect of the porosity parameter M and the suction parameter S on the profile of F .

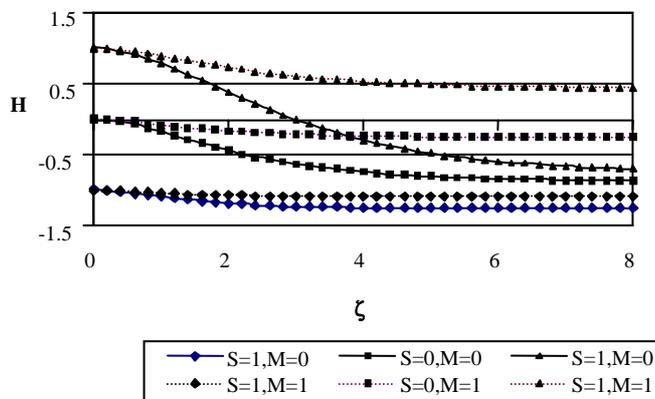


Figure 4. Effect of the porosity parameter M and the suction parameter S on the profile of H .

Figure 5 presents the influence of the axial flow at the disk surface on the steady state temperature profile for the porosity parameter $M = 0$ and 1 and for $Pr = 0.7$. Increasing M increases the temperature θ as a result of the effect of the porosity in preventing the fluid at near-ambient temperature from reaching the surface of the disk. Consequently, increasing M increases the temperature as well as the thermal boundary layer thickness. The influence of the porosity parameter M on θ becomes more apparent for the injection case than for the suction case.

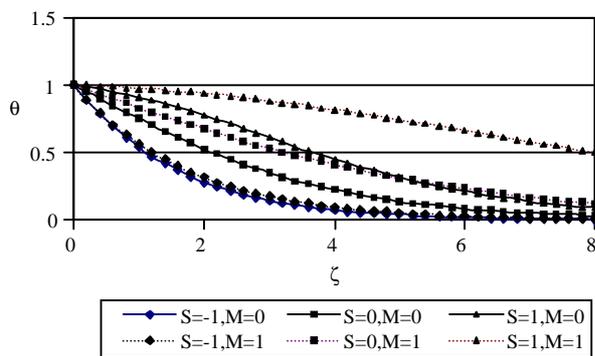


Figure 5. Effect of the porosity parameter M and the suction parameter S on the profile of θ .

The action of fluid injection is to fill the space immediately adjacent to the disk with fluid having nearly the same temperature as that of the disk. As

the injection becomes stronger, the blankets extend to greater distances from the surface. As shown in Figure 5, these effects are manifested by the flattening of the temperature profile adjacent to the disk. Thus, the injected flow forms an effective insulating layer, decreasing the heat transfer from the disk. Suction, on the other hand, serves the function of bringing large quantities of ambient fluid into the immediate neighborhood of the surface of the disk. As a consequence of the increased heat-consuming ability of this augment flow, the temperature drops quickly as we proceed away from the disk.

Conclusion

In this study the steady flow induced by a rotating disk with heat transfer in a porous medium was studied in the presence of uniform suction and injection. The results indicate the restraining effect of the porosity on the flow velocities and the thickness of the boundary layer. On the other hand, increasing the porosity parameter increases the temperature and thickness of the thermal boundary layer. It is of interest to see the effect of the porosity of the medium on the suppression of the crossover of the axial component of velocity in the non-porous case with uniform injection. The porosity of the medium has, in general, a more apparent effect on the flow and temperature fields in the case of uniform injection than in the case of uniform suction.

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