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Modelling of Fracture Toughness in Steel Laminates with a Finite Element Method

Mehmet ŞİMŞİR, Tayfur ÖZTÜRK, Mustafa DORUK
Middle East Technical University, Metallurgical and Materials Engineering Department, Ankara-TURKEY
e-mail: m.simsir@cumhuriyet.edu.tr

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Abstract

The fracture toughness of laminates in which both phases are ductile is modeled with finite element analyses in terms of J_{IC}. The models employed were an exact analogue of compact tension test pieces loaded longitudinally at pinholes, transverse to the crack plane. This study focused on the verification of the model in monolithic samples, the simplest form of which was 2-dimensional (plane strain), and the most elaborate version of which was a 3-D layered structure. Adaptation of the model to laminates showed that it is possible to predict delamination if the individual layers are glued together with a certain interfacial strength. The fracture toughness of laminates, J_{IC}, is predicted with and without delamination with the use of a fracture criterion based on a critical value of load line displacement. This critical value derived from the experiment, and the fracture toughness of steel laminates with layer properties of low-C and medium-C steels are predicted successfully for 2 volume fractions. It is further found that the fracture criterion, which was based on a critical value of LLD, can well be based on a critical value of plastic zone size. This has the advantage that the critical value of plastic zone size and hence the J integral can be predicted with FEM analysis.

Key words: Finite element method, J integral, Laminates, Interfacial strength, Delamination, Load line displacement, Plastic zone size.

Introduction

The modeling of fracture toughness via fine element analysis has been of interest for several decades (Tracey, 1973; Parks, 1974; Barsoum, 1976; Rybicki and Kanninen, 1977; Blackburn and Hellen, 1979; De Lorenzi, 1982; Raju, 1987; Cordes *et al.,* 1995). Most of these studies have focused on the prediction of the fracture toughness of essentially brittle materials, in terms of K_{IC}. For materials that behave in an elasto-plastic manner, the J integral, as proposed by Rice (1968), is the most common method (McMeeking, 1977; Parks, 1977; Sakata *et al.,* 1983; Dodds and Read, 1985; Sivaneri *et al.,* 1991; Fraisse and Schmit, 1993; Freg and Zhang, 1993; Stump and Zywicz, 1993).

The J integral method simply refers to a change in energy stored in the material when the crack advances a unit length. The fracture toughness of the material therefore is a critical value of this energy so that the crack grows in a stable manner, before catastrophic failure.

The prediction of the J line integral for a stationary crack was first carried out by Chan *et al.,* (1970). Since then many studies have been carried out concerning fracture criteria adaptation and application to a finite element method over the years. Various fracture criteria have been proposed to predict the fracture toughness, J_{IC}, based on a critical value of crack growth (Newman, 1985; Shivakumar and Newman, 1989; Kuang and Chen, 1996), crack tip opening displacement (CTOD) (Shih *et al.,* 1979), crack tip opening angle (Shih *et al.,* 1979), strain energy density (De Giorgi *et al.,* 1989), strain energy release rate (Cordes and Yazıcı, 1993), crack mouth opening displacement (CMOD) (Cordes *et al.,* 1995), etc. While in earlier studies carried out by Rice and Johnson (1970), Shih (1974) and Rice (1976) the crack tip remains stationary in some other studies (Shih *et al.,* 1979; Dhar *et al.,* 2000) crack tip blunting was allowed with the use of special elements. Crack growth was modeled quite successfully, especially in later studies (Dodds and Read, 1985; Dhar *et al.,* 2000) with a variety of techniques (Kobayashi, 1973; Andersson, 1974; Light *et al.,* 1975; De Koning, 1977), but the prediction requires the use of experimental data, against which the model is calibrated.

The current work follows an experimental study on steel laminates reported by Şimşir *et al.* (2004), and attempts to predict fracture toughness in the same system. The geometry under study is 2-phase laminates of layers, one with 4 times the strength of the other, and both ductile. The model is an exact analogue of the test piece used in the experimental study, with a fracture being investigated in a crack divider orientation.

Numerical procedures

Finite element modeling (FEM) as implemented in the Marc software was used throughout this study. The method is applied to a compact tension specimen with geometry as in ASTM-E 813. The sample had parameters of $a_o/W = 0.65$ and $W/B = 4$ (where a_o is the original crack length, W is the width of the specimen, and B is the thickness of the specimen). This is the same geometry as that used in the experimental determination of fracture toughness (Şimşir *et al.,* 2004). The model was loaded in terms of displacement at the pinhole along the y direction. Typically, the total displacement was one-hundredth of the crack length a_o, which was imposed typically over 100 increments. In the evaluation, Updated Lagrange and Large Strain Additive approaches (Bleackley and Luxmoore, 1983; Dodds and Read, 1985; Dhar *et al.,* 2000) were used since the material modeled involves severe plastic deformation, especially in front of the crack tip. Convergence is checked by the Full Newton-Raphson Iteration method (Dodds *et al.,* 1988): a relative displacement (i.e. the maximum displacement of the last iteration is small compared to the actual displacement change of the increment) value of 0.01 is assumed to be sufficient. Mesh sizes and density varied according to geometry (see below). Elements at the crack tip had the smallest size, and had a value of 5×10^{-4} the crack length size, a_o. These elements at the crack tip were modified in all cases by the "one-quarter method" (Barsoum, 1976; Raju 1987, 1988).

In order to determine the J integral, a radius, r, of line integration was specified. The value for the radius was varied over a range. The smallest value was equal to the crack tip element size. The largest had a size covering nearly the whole of the uncracked ligament. A total of 9 radii were used. Normally, the J integral increases with an increase in r. Beyond a certain value of r, J is saturated. This value, i.e. far field J, is taken as the J integral value.

The approach in this study was sequential. First 2-dimensional analyses were carried out assuming that the model deforms in a plane strain condition. Subsequently 3-dimensional analyses were carried out, without imposing the condition of plane strain. Finally, a 3-dimensional model were adapted to laminates by the introduction of a layered structure.

In the 2-D analysis, half of the test piece was modeled (Figure 1a). Because of the symmetry, displacement in the y-direction of nodes between C and D at the symmetry line (x-direction) was set to zero. To ensure equilibrium, the node at the very edge of the symmetry line, D, was stationary, i.e. displacements in the x- and y- directions were set to zero. Meshing was done manually, since different mesh sizes were necessary at the crack tip region and elsewhere. An 8-node quadrilateral plain strain element was used[1]. In the model a total of 64 elements and 242 nodes were used.

In the 3-D analysis, the model was an expanded version of the 2-D model by the introduction of the z-direction (Figure 1b). The boundary conditions were the same. Only one fourth of the sample was analyzed since there are symmetry planes, shown by XY and XZ. The XZ plane is placed at the midthickness of the model. Displacements at the XZ plane in the z-direction were zero. The model is meshed with 20 node brick elements[2]. The meshing is carried out automatically. A total of 900 elements with 5200 nodes were used in the analysis.

The 3-D analysis given above refers to monolithic materials. To adopt this for layered materials, the model depicted in Figure 1c was used. In terms of

[1]Element 27 in Marc software

[2]Element 21 in Marc software

boundary conditions, the model is identical to the 3-D one. However, the model is made up of layers of different mechanical properties. The choices of element type and size were the same as those in the 3-D model. Typically, there were 10 elements across the thickness of each layer. Where the layers were in contact, the elements on either side were subdivided into 10 subelements. The model had a total of over 1500 elements with 8000 nodes.

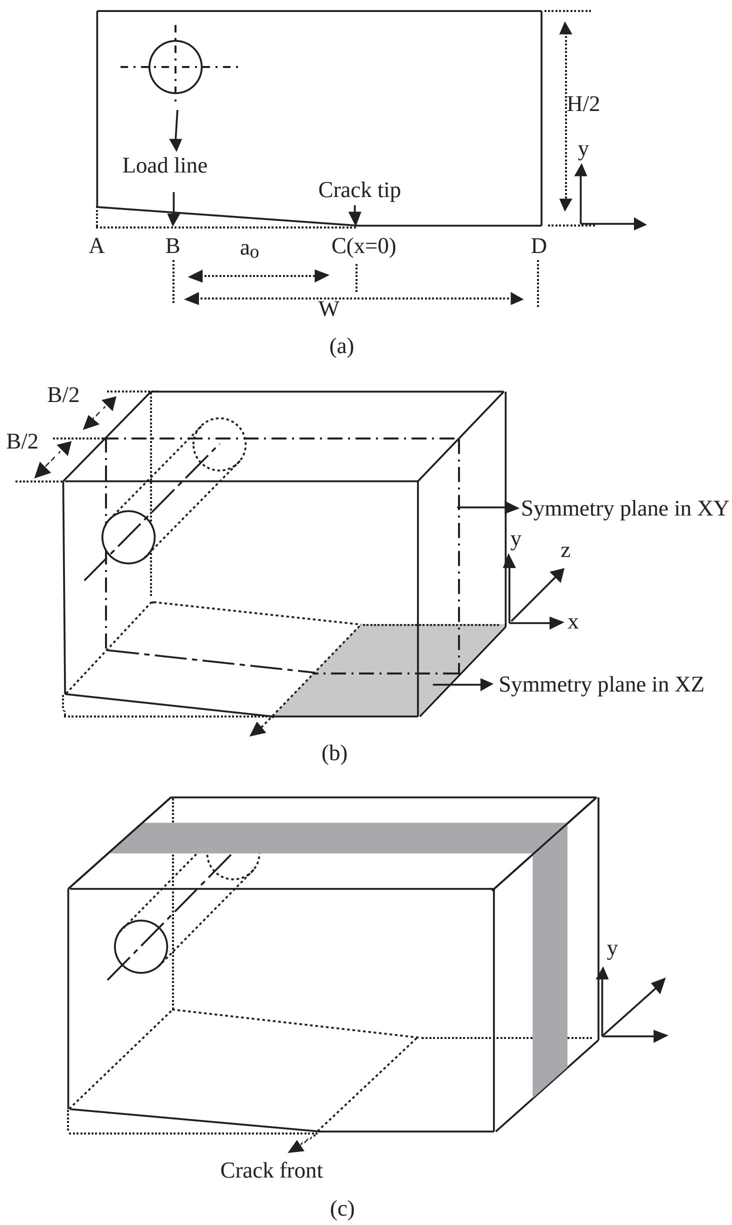


Figure 1. "Compact Tension" models used in the prediction of fracture toughness. B (x = -33.8) refers to position of the load line, C (x = 0) is the crack tip and C-D (x = 0- 18.2) is the uncracked ligament. a_o/W has a value of (BC/BD) = 0.65. a) for 2-D analysis. The model refers to one half of the "test piece". b) for 3-D analysis. The model refers to one-fourth of the test piece. Symmetry planes in xz and xy are shown. c) 3-D layered analysis, same as (b). Shading refers to layers of different properties.

The layers making up the model are attached to each other by "gluing". In this way nodes on either side of the interface are attached and behave as one, except when a specified value of interfacial stress or force is reached. When this stress is reached, the layers are separated and each node behaves independently. A value for "separation distance" was taken as 5% of the smallest element size [3].

Where the J value varied across the thickness of the test piece, i.e. in 3-D analysis and layered 3-D model, an averaging procedure was used. For this purpose, J values were integrated, using the trapezoidal rule, from the surface to the center and divided by half of the thickness.

Fracture criterion

In the analysis, the fracture criterion was evaluated based on an assumption that there is a critical value for load line displacement. This is unlike the widespread approach adopted in many studies, which bases the criterion on a critical value of crack growth. This was necessary since the modeling implemented in this work is enabled to predict the crack growth, i.e. crack length remains the same throughout the loading in the current analysis. However, the critical value of load line displacement was taken as that observed experimentally in the materials concerned (see below), corresponding to a crack extension of 0.2 mm.

Results and Discussion

Calibration and Verification of the Model: In order to verify the method of analysis, first monolithic materials were investigated using 2-D and 3-D models. First the model was calibrated, in order to establish the critical value of load line displacement, with respect to experimental data from the literature. HY 130 steel, which was the subject of an extensive cooperative program in the 80s, was chosen for this purpose (Clarke *et al.,* 1980). The material properties of this steel, taken from Kuang and Chen (1996), are given in Table 1.

J versus Δa data for HY130 steel are available from Clarke *et al.* (1980). The steel has a fracture toughness value of 165 kJ/m^2, which corresponds to a J-integral value at a crack growth value of 0.2 mm. J integral versus load line displacement was determined via 2-D analysis (Figure 2a). The same value of fracture toughness in this curve corresponds to a load line displacement value of 0.82 mm. Thus in moving from 2-D to 3-D and to a layered model, LLD(cr) = 0.82 mm was taken as the critical value.

In the 3-D analysis, J values vary across the thickness. It is seen that J values are lower at the surface (Sakata *et al.,* 1983; Dodds *et al.,* 1988), where the conditions are plane-stress, but increase inside the sample, reaching a plateau at 1/3 the thickness (Figure 3). J values averaged across the thickness versus LLD are given in Figure 2b. Using the LLD(cr) = 0.82 mm, J_{IC} predicted for HY130 steel is 150 kJ/m^2. This value is quite close to the previous value predicted with the 2-D analysis. A slight decrease is due to the variation of J across the sample thickness.

Table 1. Mechanical properties for HY130 steel and medium-C (0.5% C) steel. Data for Hy130 steel are taken from Kuang and Chen (1996). κ and n values describe the true stress-true strain relationship in the form $\sigma = \kappa\varepsilon^n$.

	σ_{ys} MPa	σ_{UTS} MPa	κ MPa	n	% Elongation at fracture
HY130 Steel	975	1030	1140	0.025	-
Medium-C steel (0.5% C)	409	641	1163	0.2144	33.8

[3] Because the separation of layers, i.e. delamination, is an important process in the failure of layered composites, a separate analysis was carried out to establish a valid value for "separation distance". The model used in the analysis was 2-dimensional consisting of 2 parts. Half of the model had material properties of medium-C steel, while the other half had properties of low-C steel. The parts were joined together along a centerline in a continuous manner except for a midpart, which was "glued". Meshed with 8 node plane strain elements, the model was deformed by longitudinal displacements, transverse to centerline at the edges. Interfacial strength for the "glued region" is specified as what was measured experimentally (118 MPa, i.e. interfacial shear strength converted to tensile stress using the Tresca criterion) (Şimşir, 2004). Longitudinal normal stresses were monitored during loading at various values of separation distance. It is found that the delamination is predicted at the expected value of the normal stress when the separation distance was 5% of the (smallest) element size.

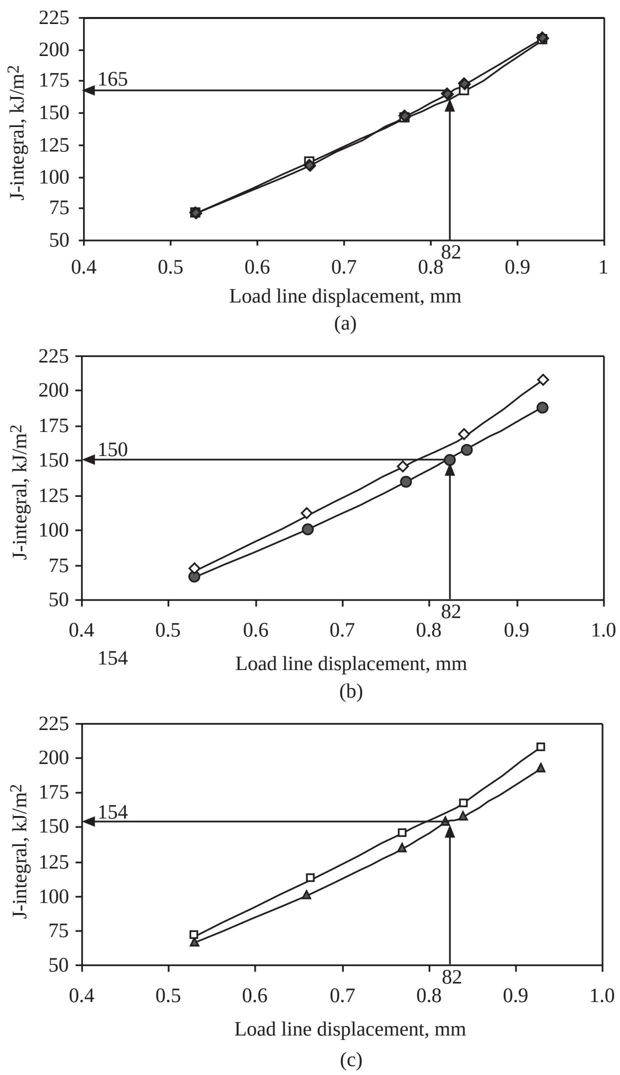


Figure 2. J values versus load line displacement predicted for HY 130 steel. Open symbols show experimental values taken from Clarke *et al.* (1980) and solid symbols are those predicted with, a) 2-D analysis, b) 3-D analysis, and, c) Layered 3-D analysis.

In the 3-D layered model, J values vary across the total thickness as well as across the individual layers (Figure 4). Additionally there is scatter in the J values at the interface. In calculating the average, 2 J values on either side of the interface were omitted. Variation of averaged J values as a function of LLD is given in Figure 2c. Using LLD = 0.82 mm, the model yields a fracture toughness value of J_{IC} = 154 kJ/m^2.

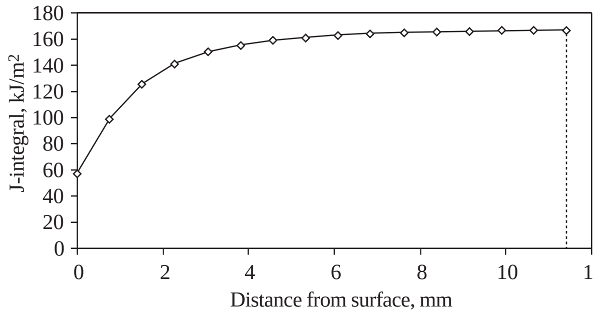


Figure 3. Variation of J integral values (from 3-D analysis) from surface to center for HY 130 steel. Note that J values increase from surface to center.

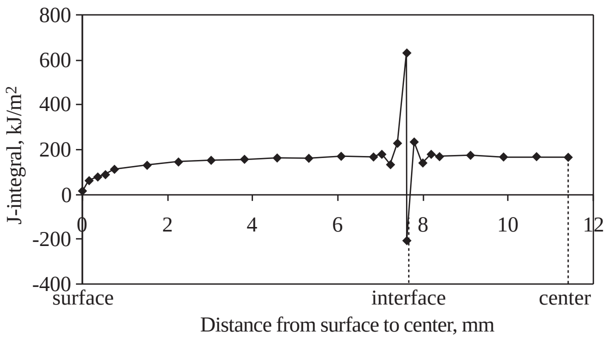


Figure 4. Variation of J integral values (from layered analysis) from surface to center predicted for HY 130 steel. Note that there is a severe fluctuation of J integral values at the interface between the layers.

The values of 165 kJ/m^2, 150 kJ/m^2 and 154 kJ/m^2 predicted for HY130 steel from 2-D, 3-D and 3-D layered models are quite close to each other and thus the 3-D layered model with the details implemented in this work (see above) can now be used for the modeling of laminates.

This point was further checked with a medium–C steel (0.5% C). The properties of this steel are reported in Table 1. A procedure described in Şimşir (2004) was used. The crack growth-load line displacement curve is given in Figure 5. Using Δa(cr) = 0.2 the steel had a fracture toughness value of 136 kJ/m^2. This value of crack growth corresponds to LLD(cr) = 1.09 mm (Figure 5). Using the 3-D layered model, the predicted value of fracture toughness at LLD(cr) = 1.09 mm was 112.5 kJ/m^2. This value was considered quite acceptable.

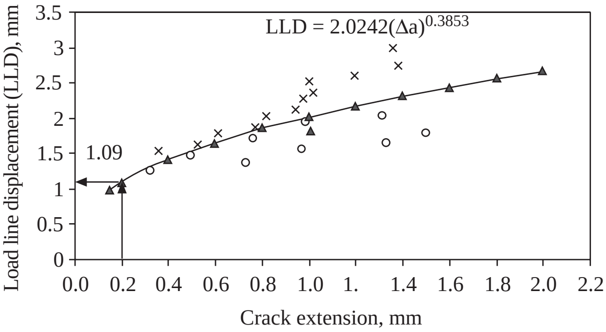


Figure 5. LLD versus crack extension data measured experimentally for medium-C (0.5% C) steel. x and o refer to different samples, ▲ refers to the average values.

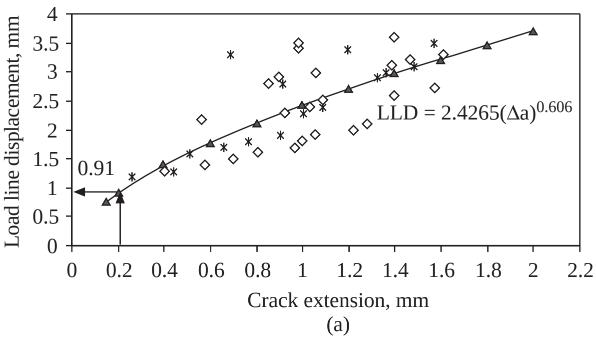
Prediction of Fracture Toughness: Having verified the validity of the model, the fracture toughness of steel laminates is evaluated. The steel laminates in question are an exact analogue of those studied experimentally (Şimşir *et al.,* 2004). Thus they are made up of layers of flow characteristics the same as those measured for medium-C (0.6% C) and low-C (0.1% C) steels (Şimşir *et al.,* 2004). Two laminates were investigated with the stacking sequence in Table 2 with volume fractions of Vr = 0.4 and Vr = 0.8. The layers differ from each other by a factor of 4 in terms of yield strength (Şimşir *et al.,* 2004). Young's modulus is taken as 200 GPa, and Poisson's ratio v = 0.3.

Table 2. Stacking sequence of layers for steel laminates (Şimşir *et al.*, 2004).

Laminate	Stacking Sequence *M: medium-C steel (2.5 mm), L: low-C steel (1.5 mm)*
Vr = 0.4	L L M L M L L
Vr = 0.8	M M L M L M M

In order to determine J_{IC}, the critical value of LLD must first be determined. This value was determined experimentally on laminates from their LLD versus crack extension relationship via a compliance method (Şimşir *et al.,* 2004). The relationship reported in Figure 6a and b yielded values of LLD(cr) = 0.91 mm, and LLD(cr) = 1.04 mm for Vr = 0.4 and Vr = 0.8 laminates, respectively.

Fracture toughness was predicted for 2 conditions. In one, the strength of the interface was extremely high, and so delamination was not allowed in the laminates. In the other, the interfacial strength of the layers was assigned a value of 118 MPa, taken from experimental work (Şimşir, 2004).



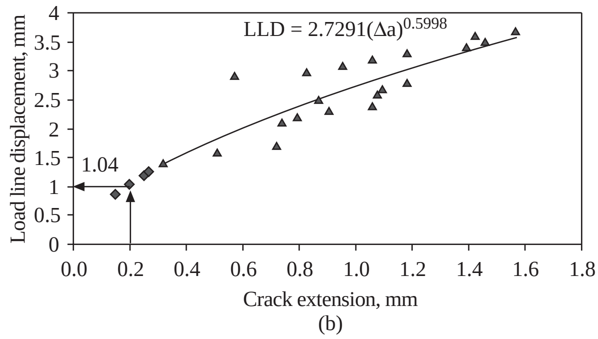


Figure 6. LLD versus crack extension data for steel laminates. a) for Vr = 0.4 × and ◇ refer to different samples, ▲ shows the average values. b) for Vr = 0.8.

J values predicted for the laminates vary across the thickness of the sample. A typical example is given Figure 7a, which refers to Vr = 0.4 at the critical value of LLD(cr) = 0.91mm. J values are lower in the softer layer and increase in the hard layer, with an overall pattern of rising J as it is moved from the surface of the model to its center.

Prediction yields a J_{IC} value of 98.5 kJ/m^2 for a steel laminate with Vr = 0.4. The value derived for Vr = 0.8 is 156.6 kJ/m^2. These values are derived for the condition where no interface separation was permitted.

Using the value of 118 MPa as interfacial strength, the model allows delamination. Variation of the J integral across the model thickness is given in Figure 7b. J values of the model without delamination are smaller than those of the model with delamination. Figure 8 shows the delamination pattern in the model for Vr = 0.4 after LLD = 0.91 mm. At the critical values of LLD both samples Vr = 0.4 and Vr = 0.8 showed delamination. It is found that delamination occurs within a volume in front of the crack tip with d/B = 0.8 and d/B = 0.98 (d: delamination length, B: thickness of the sample) for Vr = 0.4 and Vr = 0.8, respectively. The delamination in the laminate with Vr = 0.4 is less than that of the laminate with Vr = 0.8.

As reported in Şimşir *et al.* (2004), delamination was also observed in the experiments. The values were d/B = 0.60 for Vr = 0.4 and d/B = 0.34 for Vr = 0.8. These values are smaller than those predicted by the current analysis.

With delamination, the model predicts a J_{IC} value of 100 kJ/m^2 for Vr = 0.4 and 165 kJ/m^2 for Vr = 0.8. These values should be compared with earlier values of 98 kJ/m^2 and 156 kJ/m^2, which were obtained without delamination. Thus as expected when delamination is allowed, the prediction leads to higher values for fracture toughness.

J_{IC} predicted in the current analysis with values of 100 and 165 kJ/m^2 should be compared with experimental values measured for the laminates. Experimentally measured values were 97 and 148 kJ/m^2. The values are quite close to one another. More importantly, it is seen that the variation of fracture toughness with volume fraction follows the same trend as the experiment.

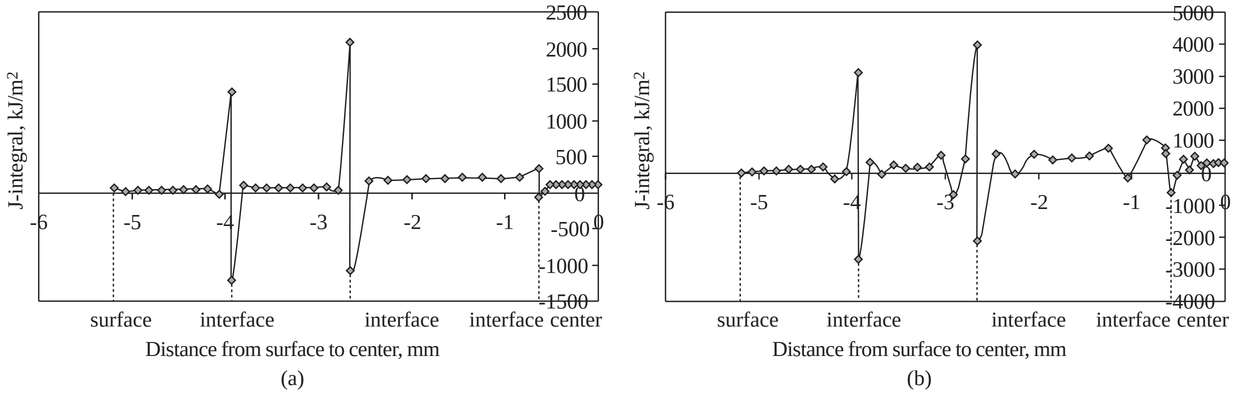


Figure 7. Variation of J integral values from surface to center for Vr = 0.4 laminate. M refers to medium-C and L refers to low-C layer. a) without delamination, b) with delamination,

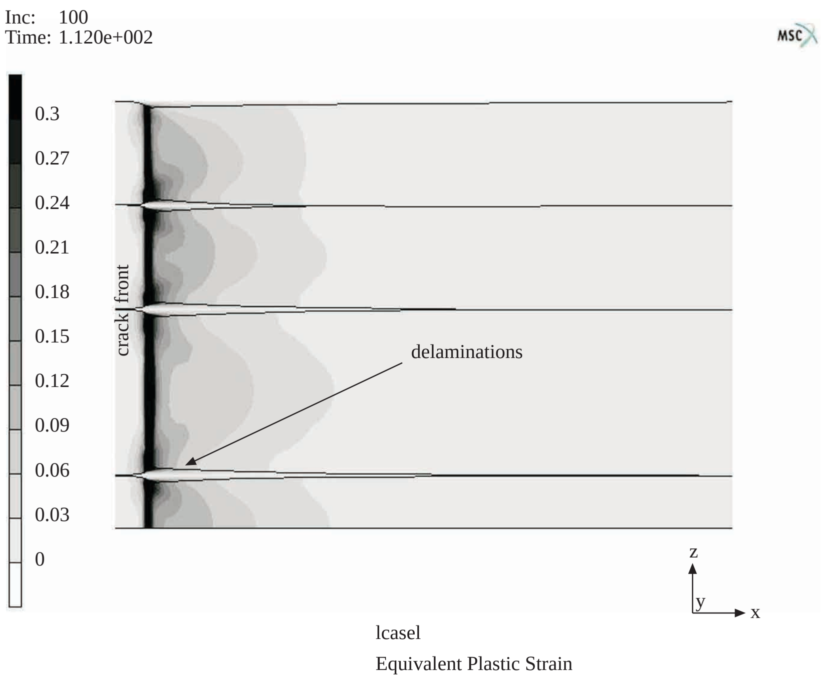


Figure 8. Delamination in laminate with Vr = 0.4 at a section in the crack plane. Values indicated refer to equivalent plastic strain; only values up to $\varepsilon_{eqv} = 0.3$ are shown.

The results show that the fracture toughness of laminates can be predicted successfully with finite element analysis without the need for crack growth modeling. However, a difficulty in the current approach is that the critical value of LLD should be established experimentally, i.e. the need for an experiment cannot be eliminated.

An attempt has been made to determine whether it would be possible to predict fracture toughness based on a criterion other than LLD in a manner such that the experiment would not be required. Figure 9a and b shows the variation of plastic zone size as a function of LLD in 2-D analysis for HY 130 steel and medium-C (0.5% C) steel, respectively. Here plastic zone size r* is the minimum value of r, beyond which the J integral is saturated. In order to improve the accuracy of evaluation, r* refers to 0.95 of the J integral at a given value of LLD.

As seen in Figure 9, in LLD versus r* there is initially a rapid increase, but after a certain value of LLD r* saturates. At saturation conditions r* has a value of 0.30, which corresponds to an LLD of 1.12 mm for 0.5% C steel. This value should be compared with the actual critical LLD value of 1.09 derived from experimental work. Thus a prediction based on experimentally measured LLD(cr) can well be based on r* with the advantage that the latter can be determined by FEM analysis without the need for the experiment. This is also verified with HY130; values in this case were r* = 0.66, which corresponded to an LLD of 0.74 mm (the actual value was 0.82 mm).

Results of similar analysis for the laminates are given in Figure 9c and d. The critical plastic zone size values predicted are r* = 0.23 and r* = 0.43 mm. J_{IC} values corresponding to these zone sizes have values of 97 and 162 kJ/m^2 for the laminate with Vr = 0.4 and Vr = 0.8, respectively. These values are quite close to those (97 and 148 kJ/m^2) derived from experimentally measured LLDcr's.

With the introduction of a critical plastic size as the fracture criterion, the fracture toughness of laminates can be predicted successfully by finite element analysis without the need for the experiment. This is particularly useful for comparative evaluations of fracture toughness for structural optimization purposes, i.e. size and volume fraction of reinforcement. These parameters can be modified at ease together with interfacial strength so as to establish conditions that would maximize the laminate toughness.

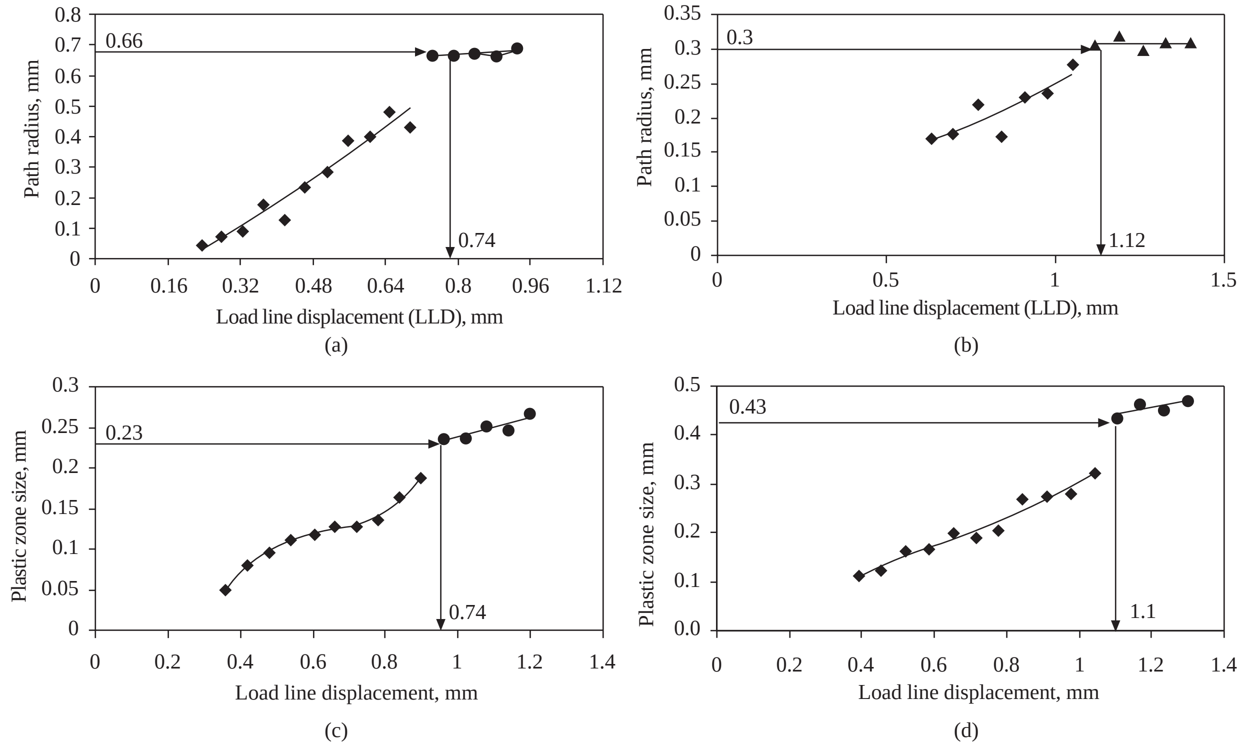


Figure 9. Variation of plastic zone size with LLD, a) for HY 130 steel, b) for medium-C (0.5% C) steel, c) for steel laminate with Vr = 0.4, d) for steel laminate with Vr = 0.8.

Conclusion

The present study aimed at modeling the fracture toughness, in terms of J_{IC}, of laminates made up of 2 ductile phases. The models employed were an exact analogue of compact tension test pieces loaded longitudinally at pinholes. Unlike experiments, crack tip in the models remained stationary. The study has focused on verification of the model in monolithic samples, the simplest form of which was 2-D (plane strain), and most elaborate of which was a 3-D layered structure.

Adaptation of the model to laminates showed that it is possible to predict delamination if the individual layers are glued together with a certain interfacial strength. The fracture toughness of laminates J_{IC} is predicted with and without delamination with the use of a fracture criterion based on a critical value of load line displacement. This critical value derived from the experiment, and the fracture toughness of steel laminates with layer properties of low-C and medium-C steels are predicted successfully for 2 volume fractions.

It is further found that a fracture criterion based on a critical value of LLD can well be based on a critical value of plastic zone size. This has the advantage that the critical value of plastic zone size can be predicted by FEM analysis.

It is concluded that this method can be used successfully for the comparative evaluation of fracture toughness for the purpose of structural optimization, i.e. so as to identify the size and volume fraction of reinforcement as well as the value of interfacial strength between the layers that would maximize fracture toughness.

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