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Soil-Structure Interaction for a Wedge-Shaped Structure Supported by a Flexible Embedded Foundation for Incident SH Waves

Abdul HAYIR
İstanbul Technical University, Faculty of Civil Engineering,
İstanbul-TURKEY
e-mail: ahayir@itu.edu.tr

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Abstract

A simple model of a wedge-shaped structure supported by a flexible foundation, embedded in an elastic half-space and excited by incident plane SH waves, is considered. For this model, a closed-form solution can be derived using the wave expansion method. The wave functions in the structure are such that they satisfy the zero-stress condition on its "free" surfaces automatically. The same holds for the waves in the flexible foundation and for the scattered waves in the soil. The coefficients of the expansion are determined by imposing the continuity of displacements and stress conditions at the structure-foundation and foundation-soil interfaces. This requires transformation of wave fields from one cylindrical coordinate system to another, accomplished with the help of Graf's Addition Theorem, as well as transformation of wave fields with given periodicity in the angular coordinate (such as satisfaction boundary conditions) into wave fields with different periodicity (so that the continuity conditions are applied) by Fourier series expansion. Numerical results are evaluated for different material properties and for different incident angles.

Key words: SH waves, Dynamic soil-structure interaction, Wave passage effects, Dynamic interaction, Flexible foundation.

Introduction

A common assumption in modeling soil-structure interactions is that the foundation is rigid. This reduces the number of degrees-of-freedom, additional to those of the fixed-base model, and simplifies the computations. One consequence of this simplification is overestimation of the energy scattered by the foundation, i.e. overestimation of the radiation damping.

Soil-structure interaction during strong motion is an active area of research, particularly in relation to seismic effects on underground and embedded structures. Extensive literature reviews on the subjects of solution techniques for ground motion amplification and vibrations of structures due to incident and scattered waves are available [see Diravinski (1983)and Sanchez-Sesma (1987), for instance where attention is directed to the wave functions expansion analytic method for solving wave equations and the numerical collocation techniques for satisfying boundary conditions]. Early research began with the scattering of SH waves by inclusions of circular geometry and obtained analytical solutions involving cylindrical wave functions. Luco (1969) and Trifunac (1972) studied the interaction of a shear wall mounted on an embedded cylindrical foundation. These models were subsequently extended to include circular tunnels by Lee and Trifunac (1982). In the SH waves model, only anti-plane translations of structures and inclusions are considered. Sanchez-Sesma *et al.* (1982) using a boundary method for elastic wave diffraction, obtained a solution for irregular surfaces in the SH case.

More work involving wave functions expansions and collocation techniques includes studies of 3-dimensional SH wave diffraction by a hemispherical valley [see Sanchez-Sesma *et al.* (1989)], surface motion of a semi-cylindrical alluvial valley for incident SH waves (Trifunac, 1971) and a closed-form solution of 2-dimensional scattering of plane SH waves by a cylindrical hill of a circular-arc cross section in a half space (Yuan and Liao,1996).

In this paper, a simple model of a wedge-shaped structure supported by a flexible foundation, embedded in an elastic half-space and excited by incident plane SH waves, is considered and numerical results are obtained. The numerical results include not only the wedge-shape structure, but also the foundation and the half-space, whereas the previous study on the same model (Todoroska *et al.*, 2001) concentrated on the structure. Hayir *et al.* (2001) solved the same wedge resting on the flexible interface for SH waves.

To arrange the system of equations, it is necessary to obtain displacement fields in the same coordinate system and periodicity. Using Graf's Addition Theorem and the Fourier transformation, the displacement field of soil is transformed from (R,θ) to (R_1, θ_1) and displacement of the structure is reduced to the same periodicity, respectively.

Model and Problem Formulation

The model consists of a structure sitting on a flexible foundation imbedded into a half-space, and excited by a plane horizontally polarized monochromatic shear wave with frequency ω, and angle of incident γ as shown in Figure 1. The structure, the flexible foundation and the half-space are homogeneous and isotropic with shear wave velocities, β_b, β_f and β_s respectively. A linear stress-strain relationship is assumed for both structure and flexible foundation, and for the flexible foundation and the half-space. Two rectangular coordinate systems (x, z) and (x_1, z_1), and 2 corresponding polar coordinate systems (R,θ) and (R_1, θ_1) are employed in this study.

The displacement of incident, $U^{(i)}$, and reflected, $U^{(r)}$, waves can be expressed in the (R_1, θ_1) polar coordinate as

$$U^{(i)} = U_0\, e^{i\,k_s\,[\sin\gamma\, x_1 - \cos\gamma (z_1 - d)] - i\,\omega\, t} \tag{1}$$

and

$$U^{(r)} = U_0\, e^{i\,k_s\,[\sin\gamma\, x_1 + \cos\gamma (z_1 - d)] - i\,\omega\, t} \tag{2}$$

where U_0 is the amplitude of incident and reflected waves and will be taken as 1 in this study, $k_s = \omega/\beta_s$ is the wave number in the half-space, d is height of the wedge, γ is the incident wave angle, and i and r represent incident and reflected waves, respectively.

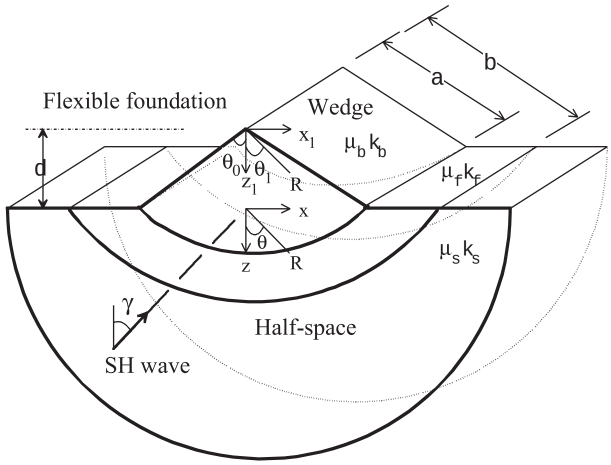

Figure 1. Model of the problem.

For convenience while applying the continuity conditions at $R_1 = b$, Eqs (1) and (2) can be expanded in Bessel-Fourier series in the (R_1, θ_1) polar coordinate system as

$$U^{(i)} = e^{i\,k_s\,d\,\cos\gamma} \sum_{m=0}^{\infty} \varepsilon_m\, (-i)^m J_m\,(k_s R_1)\, \cos m \\ (\theta_1 + \gamma)\; e^{-i\,\omega\,t} \tag{3a}$$

$$U^{(r)} = e^{-i\,k_s\,d\,\cos\gamma} \sum_{m=0}^{\infty} \varepsilon_m\, (i)^m J_m\,(k_s R_1)\, \cos m \\ (\theta_1 - \gamma)\; e^{-i\,\omega\,t} \tag{3b}$$

Free-field motion can be written as follows [see Hayir *et al.* (2001)]

$$U^{ff} = U^{(i)} + U^{(r)} \tag{4}$$

The displacement in the structure and half-space has to satisfy the 2- dimensional wave equation in which the (R_1, θ_1) coordinate system has the form:

$$\frac{\partial^2 U}{\partial R_1^2} + \frac{1}{R_1}\frac{\partial U}{\partial R_1} + \frac{1}{R_1^2}\frac{\partial^2 U}{\partial \theta_1^2} = \frac{1}{\beta^2}\frac{\partial^2 U}{\partial t^2} \tag{5}$$

where β is wave velocity and t is the time variable. The zero stress boundary conditions are

$$\tau^{(s)}_{\theta y} = 0 \qquad R_1 \geq b \qquad \text{at } \theta = \pm\frac{\pi}{2} \tag{6a}$$

$$\tau^{(b)}_{\theta_1 y} = 0 \qquad \text{at } \theta_1 = \pm\theta_0 \qquad 0 \leq R_1 \leq a \tag{6b}$$

$$\tau^{(f)}_{\theta y} = 0 \qquad b \geq R_1 \geq a \qquad \text{at } \theta = \pm\frac{\pi}{2} \tag{6c}$$

where $\tau^{(s)}_{\theta y}$,$\tau^{(f)}_{\theta y}$ and $\tau^{(b)}_{\theta_1 y}$ represent angular shear stresses in the y direction for half-space, foundation and wedge, respectively, and (R,θ) is the polar coordinate for half-space and foundation, θ_0 is half of the top angle of the wedge, (R_1, θ_1) is the angular coordinate for the wedge, a is the radius of the wedge and b is the radius of the foundation.

The general solution of scattering displacement satisfying Eq. (5) in the half-space and boundary condition (6a) is written

$$\begin{aligned} U^{(R)}(R,\theta) = \sum_{n=0}^{\infty} \Big[& A_n^R H_{2n}^{(1)}(k_s R)\cos(2n\theta) \\ & + B_n^R H_{2n+1}^{(1)}(k_s R)\sin(2n+1)\theta \Big] e^{-i\omega t} \end{aligned} \tag{7}$$

where A_n^R and B_n^R are constants to be determined and $H_n^{(1)}(.)$ are the Hankel functions of the first kind with order n.

The displacement in the structure,$U^{(b)}$, satisfying Eq (6b), can be expanded in a series of its eigen functions as follows:

$$\begin{aligned} U^{(b)} = \sum_{n=0}^{\infty} \Big[& C_n J_{\frac{2n\pi}{2\theta_0}}(k_b R_1)\cos\left(\tfrac{2n\pi}{2\theta_0}\theta_1\right) \\ & + D_n J_{\frac{(2n+1)\pi}{2\theta_0}}(k_b R_1)\sin\left(\tfrac{(2n+1)\pi}{2\theta_0}\theta_1\right) \Big] e^{-i\omega t} \end{aligned} \tag{8}$$

where C_n and D_n are constants to be determined and $J_n(.)$ is the Bessel function of the first kind with order n.

The solution for the flexible foundation, $U^{(f)}$, which satisfies Eq. (5) and boundary condition (6c) can be written as

$$\begin{aligned} U^{(f)} = \sum_{n=0}^{\infty} & H_{2n}^{(1)}(k_f R) E_n \cos(2n\theta) \\ & + H_{2n+1}^{(1)}(k_f R)\, G_n \sin(2n+1)\theta \\ & + H_{2n}^{(2)}(k_f R)\, e_n \cos(2n\theta) \\ & + H_{2n+1}^{(2)}(k_f R)\, g_n \sin(2n+1)\theta \end{aligned} \tag{9}$$

where E_n, e_n , G_n and g_n are constants to be determined, and $H_n^{(1)}(.)$ and $H_n^2(.)$ are the Hankel functions of the first and second kind with order n respectively. The unknown coefficients in Eqs (7), (8) and (9) can be determined by applying the appropriate continuity conditions in Hayir *et al.* (2001) and Todorovska *et al.* (2001).

Numerical Examples

The results are presented in terms of dimensionless parameters. As a dimensionless frequency

$$\eta = \frac{2a}{\beta T} = \frac{\omega a}{\pi \beta} \tag{10}$$

is used, which is the ratio between the width of the wedge and the wavelength of shear waves in the half-space. For dimensionless calculation, a_1 is taken as $a_1 = a\cos\theta_0$.

Figure 2a for $\eta = 0.5$ and Figure 3a for $\eta = 1$ demonstrate displacements in the radial direction associated with the depth of the flexible foundation for $\mu_f/\mu_b = 6$, $\mu_f/\mu_s = 6$, $k_f/k_b = 1/2$, $k_f/k_s = 1/2$, $N = 6$, $\theta_1 = 0^0$, and $\varsigma = b/a = 1.1$, 2 and 3. Figure 2b and Figure 3b represent dimensionless displacements at $\theta_1 = \theta_0$ for the same parameters as in Figure 2a and Figure 3a. The results show that as the depth of the foundation becomes greater, the displacements of the structure become smaller.

Figure 4a and Figure 5a show dimensionless displacements associated with the depth of the flexible foundation for $\mu_f/\mu_s = 1/6$, $k_f/k_b = 2$, $k_f/k_s = 2$, $N = 6$, $\theta_1 = 0^0$, and $\varsigma = b/a = 1.1$, 2 and 3. Figure 4b and Figure 5b illustrate dimensionless displacements at $\theta_1 = \theta_0$ for the same parameters as in Figure 4a and Figure 5a The results show that displacements are much larger than in the case of harder material. Figures 5 a and b show free surface displacements according to different incident angles in the case of harder flexible foundations. It can be seen from the graphics that the displacements are small.

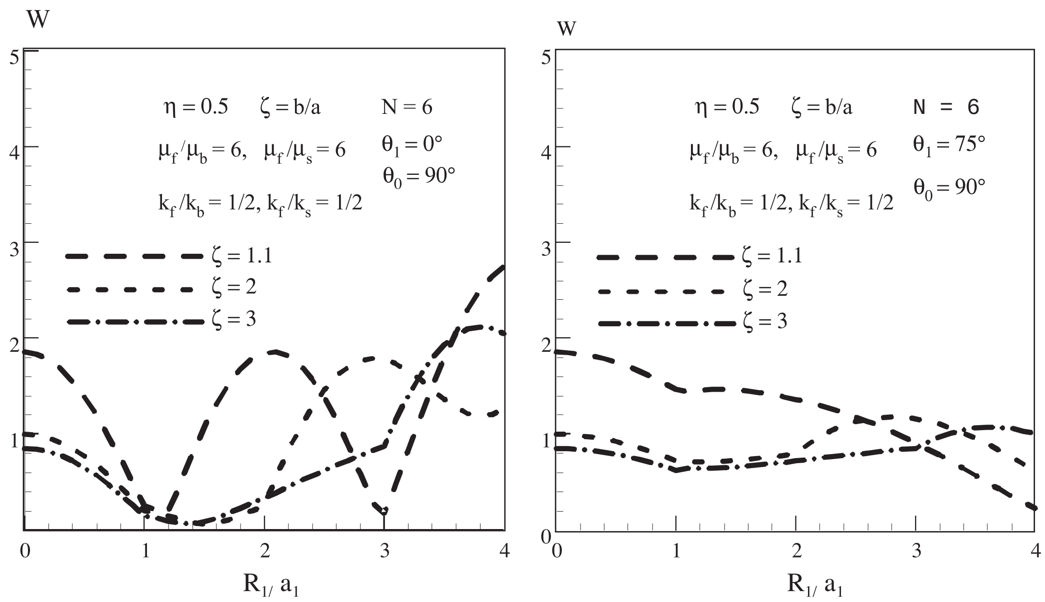

Figure 2. Dimensionless displacements versus depth of the flexible foundation, R_1/a_1, for $\eta = 0.5$, $\mu_f/\mu_b = 6$, $\mu_f/\mu_s = 6$, $k_f/k_b = 1/2$, $k_f/k_s = 1/2$, $N = 6$, and $\varsigma = b/a = 1.1,\ 2$ and 3.a) $\theta_1 = 0$, b) $\theta_1 = 90$.

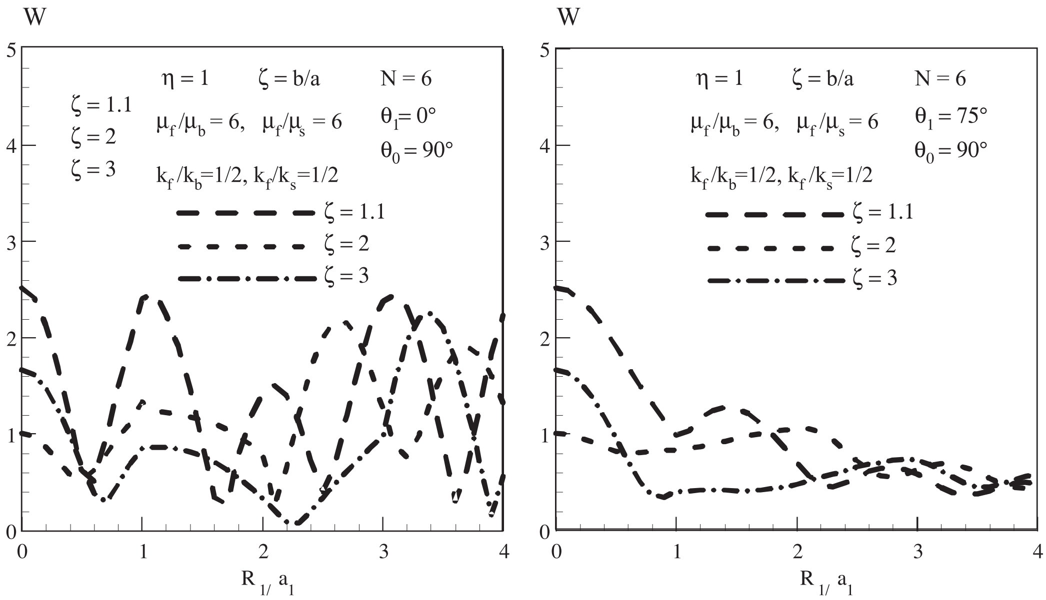

Figure 3. Dimensionless displacements versus depth of the flexible foundation, R_1/a_1, for $\eta = 1$, $\mu_f/\mu_b = 6$, $\mu_f/\mu_s = 6$, $k_f/k_b = 1/2$, $k_f/k_s = 1/2$, $N = 6$, and $\varsigma = b/a = 1.1,\ 2$ and 3.a) $\theta_1 = 0$, b) $\theta_1 = 90$.

Figure 6a for $\eta = 0.5$ and Figure 6b for $\eta = 1$ show free surface displacements according to different incident angles in the case of $\mu_f/\mu_b = 6$, $\mu_f/\mu_s = 6$, $k_f/k_b = 1/2$, and $k_f/k_s = 1/2$. It can be seen from the graphics that the displacements are much smaller than in the case of the soft flexible foundation.

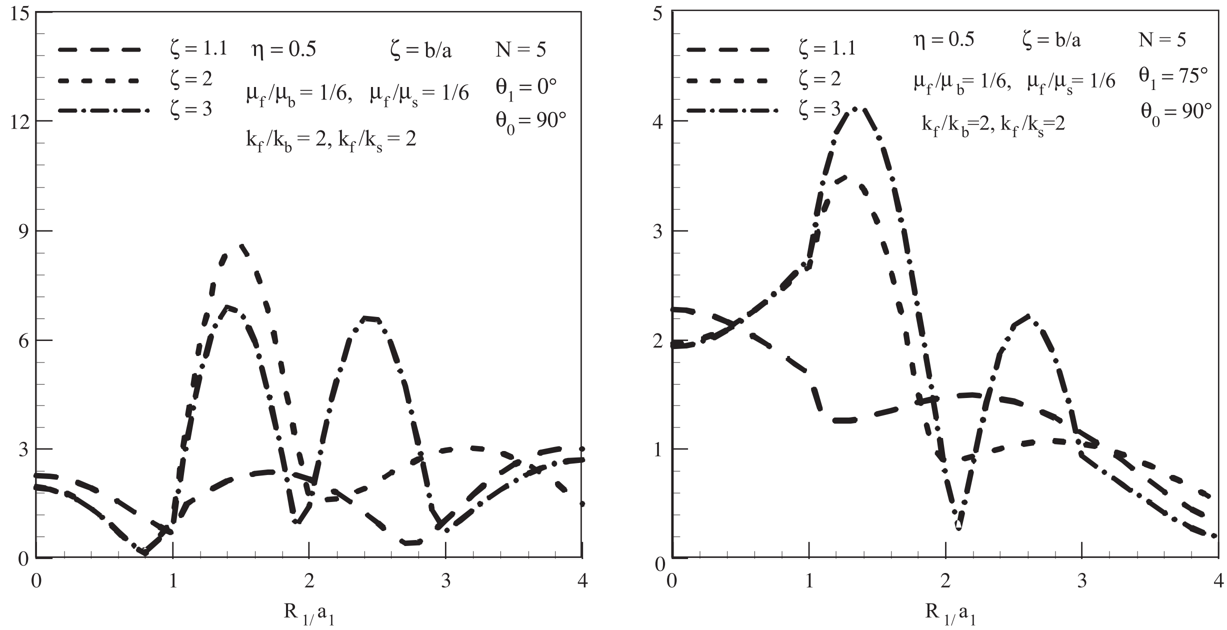

Figure 4. Dimensionless displacements versus depth of the flexible foundation, R_1/a_1, for $\eta = 0.5$, $\mu_f/\mu_b = 1/6$, $\mu_f/\mu_s = 1/6$, $k_f/k_b = 2$, $k_f/k_s = 2$, $N = 5$, and $\varsigma = b/a = 1.1,\ 2$ and 3.a) $\theta_1 = 0$, b) $\theta_1 = 90$.

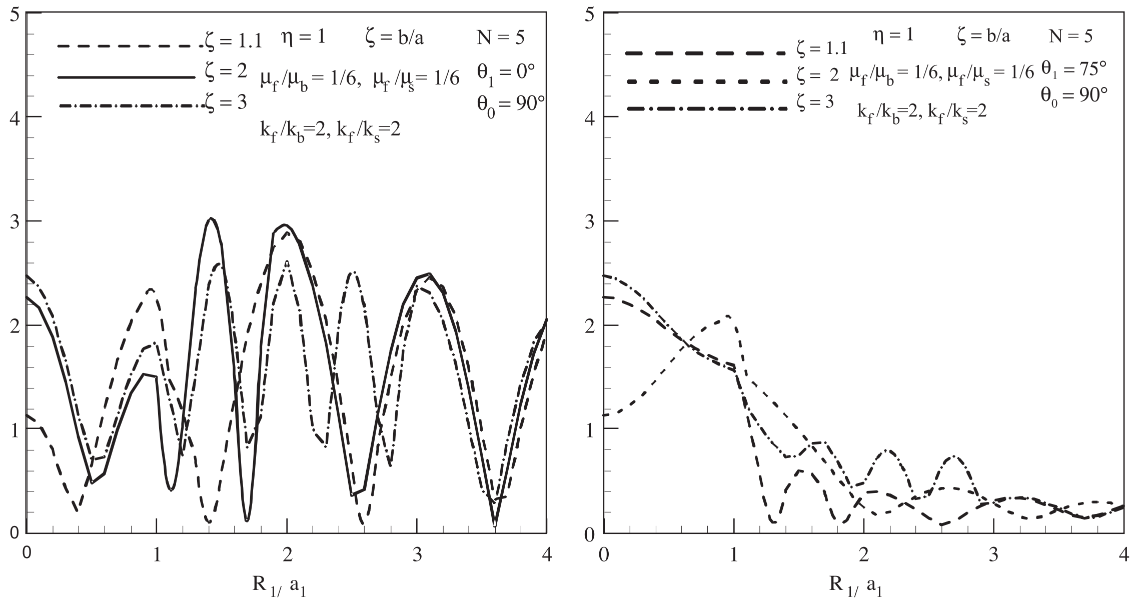

Figure 5. Dimensionless displacements versus depth of the flexible foundation, R_1/a_1, for $\eta = 1$, $\mu_f/\mu_b = 1/6$, $\mu_f/\mu_s = 1/6$, $k_f/k_b = 2$, $k_f/k_s = 2$, $N = 5$, and $\varsigma = b/a = 1.1,\ 2$ and 3.a) $\theta_1 = 0$, b) $\theta_1 = 90$.

Figures 7a and b show free surface displacements according to different incident angles in the case of $\mu_f/\mu_b = 1/6$, $\mu_f/\mu_s = 1/6$, $k_f/k_b = 2$, and $k_f/k_s = 2$. It can be seen from the graphics that the displacements are much larger than in the case of harder flexible foundation.

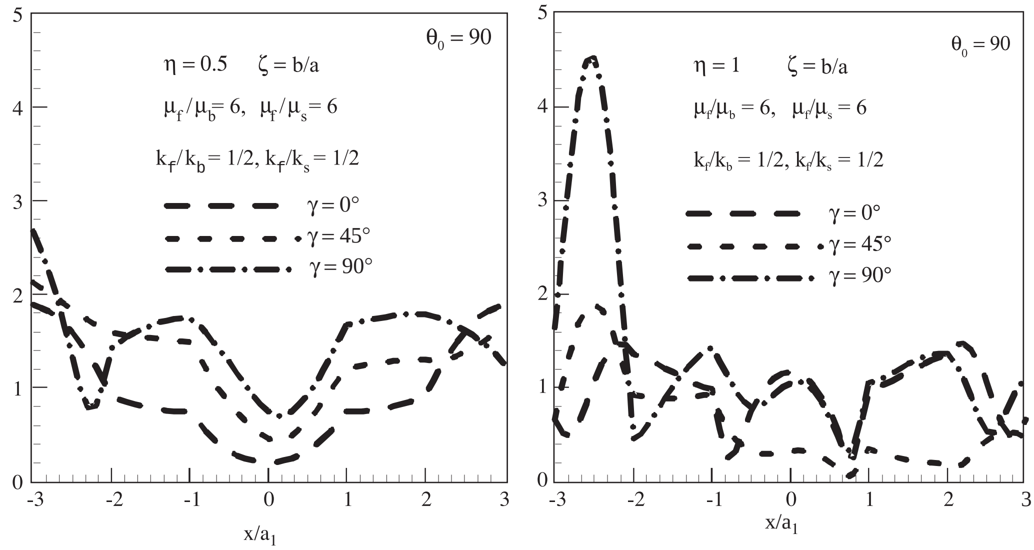

Figure 6. Free surface displacements according to different incident angle in the case of $\mu_f/\mu_b = 6,\ \mu_f/\mu_s = 6,$ $k_f/k_b = 1/2, \quad k_f/k_s = 1/2,\ \varsigma = 2,\ \gamma = 0,\ 45$ and 90 a)$\eta = 0.5$, b) $\eta = 1$.

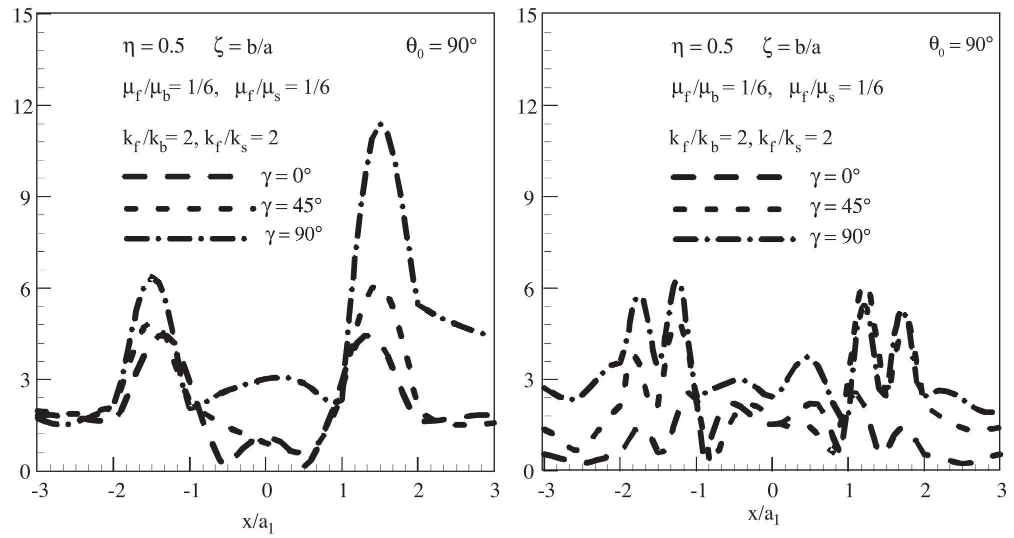

Figure 7. Free surface displacements according to different incident angles in the case of $\mu_f/\mu_b = 1/6,\ \mu_f/\mu_s = 1/6,$ $k_f/k_b = 2, \quad k_f/k_s = 2,\ \varsigma = 2,\ \gamma = 0,\ 45$ and 90 a)$\eta = 0.5$, b) $\eta = 1$.

Displacement residuals (ε) can be expressed as follows:

$$\varepsilon = \frac{U^b - U^f}{U_0}, \tag{11}$$

Figure 8a for $\mu_f/\mu_b = 1/6$, $\mu_f/\mu_s = 1/6$, $k_f/k_b = 2$ and $k_f/k_s = 2$ and Figure 8b for $\mu_f/\mu_b = 6$, $\mu_f/\mu_s = 6$, $k_f/k_b = 1/2$ and $k_f/k_s = 1/2$ show displacement residuals between the wedge and flexible foundation.

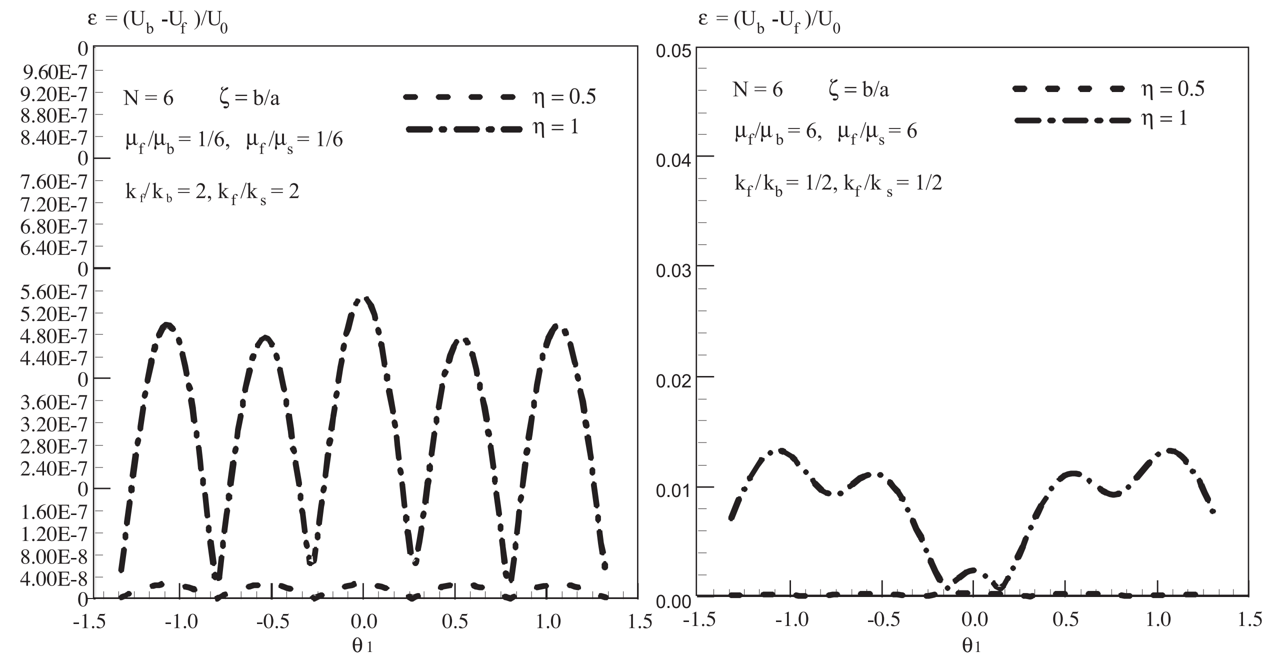

Figure 8. The displacement residuals between the wedge and soft flexible foundation for $\varsigma = 2$, $\gamma = 0^0$ a) $\mu_f/\mu_b = 1/6$, $\mu_f/\mu_s = 1/6$, $k_f/k_b = 2$, $k_f/k_s = 2$, b) $\mu_f/\mu_b = 6$, $\mu_f/\mu_s = 6$, $k_f/k_b = 1/2$, $k_f/k_s = 1/2$.

Conclusions

In this problem, structure, foundation and half-space are all deformed by SH waves. It is therefore very complicated to set the mathematical model up and solve it. Due to the increased complexity of the problem, models usually consider either kinematic interaction or dynamic interaction. In this paper, a simple structure is considered, but both kinematic interaction and dynamic interaction are taken into account. In the light of this model the following conclusions are obtained: a) depth of the foundation reduces the energy going into the structure, and b) rigidity of the flexible foundation affects the free surface and the wedge displacements. The displacements for harder flexible foundations are much smaller than those for soft ones. In conclusion, to prevent vital damage in the structure in the case of SH waves, it is suggested that the foundations be constructed more rigidly than the soil and the structure.

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