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A Reliability Model for Bridge Abutment Scour

A. Melih YANMAZ, Tuğsan ÇELEBİ
Middle East Technical University, Department of Civil Engineering, Ankara-TURKEY
e-mail:myanz@metu.edu.tr

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Abstract

Deterministic scour prediction equations for bridge abutments consider only the effects of hydraulic parameters and do not take the uncertainties of scouring parameters into account. Treatment of these uncertainties would provide the means for risk evaluation in bridge foundation design. Herein, a static reliability model is developed for the assessment of local scouring reliability around bridge abutments having relatively short lengths. This model is based on resistance-loading interference incorporating dependent parameters. In the model, the relative abutment footing depth, which can be considered at least the relative maximum scour depth, and the linear combination of the relative approach flow depth and Froude number are defined as the system resistance and external loading, respectively. By examining the statistical randomness of extensive laboratory data, a bivariate lognormal distribution is found to represent the joint probability density function of dependent resistance and loading. Reliability expressions are developed in terms of resistance. In an example, it is shown that the results of the proposed model and a Monte Carlo simulation are in good agreement. It is also observed that the execution of this model is less time consuming than the Monte Carlo simulation.

Key words: Reliability, Bridge, Abutment, Scour, Resistance, Loading, Safety factor, Return period, Monte Carlo simulation.

Introduction

The overall performance of a bridge crossing a wide river should be assessed regarding hydraulic, geotechnical and structural aspects jointly. Insufficient evaluations of the interactions among these aspects may generate a high level of uncertainty and risk in bridge design. However, it is almost impossible to estimate the overall uncertainty since the aforementioned interactions are not understood clearly. The scope of the present study is, therefore, limited to the assessment of bridge failures due to excessive scouring at abutments. This study is a continuation of previous work carried out by Yanmaz and Çiçekdağ (2001), in which a composite reliability model was developed for the assessment of local scour around cylindrical bridge piers. Owing to the random nature and complexity of the overall scouring phenomenon through the bridge opening, there exist hydraulic uncertainties leading to an unavoidable risk in bridge foundation design. Since the traditional design approach is based on the use of deterministic scour equations, a reliability-based assessment of bridge scouring is required to examine the relationship between the safety factor and reliability, which are key parameters for decision-making in bridge foundation design.

This paper concerns the development of a new reliability model for the assessment of local scour around bridge abutments. The basic concepts of reliability theory based on resistance-loading interference and the treatment of uncertainties associated with hydraulic variables are introduced in Sections 2 and 3, respectively. The mechanism of scour around bridge abutments is discussed in Section 4. The consecutive steps of the model development are provided in Section 5. Section 6 summarizes the application of the Monte Carlo simulation technique, which can be utilized for the verification of the proposed model. The utilization and verification of the model are illustrated in the application, which is presented in Section 7.

Overview of composite reliability theory

Prior to the presentation of the framework for the model development, the following basic definitions of reliability theory (based on resistance-loading interference) will be given for the sake of completeness.

The loading, x, on a system is the measure of the impact of external events. The overall loading can be expressed as a linear combination of n independent loads, $x = x_1 + x_2 \cdots + x_n$ (Yen *et al.*, 1986). The resistance, y, is the measure of the ability of the system to withstand the loading. Therefore, the reliability, α, of a system can be defined by the probability that the resistance of the system is greater than or equal to the loading. If the loading and resistance are dependent variables, the composite system reliability is given by (Mays and Tung, 1992)

$$\alpha = \int_0^{\infty}\int_0^{y} f_{x,y}(x,y)dxdy \tag{1}$$

where $f_{x,y}$(x,y) is the joint probability density function of the loading and resistance. When the loading and resistance are independent, Eq. (1) becomes

$$\alpha = \int_0^{\infty} f_y(y)\left[\int_0^{y} f_x(x)dx\right]dy \tag{2}$$

in which $f_y(y)$ and $f_x(x)$ are the probability density functions (PDFs) of resistance and loading, respectively. Determination of reliability using Eqs. (1) or (2) requires knowledge of the probability distributions of resistance and loading terms, which is normally unavailable for most hydraulic applications because of insufficient information.

Treatment of uncertainties in reliability analysis

In a random phenomenon, there exists uncertainty that can be categorized into 3 groups. Natural uncertainty arises from the random variability of the phenomenon and cannot be controlled. Model uncertainty stands from the approximations made in the equations representing the physical phenomenon. It can be incorporated into the model or equation by a model correction factor. Parameter uncertainty results from the randomness associated with the coefficients in the equations. Ignoring model uncertainty, the parameter uncertainty of a phenomenon, W, which is a function of n random independent variables $x_1, x_2, \ldots, x_n$, can be estimated by first order analysis using (Tung and Mays, 1980)

$$\begin{aligned}\Omega_W^2 = & \left(\frac{\partial W}{\partial x_1}\right)^2_{W=\overline{W}}\left(\frac{\overline{x}_1}{\overline{W}}\right)^2\Omega_{x_1}^2 \\ & +\left(\frac{\partial W}{\partial x_2}\right)^2_{W=\overline{W}}\left(\frac{\overline{x}_2}{\overline{W}}\right)^2\Omega_{x_2}^2 \\ & +.....+\left(\frac{\partial W}{\partial x_n}\right)^2_{W=\overline{W}}\left(\frac{\overline{x}_n}{\overline{W}}\right)^2\Omega_{x_n}^2\end{aligned} \tag{3}$$

where Ω_i is the coefficient of variation and the bar sign stands for the average values of the parameters. The overall uncertainty computed from Eq. (3) can be incorporated into reliability expressions as a parameter of the PDF that represents loading or resistance. The accuracy of the reliability is highly dependent on the correct choice of PDFs and the coefficients of variation of variables. To this end, sets of elaborate measurements are needed as well as the judgment of the hydraulician. The coefficients of variations in hydraulic parameters, such as flow depth and velocity, reflect possible errors in the measurement of these variables, which may be small in elaborate laboratory conditions. However, these values may attain somewhat larger values in prototype conditions, depending on the location of measurement, the precision of the instrument, and human-induced errors. The coefficients of variation of geometric variables such as abutment length may arise due to construction or measurement error, which is normally very small. There are a limited number of reports in the literature concerning the order of magnitudes of coefficients of variation of hydraulic and geometric parameters. By examining the available data on the variation of some hydraulic variables, Johnson (1996) presented limited information on the coefficients of variation and associated probability distributions of some hydraulic variables. In a recent study, Yanmaz (2000) carried out an uncertainty analysis for a diversion canal. Furthermore, another uncertainty analysis has also been conducted by Yanmaz and Çiçekdağ (2001) for bridge pier scouring. Suitable values can be assigned to the coefficients of variation of relevant variables with reference to these studies.

The literature is not rich in models assessing the risk of bridge scour, mainly due to the lack of relevant laboratory and prototype information. Johnson's (1992, 1999) studies are based on the risk assessment of scour using the Monte Carlo simulation technique. In a previous study carried out by Yanmaz and Çiçekdağ (2001), a reliability model based on resistance-loading interference was developed for bridge pier scouring. The objective of the present study is to extend the previous methodology developed by Yanmaz and Çiçekdağ (2001) for the reliability-based assessment of bridge abutment scouring. The development of such a model depends on a correct interpretation of local scour around bridge abutments. To this end, the next section is devoted to the explanation of the scour mechanism.

Abutment scour prediction

The protrusion of a bridge abutment into the channel causes the separation of incoming flow at the upstream face of the abutment. This leads to the creation of a vortex system that moves downstream by eroding the loose bed past the abutment (see Figure 1). A dimensional analysis of the governing parameters affecting the maximum clear water scour depth around an abutment, d_s, placed in a channel having negligible contraction effects and for sufficiently long flow duration yields

$$\frac{d_s}{d_0} = f(F_r, \frac{L}{D_{50}}, \frac{L}{d_0}, \sigma_g, K_s, K_\theta, K_G) \qquad (4)$$

where d_0 = depth of approach flow; F_r = Froude number which is given by $u/\sqrt{gd_0}$, where u is the mean approach flow velocity; g = gravitational acceleration; L = length of abutment perpendicular to the flow direction; σ_g = geometric standard deviation of sediment size distribution; K_s = shape factor; K_θ = adjustment factor for alignment of the abutment to the main flow; and K_G = factor representing the effects of approach channel geometry. The general form of Eq. (4) is relatively complex. Therefore, some terms, which are of secondary importance, can be ignored with reference to the following simplifications. For $L/D_{50} > 25$, where D_{50} is the median sediment size, the scour depth is independent of sediment size (Melville, 1997). For this range, individual bed particles are large relative to the groove excavated by the downflow and erosion is impeded because the porous bed dissipates some of the energy downflow. Therefore, the effect of L/D_{50} is ignored for $L/D_{50} > 25$, which reflects most of the common prototype conditions (Melville, 1997). For uniform sediment, $\sigma_g = 1.0$. Short abutments placed perpendicular to the shoreline would indicate $K_\theta = 1.0$ and $K_G = 1.0$. Therefore, for a given shape of abutment, the functional relation given by Eq. (4) reduces to

$$\frac{d_s}{d_0} = f_1\left(F_r, \frac{L}{d_0}\right) \qquad (5)$$

Equation (5) forms the basis of most abutment scour prediction equations reported in the literature. Table 1 summarizes some of the scour equations as well as their application conditions. Among these

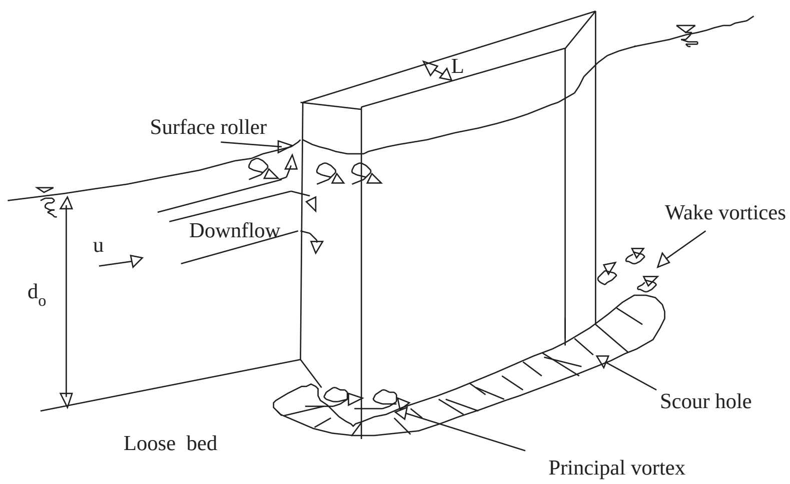

Figure 1. Flow pattern around a wing-wall abutment.

Table 1. Some abutment scour prediction equations.

Researcher (1)	Standard Equation (2)	Equation No. (3)	Characteristics (4)
Liu *et al.*, 1961	$\frac{d_s}{d_0} = 1.1 \left(\frac{L}{d_0}\right)^{0.40} F_r^{0.33}$	(6)	Spill through, live bed (L/d$_0$) < 25
Liu *et al.*, 1961	$\frac{d_s}{d_0} = 2.15 \left(\frac{L}{d_0}\right)^{0.40} F_r^{0.33}$	(7)	Vertical wall, live bed, (L/d$_0$) < 25
Laursen, 1963	$\frac{d_s}{d_0} = 1.89 \left(\frac{L}{d_0}\right)^{0.5}$	(8)	Vertical wall, maximum clear water
Laursen, 1980	$\frac{d_s}{d_0} = 1.5 \left(\frac{L}{d_0}\right)^{0.48}$	(9)	Vertical wall, live bed
Froehlich, 1989 (HEC-18 equation)	$\frac{d_s}{L'} = 2.27 K_s K_\theta \left(\frac{d_0}{L'}\right)^{0.57} F_r^{0.61}$	(10)	All abutment types, clear water and live bed
Lim, 1997	$\frac{d_s}{d_0} = 1.8 \left(\frac{L}{d_0}\right)^{0.5}$	(11)	Vertical wall, clear water
Melville, 1997	$d_s = K_{yL} K_I K_d K_s K_\theta K_G$	(12)	All abutment types, clear water and live bed

equations, the HEC-18 equation (Richardson and Davis, 2001) and Melville's equation (1997) will be further discussed because of their popularity and reasoning based on comprehensive analyses of scouring parameters.

Most of the abutment-scour prediction equations reported in the literature use the length of the abutment projected normal to the flow as an independent variable. This may be applicable only to short abutments for which upstream flow distribution due to channel constriction is negligible. In reality, for long embankments in floodplains, there is a large ineffective flow zone upstream of the embankment adjacent to the banks of the floodplains, which has a retarding effect on the scour around abutments (Froehlich, 1989; Melville, 1997). Richardson and Davis (2001) propose Froehlich's (1989) equation as a standard equation in HEC-18, which is a comprehensive technical manual for dealing with a bridge scour problem (Eq. (10) in Table 1). In this equation, L' = length of active flow obstructed by the embankment; and K_θ = adjustment factor for flow alignment, which is given by $(\theta/90)^{0.13}$ where θ is in degrees. Equation (10) is proposed for both live-bed and clear water abutment scour conditions. For design purposes, Froehlich (1989) recommended including a constant +1 on the right side of Eq. (10).

The equation proposed by Melville (1997) is conservative in nature as it is based on several adjustments, which are derived by enveloping the relevant laboratory data (Eq. (12) in Table 1). In this equation, K = adjustment factors accounting for the effects of various parameters, such as K_{yL} = factor to account for the combined effects of flow depth and abutment length; K_I = flow intensity and bed armoring factor; and K_d = sediment size factor. Melville (1997) classified abutments as short ($L/d_0 < 1$) or long ($L/d_0 > 25$), and suggested K_{yL} = 2L for the former case and $K_{yL} = 10d_0$ for the latter. For intermediate abutment lengths, the relation $K_{yL} = (d_0 L)^{0.5}$ has been proposed. For uniform bed material under clear water conditions, $K_I = u/u_c$, where u_c is the mean threshold velocity. The shape factors K_s = 1.0, 0.75, and 0.45 were suggested by Melville (1997) for vertical wall, wing wall, and spill through abutment having side slopes of 1.5H:1V, respectively. The effect of flow alignment has been taken into account by K_θ, which is given relative to the $\theta = 90°$ case. For $\theta > 90°$, $K_\theta > 1.0$, e.g, K_θ = 1.06 for $\theta = 120°$ (Melville, 1997). It can be stated that Melville's (1997) equation also considers the effects of the governing parameters expressed in Eq. (5).

Framework for reliability model development

The main objective of this study is the development of a composite reliability model for bridge abutment scour using the resistance-loading interference. The model development will be carried out step-by-step, with the relevant phases explained through the following subsections.

Assignment of resistance and loading parameters

The preliminary step in the development of the reliability model is the identification of appropriate loading and resistance terms, which reflect the physics of the scouring phenomenon. The linear combination of the relative approach flow depth and Froude number, $x = d_0/L + F_r$, is assumed to express the effects of external forces acting on the system. The physical significance of the loading term, x, is based on the fact that the parameters comprising x are involved in the scouring process (see equations in Table 1) and are basic variables in the forces acting on bridge abutments. The approach flow depth is an important variable involved in pressure, body, and inertia forces. The abutment length, L, which is the measure of degree of channel constriction, is required in the dynamic drag and lift forces acting on the abutment and is also used in the momentum equation in close vicinity to the bridge opening. The Froude number reflects the effects of inertia and body forces (Yanmaz and Çiçekdağ, 2001). Therefore, the linear combination of these parameters is assumed to reflect the combined effects of external loads acting on bridge abutments. The depth of the abutment footing, d_f, below the mean bed level, which can be obtained from the maximum possible scour depth at an abutment, dictates the level of system resistance with respect to erosion. In this study, the relative system resistance is assumed to be $y = d_0/d_s$. With the selection of d_0/d_s (instead of d_s/d_0), all the values of the calibration data are obtained greater than unity, which facilitated logarithmic transformations in the frequency analysis. To obtain the form of the reliability expression, probability distributions of the resistance and loading must be determined through frequency analysis.

Treatment of uncertainties

The deterministic scour prediction equations reported in the literature are simplistic in that they do not consider model or parameter uncertainty. The local scour mechanism around bridge abutments is relatively complex and so no single method that is valid for universal conditions concerning flow, sediment, river, and abutment characteristics has yet been developed. All of the methods proposed in the literature are based on several simplifying assumptions, and hence are valid only under certain conditions. Therefore, it can be concluded that there is no exact equation expressing the depth of local scour precisely. Although the forms of the equations proposed in the literature are similar, the results of these equations differ widely from each other because of variations in the derivational conditions. Therefore, the total uncertainty in the scouring mechanism cannot be quantified for a general case. Instead, a specific equation can be used to estimate the level of uncertainty, which may give a value for the conditions of derivation of the model. It is believed that Melville's (1997) comprehensive approach based on elaborate experimental data considers several aspects of the phenomenon sufficiently. To this end, the scour equation proposed by Melville (1997) can be used for the uncertainty analysis of the scouring parameters. Ignoring the model bias and using Eq. (3), the total parameter uncertainty in the resistance term, $y = d_0/d_s$, can then be determined for clear water conditions with uniform bed material as follows

$$\Omega_y = (0.25\Omega_{d_0}^2 \quad +0.25\Omega_L^2 + \Omega_u^2 + \Omega_{u_c}^2 + \Omega_{K_d}^2 + \Omega_{K_s}^2 + \Omega_{K_\theta}^2 + K_G^2)^{1/2} \tag{13}$$

The coefficients of variation of parameters in Eq. (13) reflect the possible variations in several geometric and hydraulic parameters. Values can be assigned to the coefficients of variations with reference to Johnson (1996, 1999), Yanmaz (2000), and Yanmaz and Çiçekdağ (2001) as follows: $\Omega_{d_0} = 0.24$, $\Omega_u = 0.3$, $\Omega_{u_c} = 0.35$, $\Omega_{K_s} = 0.15$, $\Omega_{K_\theta} = 0.15$, $\Omega_{K_d} = 0.15$, $\Omega_L = 0.01$ and $\Omega_{K_G} = 0.2$. The overall coefficient of variation of the relative resistance can then be obtained from Eq. (13) as 0.5778.

Derivation of resistance-loading relations

The philosophy behind this study is to consider the statistical randomness of a wide range of data reflecting the sources of possible errors implicit in the measurements of several researchers. The available statistical information will thus represent a broad range of uncertainty on the variations of scour depth, abutment length, bed material uniformity, and flow condition. To examine the uncertainty of the statistical randomness of the scour variables, the data reported in the literature should be re-interpreted by examining the frequency distributions of the governing scouring variables. The calibration data used in the development of the model are summarized in Table 2. In this study, extensive experimental data compiled from Liu *et al.* (1961), Gill (1972), Wong (1982), Rajaratnam and Nwachukwu (1983), Kwan (1984), Tey (1984), Kandasamy (1985), Kwan (1988), Dongol (1994), Lim (1997) and Lauchlan *et al.* (2001) have been analyzed. The relative abutment lengths of the calibration data, L/d_0, are less than 17.4 and 13.0 for vertical and wing wall abutments, respectively. The regression equations fitted to the experimental data of vertical and wing wall abutments are given by Eqs. (14) and (15), respectively.

$$y = 0.5976 \ln x + 0.7871 \quad 0.346 < x < 4.171 \quad (14)$$

$$y = 0.8276 \ln x + 0.9484 \quad 0.465 < x < 1.783 \quad (15)$$

for which the correlation coefficients are 0.67 and 0.91, respectively. The functional relationships given by Eqs. (14) and (15) are presented by plotting them on the calibration data (see Figures 2 and 3). The scatter of data points in Figures 2 and 3 is assumed to reflect the level of uncertainty implicit in the scouring parameters. This uncertainty can be incorporated into the reliability equation by assigning proper coefficients of variation of parameters. The agreement of Eq. (14) is assessed for vertical wall abutments by comparing the results obtained from Eq. (14) with the scour equations proposed by Melville (1997) and Froehlich (1989) using the experimental data presented in Table 2 (see Figures 4 and 5). As

Table 2. The ranges of the calibration data.

Type (1)	Researcher (2)	L(cm) (3)	d_o(cm) (4)	D_{50}(cm) (5)	d_s(cm) (6)	F_r (7)
Vertical wall	Gill (1972)	10-30.5	3.26-10	0.09-0.15	5.79-19.17	0.236-0.737
	Dongol (1994)	15	13-60	0.09	23.8-30.4	0.171-0.301
	Lim (1997)	10-15	10-15	0.094	5.7-23	0.196-0.257
	Liu *et al.*, (1961)	30.5	7.6-15.2	0.056	20.12-41.15	0.149-0.423
	Lauchlan *et al.*, (2001)	5-60	10-20	0.08-0.10	8.2-42.1	0.136-0.323
	Rajaratnam (1983)	15.2	10.7-15.4	0.14	8.5-18.3	0.166-0.312
	Kwan (1984)	16.4-87	5-10	0.085	8.5-26.8	0.313-0.408
	Tey (1984)	16.5-30.2	5-17.5	0.082	14.6-27.0	0.249-0.381
Wing wall	Wong (1982)	60	7.5-17.5	0.062-0.167	17.2-28.5	0.275-0.478
	Tey (1984)	29.6-60	5-50	0.082	7.8-36.5	0.170-0.381
	Kwan (1988)	45	5-20	0.085	20-40	0.263-0.417
	Kandasamy (1985)	65	5	0.090	20-24.8	0.452

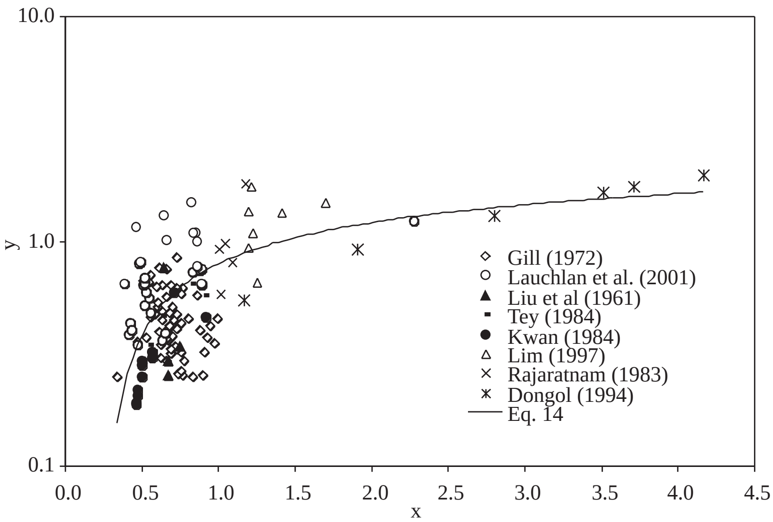

Figure 2. Variation of Eq. (14) with the calibration data for vertical wall abutments.

can be observed from Figure 4, the scour depths obtained from Melville's equation, d_{sMel}, are greater than the results of the present study, d_s, since Melville's method is conservative. The scour depths obtained from Froehlich's equation, d_{sFro}, are collected around the line of best agreement between the 2 approaches (see Figure 5).

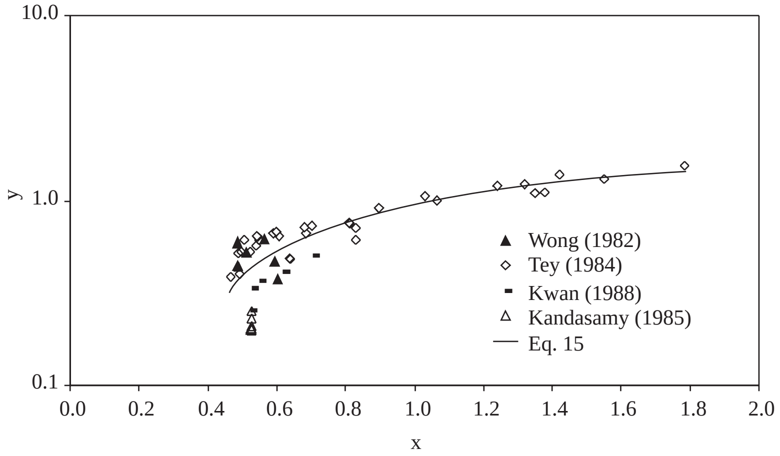

Figure 3. Variation of Eq. (15) with the calibration data for wing wall abutments.

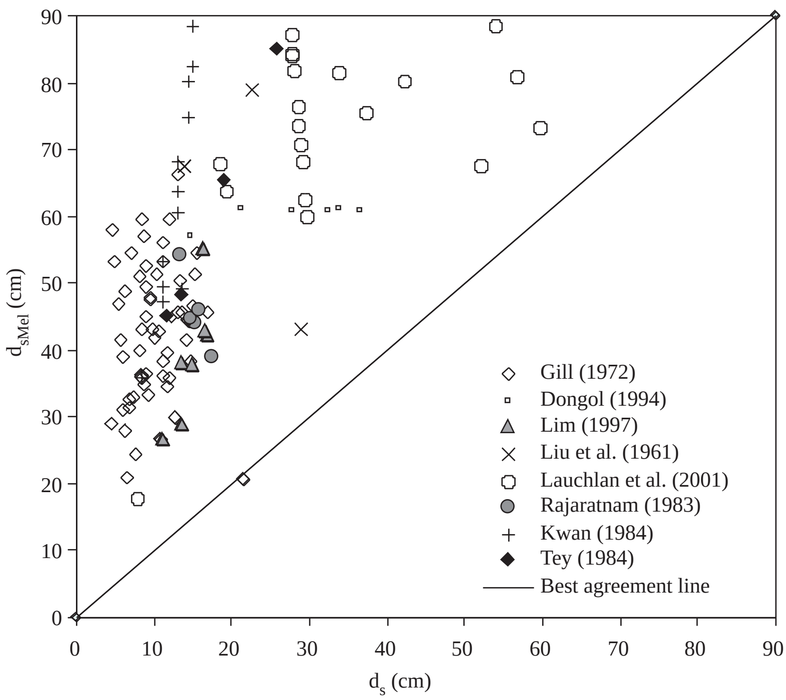

Figure 4. Correlations of scour depths obtained from Eqs. (12) and (14).

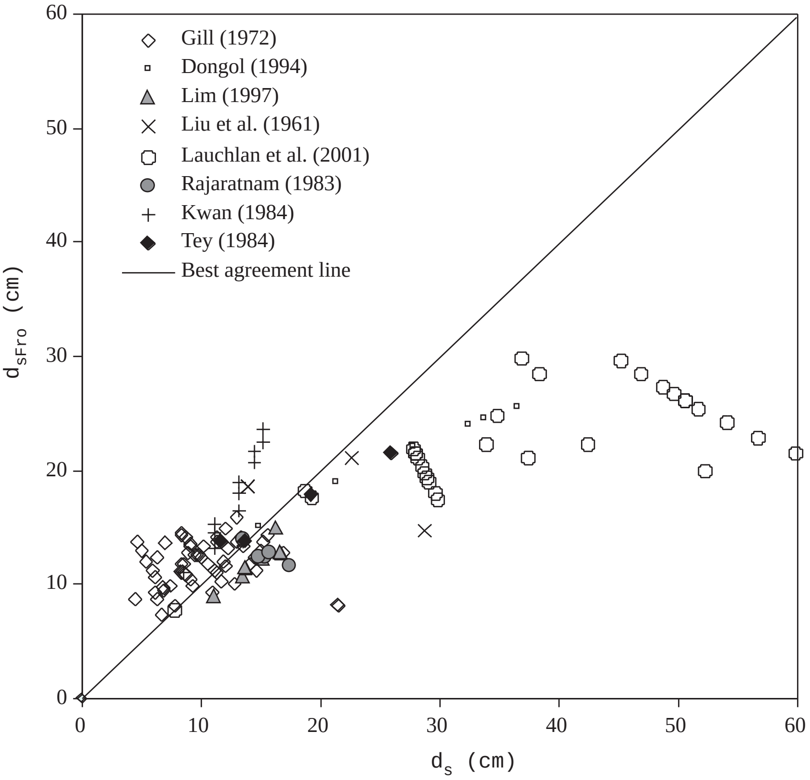

Figure 5. Correlations of scour depths obtained from Eqs. (10) and (14).

Determination of representative PDFs

In the application of reliability theory to water resources systems using resistance-loading analysis, extreme value functions should be used since the loading and resistance parameters reflect the design condition, i.e. the worst possible case. The maximum resistance under extreme loading is the main concern in decision-making for design. Several examples can be given for the use of extreme functions in hydraulic design, e.g., Mays (1979); Tung and Mays (1980, 1981), Yanmaz (2000); and Yanmaz and Çiçekdağ (2001). In recognition of the nature of the design resistance and loading terms, extreme distributions are fitted to the data sets. The experimental scour depths under clear water conditions reflect the terminal maximum depths. Therefore, the use of extreme functions for the resistance terms is also required. In this analysis, it is assumed that the experimental loading measurements reflect the terminal maximum values. Hence, the commonly used extreme distributions in hydrology, i.e. 2-parameter lognormal (LN2), 3-parameter lognormal (LN3), extremal type 1 (EV1), Pearson type 3 (PT3), and log-Pearson type 3 (LPT3) can be tested for the goodness of fit.

To observe the statistical distribution of the governing parameters, the calibration data are divided into small class intervals of the loading term, $x = d_0/L + F_r$, and the statistical distribution of the resistance term, d_0/d_s, is investigated in these ranges. With this approach, it is intended to observe the randomness of the resistance under almost constant values of loading to assess the degree of variability, which is a measure of uncertainty. Similar analyses are also carried out for loading in small intervals of resistance. Tests are carried out for the aforementioned functions to observe the goodness of each fit for x and y values of the data sets for vertical abutments using both Chi-square (χ^2) and Kolmogorov-Smirnov (D_n) techniques with a confidence level of 90%. Herein, only the results of the frequency analyses carried out for vertical wall abutments are presented for the sake of brevity. Similar analyses are also carried out for wing wall abutments (Çelebi, 2002). The results are given in Tables 3 and 4, in which N is the sample size, μ is the mean, and σ is the standard deviation. As can be seen from these tables, although almost all of the PDFs tested in the analysis could be selected for x and y, the LN2 distribution function is selected for both x and y for simplicity. Similar analyses conducted for wing wall abutments show that an LN2 function can also be selected for both x and y (Çelebi, 2002). The correlation coefficients between x and y have been obtained as 0.67 and 0.91 for vertical wall and wing wall abutments, respectively. With these values, it can be assumed that these variables are dependent. Since LN2 is a commonly accepted function for the dependent variables x and y, the joint probability density function of the dependent variables can be expressed by a bivariate LN2 function according to the central limit theorem (Yevjevich, 1972). The remaining probability density functions describing the distribution of x and y are not considered specifically since their bivariate forms are not defined mathematically. The joint probability density function of the resistance and loading is given by (Yevjevich, 1972):

$$f_{x,y}(x,y) = \frac{1}{2\pi xy\sigma_{\ln x}\sigma_{\ln y}\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2(1-\rho^2)}\left(\left[\frac{\ln x-\mu_{\ln x}}{\sigma_{\ln x}}\right]^2 - 2\rho\left[\frac{\ln x-\mu_{\ln x}}{\sigma_{\ln x}}\right]\left[\frac{\ln y-\mu_{\ln y}}{\sigma_{\ln y}}\right] + \left[\frac{\ln y-\mu_{\ln y}}{\sigma_{\ln y}}\right]^2\right)\right] \tag{16}$$

Table 3. Frequency analyses of y for various intervals of x for vertical walls.

x (1)	N (2)	μ (3)	σ^2 (4)	Accepted PDF (χ^2) (5)	Accepted PDF (D_n) (6)
0.34-0.478	11	0.401	0.0811	LN3,LPT3,LN2,EV1	All PDFs
0.478-0.530	11	0.538	0.0438	PT3,LPT3,LN2,EV1	PT3,LPT3,LN2,EV1
0.530-0.570	11	0.463	0.021	All PDFs	All PDFs
0.570-0.636	10	0.474	0.0193	All PDFs	All PDFs
0.636-0.663	10	0.564	0.0875	All PDFs	All PDFs
0.663-0.695	10	0.492	0.0557	All PDFs	All PDFs
0.695-0.740	11	0.476	0.0252	All PDFs	All PDFs
0.740-0.820	10	0.381	0.0181	All PDFs	All PDFs
0.820-0.882	10	0.835	0.1168	All PDFs	All PDFs
0.882-0.950	10	0.481	0.0256	All PDFs	All PDFs
0.950-1.200	10	0.875	0.1985	All PDFs	All PDFs
1.200-4.20	11	1.373	0.1568	PT3,LPT3,LN2,EV1	PT3,LPT3,LN2,EV1
0.34-4.20	125	0.614	0.142	PT3,LPT3,LN2	All PDFs

Table 4. Frequency analyses of x for various intervals of y for vertical walls.

x (1)	N (2)	μ (3)	σ^2 (4)	Accepted PDF (χ^2) (5)	Accepted PDF (D_n) (6)
0.180-0.260	11	0.606	0.0339	LN3,PT3,LPT3	All PDFs
0.260-0.318	11	0.658	0.0154	All PDFs	All PDFs
0.318-0.355	11	0.666	0.0217	All PDFs	All PDFs
0.355-0.395	10	0.648	0.0169	LN3,PT3,LN2,EV1	All PDFs
0.395-0.448	10	0.690	0.0273	PT3,LPT3,LN2,EV1	PT3,LPT3,LN2,EV1
0.448-0.500	10	0.714	0.0232	All PDFs	All PDFs
0.500-0.585	11	0.762	0.0426	All PDFs	All PDFs
0.585-0.642	10	0.696	0.0172	All PDFs	All PDFs
0.642-0.742	10	0.685	0.0679	All PDFs	All PDFs
0.742-0.925	10	0.750	0.0433	All PDFs	All PDFs
0.925-1.250	10	1.135	0.3151	All PDFs	All PDFs
1.250-2.000	11	2.033	1.6187	All PDFs	All PDFs
0.180-2.000	125	0.841	0.331	LPT3	LPT3,EV1

where ρ is the correlation coefficient.

Derivation of reliability expressions

With the aforementioned information, the composite system reliability is computed from

$$\alpha = \int_0^\infty \int_0^y \frac{1}{2\pi xy\sigma_{\ln x}\sigma_{\ln y}\sqrt{1-\rho^2}} \exp(-\frac{1}{2(1-\rho^2)}((\frac{\ln x-\mu_{\ln x}}{\sigma_{\ln x}})^2 - 2\rho(\frac{\ln x-\mu_{\ln x}}{\sigma_{\ln x}})(\frac{\ln y-\mu_{\ln y}}{\sigma_{\ln y}})+ (\frac{\ln y-\mu_{\ln y}}{\sigma_{\ln y}})^2))dxdy \quad (17)$$

The double integral in Eq. (17) cannot be determined analytically. Yanmaz and Çiçekdağ (2001) developed a numerical solution algorithm for Eq. (17) based on the Simpson one-third rule of numerical integration (Wylie and Barrett, 1985):

$$\int_{y_o}^{y_2}\int_{x_o}^{x_2} f_{x,y}(x,y)\,dxdy = \frac{hk}{9}[(f_{00}+f_{02}+f_{20}+f_{22}) + 4(f_{01}+f_{10}+f_{12}+f_{21}) + 16f_{11}] + E_t \quad (18)$$

where h and k are the intervals at which x and y are tabulated, respectively, $f_{ij} = f(x_o+ih, y_o+jk)$ for $0 \leq i$ and $j \leq 2$, and E_t is the error term. By applying the expression given in Eq. (18) successively, the following equation is obtained:

$$\int_{y_o}^{y_{2m}}\int_{x_o}^{x_{2n}} f_{x,y}(x,y)dxdy = \frac{hk}{9}\left(\begin{array}{l} 4\left(\sum_{i=2}^{2n-2,2}\sum_{j=2}^{2m-2,2} f_{ij} + \sum_{i=1}^{2n-1,2}\sum_{j=0}^{2m,2m} f_{ij} + \sum_{i=0}^{2n,2n}\sum_{j=1}^{2m-1,2} f_{ij}\right) + \sum_{i=0}^{2n,2n}\sum_{j=1}^{2m,2m} f_{ij} + \\ 16\sum_{i=1}^{2n-1,2}\sum_{j=1}^{2m-1,2} f_{ij} + 8\left(\sum_{i=1}^{2n-1,2}\sum_{j=2}^{2m-2,2} f_{ij} + \sum_{i=2}^{2n-2,2}\sum_{j=1}^{2m-1,2} f_{ij}\right) + \\ 2\left(\sum_{i=0}^{2n,2n}\sum_{j=2}^{2m-2,2} f_{ij} + \sum_{i=2}^{2n-2,2}\sum_{j=1}^{2m,2m} f_{ij}\right) \end{array}\right) + E_t \quad (19)$$

where 2n and 2m are the number of segments the for x- and y-axes, respectively. The expression given in Eq. (19) is limited to cases with an even number of segments and an odd number of points. Successive resistance levels are considered for x_{2n} in $0.1929 < y < 1.6404$ and $0.3111 < y < 1.427$ for vertical wall and wing wall abutments, respectively, according to the data analyzed. The intervals can be obtained from $h = (y_{2m} - y_0)/m$ and $k = (x_{2n} - x_0)/n$. In the analysis, the computations were initiated from the lower bound and y values were incremented with intervals of 0.1. Therefore, a set of y versus α values was obtained. To derive a relationship between α and y, the best-fit equations of the data sets were obtained through regression analysis. The piece-wise reliability equations for the vertical wall abutments are given as

$$\alpha = -3.0346y^3 + 6.0275y^2 - 2.5163y + 0.2863 \quad 0.1929 < y < 0.9299 \quad (20)$$

$$\alpha = 0.4703y^3 - 2.3514y^2 + 4.0503y - 1.3918 \quad 0.9299 \leq y < 1.6404 \quad (21)$$

The reliability equations for wing wall abutments are as follows:

$$\alpha = 260.09y^5 - 541.36y^4 + 408.69y^3 - 130.33y^2 + 15.292y - 0.1285 \quad 0.3111 \leq y < 0.6568 \tag{22}$$

$$\alpha = -1.5974y^3 + 3.3649y^2 - 1.2952y + 0.3817 \quad 0.6568 \leq y < 0.9911 \tag{23}$$

$$\alpha = -0.544y^4 + 3.3725y^3 - 8.0359y^2 + 8.7997y - 2.738 \quad 0.9911 \leq y < 1.427 \tag{24}$$

The variation of reliability against resistance for vertical and wing wall abutments is also shown in Figure 6. The reliability of wing wall abutment against scouring is greater than that of vertical wall abutment at a constant level of resistance. The reliability equations are valid for the no bias case.

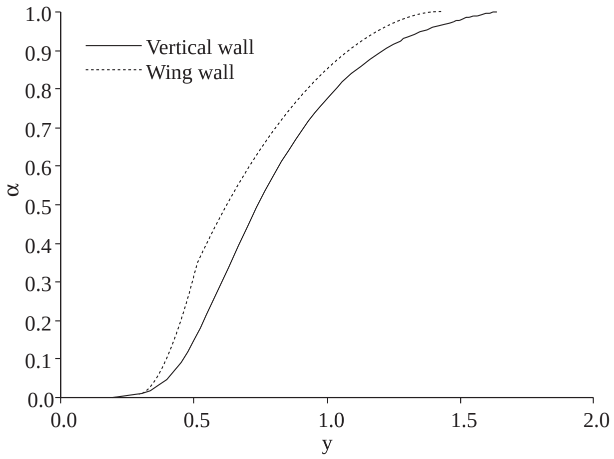

Figure 6. Variation of reliability with respect to resistance.

The results derived from the present model facilitated the mathematical formulation of the general reliability expression given in Eq. (1) by defining 2 dependent variables, x and y, according to generally accepted knowledge about bridge abutment scour. The proposed model also has versatility for considering the potential effects of sediment grading, abutment inclination, and relative sizes of abutment length and sediment by multiplying Eqs. (14) and (15) by the adjustment factors given by Melville (1997).

Reliability estimation using monte carlo simulation

For the verification of the model developed in this paper, Monte Carlo simulation is utilized. It is well known that the correct reliability of a non-linear performance function, such as the safety margin may be evaluated through large-sample Monte Carlo simulations (Ang and Tang, 1984). In this analysis, random numbers between 0 and 1 are generated for the variables having uniform distribution. These random numbers are then transformed to the desired distribution through an inverse transform method using a computer algorithm. In this analysis, random numbers can be generated for the probability of the safety margin being greater than or equal to zero. In this study, the safety margin is defined as SM $= d_f - d_s$, where d_f is the depth of the abutment footing. In the Monte Carlo analysis, the number of simulation cycles, i.e. the number of trials to generate random numbers, influences the level of reliability. The number of cycles required in a Monte Carlo simulation to determine the exact reliability must be large in order to obtain a significant sampling of simulation events such that the sample can be treated as the population. The accuracy of the mean reliability under a particular simulation cycle may be estimated by the coefficient of variation of reliability, Ω, which decreases with increasing sample size (Melchers, 2002). Therefore, simulations should be carried out several times for large cycles such that the corresponding value of Ω is relatively small. According to Johnson (1999), it is desirable to have $\Omega < 0.1$. The utilization and comparison of the proposed model and Monte Carlo simulation will be illustrated in the following example.

Application

A bridge is to be constructed at a hypothetical site. To decide on the type of suitable bridge opening, which is characterized by the geometric features of the approach embankments and type of abutments, various lengths of abutments need to be assessed from structural, geotechnical, and hydraulics viewpoints. The hydraulics aspect of the design is concerned with the analysis of flow conditions and determination of the reliability of abutment scour. The river has a rectangular cross-section of width 40 m and its bed is composed of poorly graded gravels having D_{50} = 25 mm. The mean bed slope is 0.001. Manning's roughness coefficient is 0.035. The annual series of the flows is assumed to follow an LN2 function with $\mu_{lnQ} = 4.8$ m^3/s and $\sigma_{lnQ} = 0.5$ m^3/s. For decision-making, information is required concerning the interrelation between the reliability, safety factor, and return period. The reliability computations will be carried out using the model proposed in this study. Verification of the model will also be illustrated using Monte Carlo simulation. In this example, the contraction scour and general bed scour are ignored and it is assumed that the local scour around symmetrical abutments is not influenced by the presence of possible intermediary piers.

The hydraulic computations are presented in Table 5 for various return periods, T_r, ranging from 2 to 100 years. The critical shear velocity, u_{*c}, is determined as 0.15 m/s from Shield's criterion. The discharges given in column 2 of Table 5 are obtained from the frequency analysis of the LN2 function. The corresponding approach flow depths presented in column 3 are obtained from Manning's equation. The mean flow velocity is determined from the continuity relation (column 4 of Table 5). The mean threshold velocity is given in column 5. With this information, it is determined that clear water conditions prevail at the bed level for all return periods. The corresponding Froude numbers are given in column 6.

The details of the reliability computations for vertical wall abutments are given in Table 6. Similar manipulations are also repeated for wing wall abutments. In the analysis, the abutment length, L, is considered a decision variable and the corresponding maximum depth of local scour, safety factor and reliability are determined with respect to the return period. Successive abutment lengths have been considered in the range 1.25 $m \leq L \leq 4.5$ m (column 2 of Table 6). Resistance values are obtained from Eq. (14) for vertical wall abutments. Reliability is obtained from Eqs. (20) and (21). It is assumed that a maximum value of 5.0 m can be attained for the depth of pier footing, d_f, with respect to the mean bed level. The safety factor, SF, is then defined as the ratio of d_f to the maximum value of scour depth, d_s, for a particular return period under a desired reliability level (olumn 7 of Table 6). As can be observed from Table 6, $L/D_{50} > 25$ and x is in the range of the validity of Eqs. (14) and (15) for all cases. The results of the analysis are presented in Figure 7 for the return periods of 5, 25, and 50 years. In this figure, the correlation between the reliability and safety factor under various return periods is investigated. It is shown that the reliability increases with the increasing safety factor. It can also be observed from this figure that, for a particular safety factor, the reliability of scouring around the wing wall abutment is greater than that of the vertical wall abutment. The results of the analysis are consistent with similar studies performed earlier for different applications, e.g., for culvert design by Tung and Mays (1980). In this application, the relative abutment length, L/d_0, is small enough to allow the effect of contraction scour to be safely ignored.

Since wing wall abutments yield higher reliability levels than those of vertical walls, further analysis is carried out to obtain additional information. To this end, the variation of reliability against the length of wing wall abutment is observed for various return periods (see Figure 8). As can be observed from this figure, the reliability increases with increasing return period and decreasing abutment length. The selection of a suitable abutment length is dictated by the requirement of a particular safety factor that is compatible with local site conditions.

Table 5. Hydraulic computations for the practical application.

T_r (1)	Q (m^3/s) (2)	d_o (m) (3)	u (m/s) (4)	u_c (m/s) (5)	F_r (6)
2	121.45	2.156	1.408	2.306	0.306
5	185.02	2.808	1.647	2.405	0.314
10	230.43	3.226	1.786	2.457	0.317
25	291.32	3.747	1.944	2.513	0.321
50	339.30	4.132	2.053	2.549	0.322
100	388.53	4.510	2.154	2.582	0.324

Table 6. Reliability computations for the practical application.

T_r (years) (1)	L (m) (2)	x (3)	y (4)	α (5)	d_s (m) (6)	SF (7)
5	1.25	2.560	1.349	0.94748	2.082	2.40
	1.50	2.186	1.254	0.91721	2.238	2.23
	1.75	1.918	1.176	0.88450	2.387	2.09
	2.00	1.718	1.110	0.85031	2.529	1.98
	2.25	1.562	1.054	0.81537	2.665	1.88
	2.50	1.437	1.004	0.78024	2.797	1.79
	2.75	1.335	0.960	0.74529	2.926	1.71
	3.00	1.250	0.920	0.71080	3.051	1.64
	3.25	1.178	0.885	0.67672	3.173	1.58
	3.50	1.116	0.853	0.64184	3.293	1.52
	3.75	1.063	0.823	0.60687	3.410	1.47
	4.00	1.016	0.796	0.57256	3.525	1.42
	4.25	0.975	0.772	0.53937	3.638	1.37
	4.50	0.938	0.749	0.50756	3.750	1.33
25	1.25	3.318	1.504	0.98091	2.491	2.01
	1.50	2.818	1.406	0.96183	2.664	1.88
	1.75	2.462	1.325	0.94081	2.827	1.77
	2.00	2.194	1.257	0.91804	2.982	1.68
	2.25	1.986	1.197	0.89394	3.130	1.60
	2.50	1.819	1.145	0.86889	3.273	1.53
	2.75	1.683	1.098	0.84326	3.412	1.47
	3.00	1.570	1.056	0.81732	3.546	1.41
	3.25	1.473	1.019	0.79128	3.678	1.36
	3.50	1.391	0.984	0.76532	3.806	1.31
	3.75	1.320	0.953	0.73954	3.932	1.27
	4.00	1.257	0.924	0.71406	4.055	1.23
	4.25	1.202	0.897	0.68893	4.176	1.20
	4.50	1.153	0.872	0.66351	4.295	1.16
100	1.25	3.932	1.605	0.99999	2.810	1.78
	1.50	3.331	1.506	0.98129	2.995	1.67
	1.75	2.901	1.424	0.96564	3.168	1.58
	2.00	2.579	1.353	0.94867	3.333	1.50
	2.25	2.328	1.292	0.93042	3.490	1.43
	2.50	2.128	1.238	0.91110	3.642	1.37
	2.75	1.964	1.190	0.89096	3.789	1.32
	3.00	1.827	1.147	0.87021	3.931	1.27
	3.25	1.712	1.108	0.84906	4.070	1.23
	3.50	1.612	1.073	0.82767	4.205	1.19
	3.75	1.526	1.040	0.80617	4.337	1.15
	4.00	1.451	1.010	0.78466	4.467	1.12
	4.25	1.385	0.982	0.76323	4.594	1.09
	4.50	1.326	0.956	0.74195	4.719	1.06

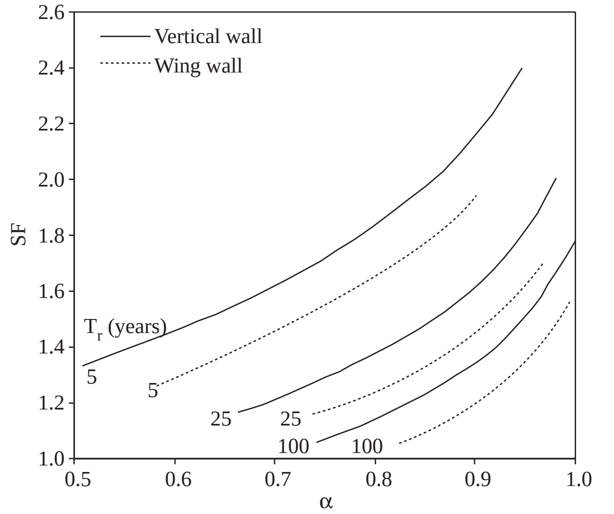

Figure 7. Variation of reliability with respect to safety factor.

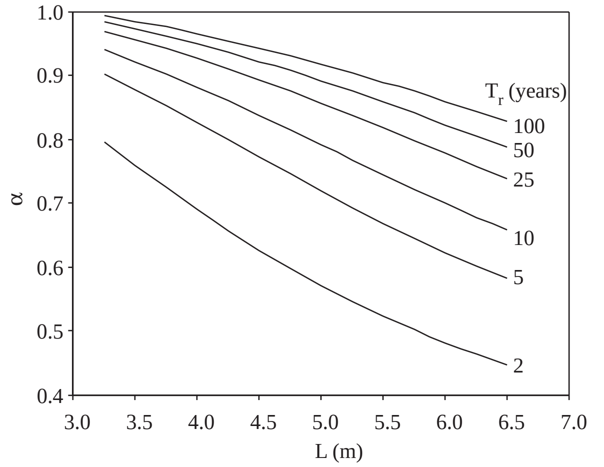

Figure 8. Relationship between reliability and length of wing wall abutments under various return periods.

For the verification of the model developed in this paper, reliability computations are also conducted using Monte Carlo simulation for wing wall abutments under $T_r = 5$ years. In the simulation, triangular distribution is assumed for the flow depth and velocity, uniform distribution for Melville's adjustment factors and lognormal distribution for the abutment length. An analysis is carried out to determine the suitable number of simulation cycles. Therefore, several runs are performed to determine the coefficient of variation of reliability, Ω, under various simulation cycles. The variation of Ω against the number of simulation cycles is shown in Figure 9, which implies that as the number of simulation cycles increases, Ω approaches a constant value, i.e. 0.02. In the computations, 4000 cycles are taken and a set of reliability values is obtained. The number of trials for a particular abutment length is dictated by the achievement of $\Omega < 0.1$. The results of reliability are plotted together with the results of the proposed model in Figure 10. This figure shows the relationship between reliability and the safety factor. The results of the composite reliability model and Monte Carlo simulation (see Figure 10) are close to each other, demonstrating that reliability increases with increasing safety factor.

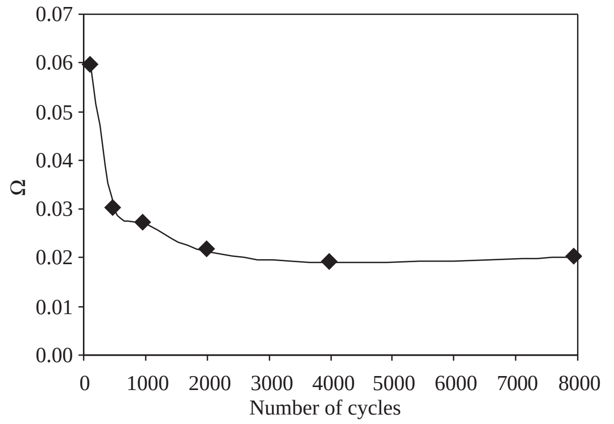

Figure 9. Coefficient of variation as a function of the number of simulation cycles.

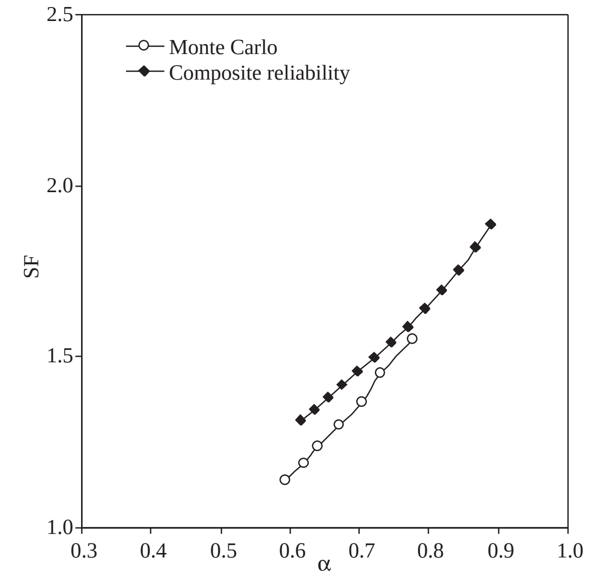

Figure 10. Variation of safety factor with respect to the reliability of wing wall abutments for $T_r = 5$ years.

Conclusions

A composite reliability model is developed for the assessment of clear water scour around bridge abutments of vertical and wing wall types using resistance-loading interference. In the model, the linear combination of the relative approach flow depth and the Froude number is considered to represent the external loading, x. The relative bottom elevation of abutment footing, which is obtained from the maximum possible depth of scour at an abutment, y, is regarded as the system resistance. The relationships between the resistance and loading for vertical and wing wall abutments are given by Eqs. (14) and (15), respectively. The statistical analysis of the data indicates that x and y are dependent and their joint PDF can be represented by a bivariate lognormal distribution. Composite reliability analysis of the dependent variables over the ranges $0.346 < x < 4.171$ and $0.464 < x < 1.783$ for vertical and wing wall abutments, respectively, using Eq. (19) yields reliability relations, Eqs. (20) through (24). These equations enable a designer to assess various levels of reliability in terms of safety factors and return periods. Therefore, various alternatives can be compared in terms of economy and safety. The model has the versatility to consider the potential effects of sediment grading, abutment inclination, and the relative sizes of abutment length and sediment by multiplying Eqs. (14) and (15) by the adjustment factors proposed by Melville (1997). The proposed model can be used for both analysis and design purposes. In the analysis, the reliability level of an existing bridge with a known foundation depth can be determined for the given flow conditions. In the case of abutment footing design, either a high safety factor is taken and the corresponding reliability of scouring is obtained or a high reliability is taken and the corresponding safety factor is determined. A detailed interpretation of the results may give guidelines concerning reliability, safety, return period, and economy.

Monte Carlo simulation is also applied for the verification of the model. Both approaches yield close results, demonstrating that reliability increases with increasing safety factor under a particular return period. The application of the dependent model is less time consuming than Monte Carlo simulation since the former model is based on the use of dimensionless explicit relations (Eqs. (20) through (24)). The final decision for the design abutment length and footing depth may be given by considering the local topographic, geologic, hydrologic, and hydraulic factors in addition to the navigational and economic requirements. Various scour countermeasures should also be implemented in the close vicinity of bridges to increase safety.

Nomenclature

D_n	Kolmogorov-Smirnov statistic;
D_{50}	median sediment size;
d_f	depth of the bottom of abutment footing;
d_s	maximum depth of scour around an abutment;
d_0	approach flow depth;
E_t	error term;
EV1	extreme value type 1 distribution;
F_r	Froude number of approach flow;
$f_{x,y}$ (x,y)	joint probability density function of loading and resistance;
f_x(x)	probability density function of loading;
f_y(y)	probability density function of resistance;
g	gravitational acceleration;
h	interval;
K_d	adjustment factor for sediment size;
K_G	adjustment factor for approach channel geometry;
K_I	adjustment factor for flow intensity and bed armoring;
K_s	abutment shape factor;
K_{yL}	adjustment factor for abutment length and flow depth;
K_θ	adjustment factor for abutment inclination with the approach flow;
k	interval;
L	abutment length;
L'	length of active flow obstructed by the embankment;
LN2	2-parameter log-normal distribution;
LN3	3-parameter log-normal distribution;
LPT3	log-Pearson type 3 distribution;
m	number of segments;
N	sample size;
n	number of segments;
PDF	probability density function;
PT3	Pearson type 3 distribution;
Q	discharge;
SF	safety factor;
SM	safety margin;
T_r	return period;
u	mean approach flow velocity;

u_c mean threshold velocity;
u_{*c} critical shear velocity;
x loading;
y resistance;
α reliability;
θ angle between the abutment axis and approach flow;
ρ correlation coefficient;
μ mean value;
σ standard deviation;
σ_g standard deviation of grain size distribution;
χ^2 Chi-square statistic;
Ω coefficient of variation

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