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Effect of Wall Shear Stress Distribution on Manning Coefficient of Smooth Open Rectangular Channel Flows

Ahmet BİLGİL
Niğde University, Civil Engineering Department, Niğde-TURKEY
e-mail: abilgil@nigde.edu.tr

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Abstract

The determination of velocity distribution in open channel flows is crucial in many critical engineering problems such as channel design, calculation of energy losses and sedimentation. In this study, velocity distribution is experimentally investigated in a smooth rectangular open channel. Wall shear stresses are calculated using measured local velocities. Assuming logarithmic velocity distribution along perpendiculars to a wetted perimeter, dimensionless wall shear stresses $K(I) = \tau_w \bar{\tau}_w$ and A and B constants unique to the cross section in the Prandtl equation were calculated. A correlation for friction coefficient in an open channel is developed using measured wall shear stress distribution. An improved relation for the Manning coefficient on smooth rectangular channels is proposed.

Key words: Open channel flow, Manning friction coefficient, Wall shear stress.

Introduction

Although many mathematical relations for mean velocity distribution have been developed for pressurized and free surface flows, there are still many points related to turbulent flow that need to be studied. It is usually difficult to develop a formulation for free surface flows. In particular, there is a lack of literature on the formulation of friction resistance. Today, the most widely used formulation by hydraulic engineers is the Manning formula. Manning (1895) studied turbulent flow in open channels and proposed the following two formulae for average velocity distribution:

$$V = C' \sqrt{gRJ_e}\left[1 + \frac{0.22}{\sqrt{P_o R}}\left(R - 0.15P_o\right)\right] \qquad (1)$$

$$V = CR^{2/3}J_e^{1/2} \qquad (2)$$

where C' and C are constants, P_o is the atmospheric pressure, R is the hydraulic radius, J_e is the hydraulic gradient, g is the acceleration due to gravity and V is the mean velocity.

Although Manning suggested the use of Eq. (1), because of its simplicity most researchers have instead used Eq. (2) (Yen, 1992). The C coefficient in Eq. (2) is expressed as

$$C = 1/n \qquad (3)$$

The parameter n in Eq. (3) called the is Manning coefficient. There are wide disagreements between researchers on the value of n and extensive research on its determination is ongoing.

Rouse (1938) and Keulegan (1938) were among the first researchers who worked on the value of n. They proposed a value considering friction factor f in the Weisbach equation and found that C' in (1) was proportional to $R^{1/6}$ leading to Eq. (2). King (1918), Powell (1949), Chow (1959), Henderson (1966), Dooge (1991), Yen (1992), Çıray (1999), Bilgil (2000) and several other researchers have proposed values for n after considering different aspects of flow conditions.

On the other hand, knowledge of wall shear stresses is important in many engineering applications such as in the design of canals, calculation of velocity distribution and sedimentation studies in open channels.

In the literature, there are few methods to calculate velocity and the wall shear stress distribution in open channels. Çıray (1970) calculated wall shear stresses distribution for turbulent flows in a rectangular channel using experimental velocity measurements and gave a relation for the velocity of any point in the cross section. Assuming logarithmic velocity distribution along perpendiculars to a wetted perimeter, Çıray proposed the following relation for velocity distribution

$$\frac{V^+(y,z)}{K^{1/2}(z)} = A + B\ln\left[K^{1/2}(z)Y^+(z)\right] \qquad (4)$$

where $V^+(y,z)\left(= U(y,z)/U_\tau\right)$, $Y^+(y,z)\left(= U_\tau y/\nu\right)$, $U_\tau\left(=\sqrt{\bar{\tau}_w/\rho}\right)$, and $K(z)=\frac{\tau_w(z)}{\bar{\tau}_w}$ represent dimensionless velocity at any point in the rectangular cross section of the uniform flow, dimensionless distance, wall shear velocity ($U_\tau(x,y)$) and local dimensionless wall shear stress respectively; A and B are unique constants for the wetted cross section, $U(y,z)$ is local velocity, y is the distance from the wall, ν is the kinematic viscosity of water, $\bar{\tau}_w$ and τ_w are mean and local wall shear stresses and ρ is fluid density. (See Figure 1). Performing a number of experiments, Çıray has proven the validity of Eq. (4) in open channels. He was also able to obtain the wall shear stress distribution as a by-product.

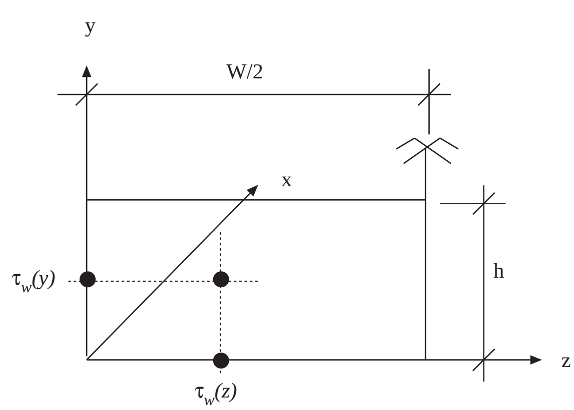

Figure 1. Coordinate system of rectangular channel uniform steady flow

Extensive research has been performed by Coles (1956), Nezu and Nakagowa (1984), Sarı (1987), Tominaga *et al.* (1989), Alkışlar (1993), Ardıçlıoğlu (1994), Knight *et al.* (1994) and Çıray (1995, 1999) on the calculation of velocity distribution in turbulent open channel flows.

The wall shear stress distribution in a planar and axially symmetric flow is uniform along the wetted perimeter and shear stress can be calculated from force balance. However, for rectangular channels with a finite aspect ratio after a certain W/h ratio ($W/h < 10$ in narrow channels), flow in the channel becomes three-dimensional and wall shear stress is not uniformly distributed due to the free surface and existence of secondary flows. Therefore, the calculation of the distribution of wall shear stress in a rectangular open channel is complex.

The purpose of this study is to develop a correlation for the friction coefficient for the turbulent flow in narrow open channels ($W/h < 10$). Therefore, an experimental set up is designed and the local shear stresses are determined from velocity measurements using different methods.

Experimental Set Up

The experimental studies were performed in a channel in Erzincan Vocational School of Atatürk University, Erzincan, Turkey (Figure 2). The channel has an adjustable base slope range of ±2%. The measurements in this study have been performed by keeping the channel base slope values between 0.0002 and 0.09 and having uniform flows. The channel has a rectangular cross section 0.21 m wide and 9 m in flow length. Since the walls and the base of the channel are made of glass, the channel surfaces were assumed to be smooth. At the beginning of the channel, there was a water inlet section of about 0.9 m in length and it is connected to the water reservoir. In order not to have an M2 profile, small stones and a screen were placed in the water inlet section. In this way, water was sent to the channel uniformly. The water discharge was adjusted by means of a flowmeter, of 1% sensitivity. Along the channel, in order to ensure uniform depth, different measuring devices were placed at four locations. By means of a lid at the end of the channel, a uniform flow was obtained. Furthermore, local flow velocity measurements were obtained by means of a pitot tube (Figure 3) placed 5 m away from the beginning of the channel inlet. A mechanism was designed to allow the pitot tube to move horizontally and vertically to any location in the channel. The pressure differences at the manometer, which was connected to the pitot tube, were obtained with a sensitivity of 0.1 mm pressure head. Average pressure differences read on manometers were recorded to a computer 20 times minute. The measurements were performed for each cm^2starting 0.5 cm from free surface. On the other hand, local velocity measurement points for different water level are given in Figure 4 in detail. The nearest measurement point was 0.5 cm away from the boundary wall. The experiments have been performed at 20 °C water temperature ($\nu = 1.01 * 10^{-6}$ m^2/s). During the experiments, the channel width (W) was constant. Nine different W/h cases with different channel slope values were tested.

The purpose of measurements is to calculate wall shear stress. The measured data were employed in a logarithmic average velocity equation proposed by Prandtl (1935) for two-dimensional turbulent flow and revised by Çıray (1970). The results of all the experimental conditions are summarized in Table 1.

Method of Shear Stress Calculation after Çıray

The average wall shear stress for a uniform flow with a free surface is calculated from the following equation:

$$\bar{\tau}_w = \gamma R J_e \tag{5}$$

where γ is the specific weight of water. Then the local wall shear stress at any point on the wall can be calculated as $\tau_w = \bar{\tau}_w K$, where K is the dimensionless wall shear stress parameter.

Table 1. Experimental data.

W/h ratio	Experiment number	Reynolds number interval ($Re = 4VR/\nu$	Discharge interval (l/s)
10.5	4	7920 – 39604	0.5 - 2.5
5.25	11	9567 – 109252	0.7 - 9
3.5	13	14368 – 168017	1.2 - 19
2.33	13	13205 – 172632	1.3 - 20
1.75	14	13082 – 167224	1.5 - 24
1.4	13	11648 – 147501	1.5 - 24
1.17	9	15285 – 132013	2.2 - 24
1	8	18859 – 119440	Mar-24
0.84	8	16734 – 105982	Mar-24

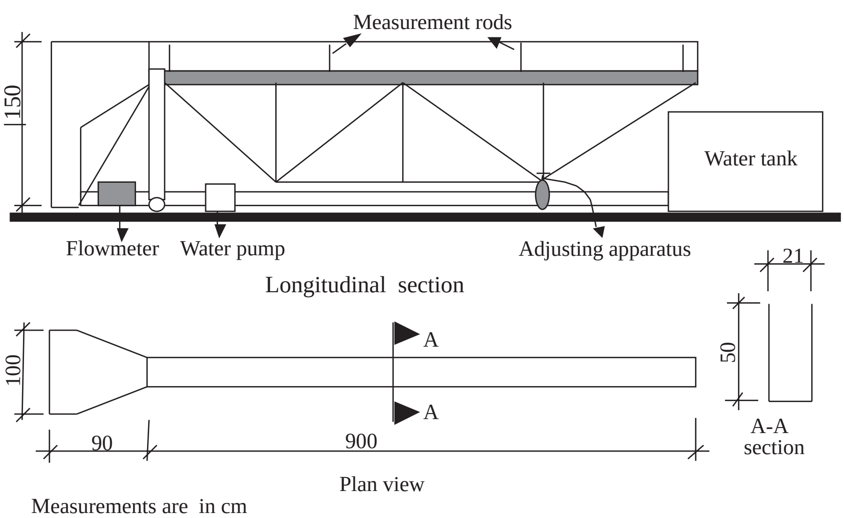

Figure 2. Schematic sketch of the experimental set up.

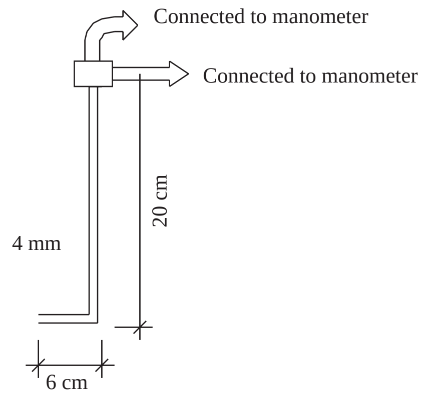

Figure 3. Pitot tube used in the experiment.

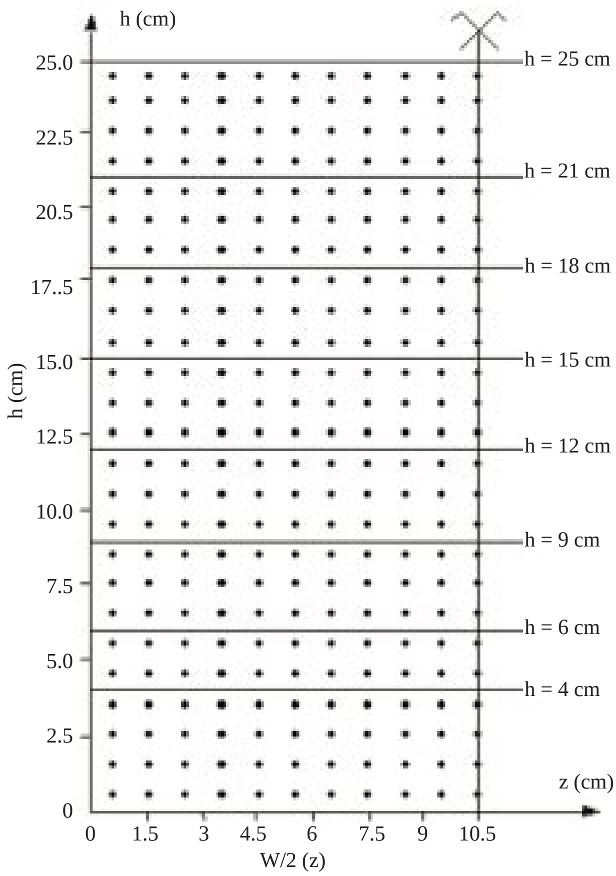

Figure 4. Local velocity measurement points for different water levels.

The compatibility equation is

$$\oint K(z)dz = L \tag{6}$$

The circle in the integral sign represents the integration over the wetted perimeter of the rectangular duct whose length is L. Eq. (6) is discretized and dz is scaled with respect to the L. Then Eq. (6) becomes

$$\sum_{i=1}^{N} K(I)\Delta z(I) = 1 \tag{7}$$

where Δz is the elementary distance on wetted perimeter and Eq. (4) assumes the form

$$\frac{V^{+}(I,J)}{K^{1/2}(I)} = A + B\ln\left[K^{1/2}(I)Y^{+}(I,J)\right] \tag{8}$$

Equation (8) is written twice for each measurement point (I, J), which means $I\rightarrow 1$ to N and $J\rightarrow 1$ to $M(I)$ where N shows the number of points on a wetted perimeter that $K(I)$ will be calculated and $M(I)$ are the number measurement points on each perpendicular (Çıray, 1999).

Therefore, the system of a non-linear equation of type 8 to be solved is $\sum_{I=1}^{N} M(I)$ to which Eq. (7) has to be added. Therefore the total number is $\sum_{I=1}^{N} M(I) + 1$. The number of unknowns is $N + 2$ where the last two correspond to A and B in Eq. (8).

These equations were solved via an iterative process. The basic idea comes from regression analysis, where for a given iteration the sum of the squares of the derivatives needs to be minimized. For the application of this technique, the number of equations must be at least equal to or greater than the number of unknowns. The details of this technique may be found in Çıray (1971).

Experimental Results

The distribution of dimensionless wall shear stress for ranges of water depth are shown in Figure 5. The connected points represent the average wall shear stress. The wall shear stresses were calculated only for half of the channel due to the symmetry at the channel axis. According to Çıray (1970), the wall shear stress at the corners and free surface is zero. Therefore, the points can be connected at these locations. The maximum value of average dimensionless shear stress was calculated as 1.15 to 1.24 for different W/h values at the channel axis. The values of maximum dimensionless wall shear stress on the vertical wall of the channel for different W/h values varied between 1.022 and 1.082.

The equations for the mean dimensionless wall shear stress were obtained for the vertical and horizontal walls of the channel. The power law relations obtained from the experimental data for the distribution of dimensionless wall shear stress can be expressed as follows:

For the vertical walls shown in Figure 5

$$K(I)_d = 0.7175y^{0.1617} \tag{9}$$

For the bottom wall of channel shown in Figure 5

$$K(I)_T = 0.6798z^{0.2744} \tag{10}$$

where y and z are the spatial coordinates shown in Figure 5. Using $X^+ = \frac{z}{W/2}$ and $K(I)_T = C_T X^{+^{n_T}}$ relations, the distribution of dimensionless wall shear stress at the bottom wall of the channel can be rewritten as

$$K(I)_T = 1.2958X^{+0.2744} \tag{11}$$

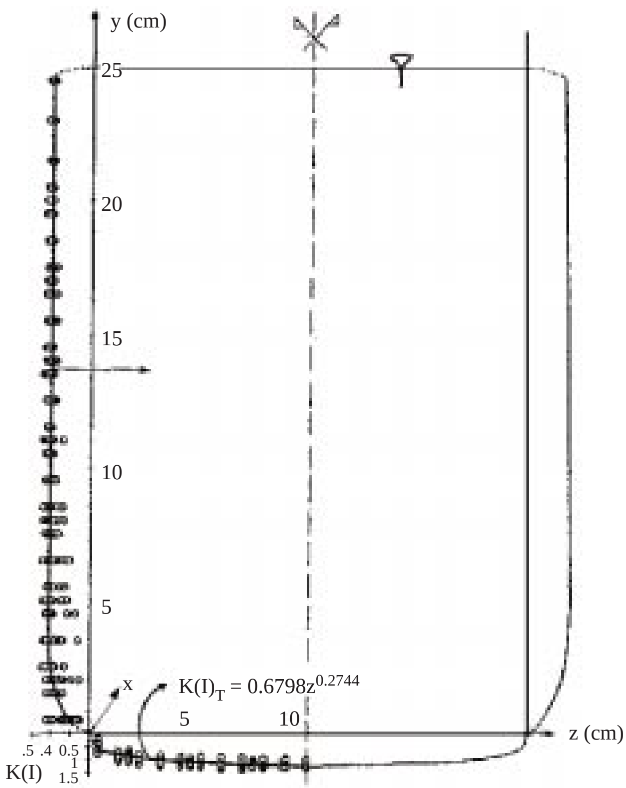

Figure 5. Measured distribution of dimensionless wall shear stress.

Logarithmic Law Constants

Variations of constants A and B given in Eq. (8) depending on discharge (Q) are shown in Figure 6. The constants A and B were calculated from the Prandtl logarithmic velocity distribution equation using experimentally measured data. The measurements were performed in a fully turbulent region. Figure 6 shows that at lower values of W/h, the constant A values decrease. This change is expected since at lower values of W/h the secondary flow cells increase. Almost a constant value of B except small oscillations supports the mixing length theory of Prandtl (1935).

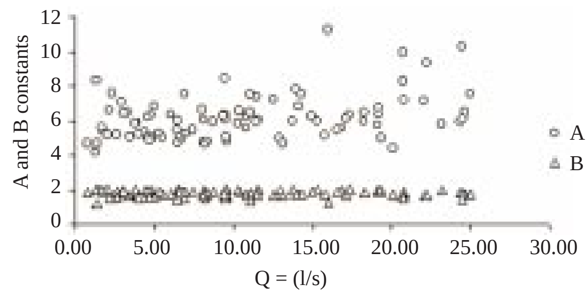

Figure 6. Variation of A and B constants with Q according to logarithmic velocity distribution.

The average values of A vary between 4.9661 and 8.0493 at different W/h values, while B values vary between 1.6630 and 1.8522. The value of B can be assumed to be a constant in the range of W/h considered. Therefore, the value of B in logarithmic velocity law can be taken as $B = 1.7559$, an average of all data. However, a similar assumption for the value of A would not be correct. The change in the value of A was observed to be exponential between $2 < W/h < 6$. The variation of A and B with W/h is summarized in Figure 7 as a function of aspect ratio.

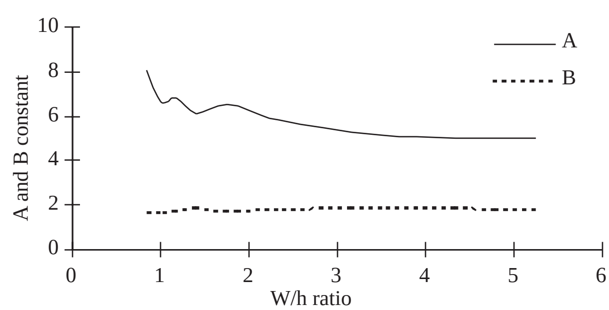

Figure 7. The change of constants A and B as a function of W/h.

Friction and Energy Losses

In wide channels, A becomes constant and since its variation is attributed to secondary currents the effect of such currents do not seem to be important in very wide channels (Figure 7). As the W/h ratio decreases, secondary flow is observed and the flow resistance is affected (Chow, 1959). The findings of this study support this opinion.

The most widely used formula for open channel flows is the Manning formula and there is still an ongoing discussion on the generality of parameter n in this formula. Therefore, a discharge formula proposed by Çıray (1999), which includes the effects of a couple of complex factors such as the secondary flows, irregular wall shear stress and W/h ratios, was used in discharge calculations. Çıray's formula, obtained from logarithmic velocity distribution (Formula 4), can be expressed as follows:

$$\frac{Q}{W\nu} = h^+ \frac{2}{n_T+2} C_T^{1/2} \left\{ A + B \ln h^+ + \frac{B}{2} [\ln C_T - (n_T+2)] \right\} \quad (12)$$

where W is the width of the channel, h is the depth of water, ν is the kinematic viscosity, $h^+ \left(= hU_\tau/\nu\right)$ is a dimensionless depth, $K(I) \left(= C_T X^{+^{n_T}}\right)$ represents the wall shear stress on the bottom of the channel, and C_T and n_T are constants. The distance z has become dimensionless by $X^+ = \frac{z}{W/2}$.

The n parameter in the Manning formula was obtained by equating Çıray's and Manning's discharge formulae as expressed by Bilgil (1998)

$$\frac{1}{n_m} = \frac{g^{1/2}}{R^{1/6}} \frac{2C_T^{1/2}}{n_T+2} \left\{ A + B \ln \left[\left(\frac{C_T}{\exp(n_T+2)} \right)^{1/2} h^+ \right] \right\} \quad (13)$$

where, n_m corresponds to parameter n in the Manning formula. This equation was developed with the assistance of the velocity distribution formed according to effects in open channels, such as irregular boundary shear stress, secondary flows, and W/h ratio. Therefore, the roughness coefficient in open channels is determined more realistically.

The mean values obtained according to the W/h ratio to be used in Eq. (13) are provided in Table 2.

According to a variety of Reynolds numbers shown in Figure 8, the distribution of the roughness coefficient of these are n=0,011 by Manning, $n_f = R^{2/3} J_e^{1/2} / V$ from flowmeter and n_m determined from Eq. (13). As seen in Figure 8, the distributions of n_f and n_m fit very well.

Table 2. The parameters, to be used in Eq. (13), according to the different W/h ratios.

Mean W/h Ratio	A Constant	B Constant	C_T	n_T
5.25	4.9661	1.7815	1.2958	0.2744
3.50	5.1245	1.8522	1.2958	0.2744
2.33	5.8623	1.8091	1.2958	0.2744
1.75	6.5410	1.7474	1.2958	0.2744
1.40	6.1041	1.8508	1.2958	0.2744
1.17	6.8244	1.7652	1.2958	0.2744
1.00	6.6726	1.7005	1.2958	0.2744
0.84	8.0493	1.6630	1.2958	0.2744

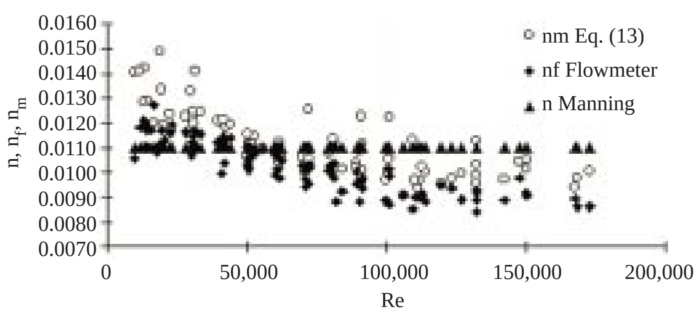

Figure 8. Variation of manning coefficients n, n_f and n_m with reynolds number.

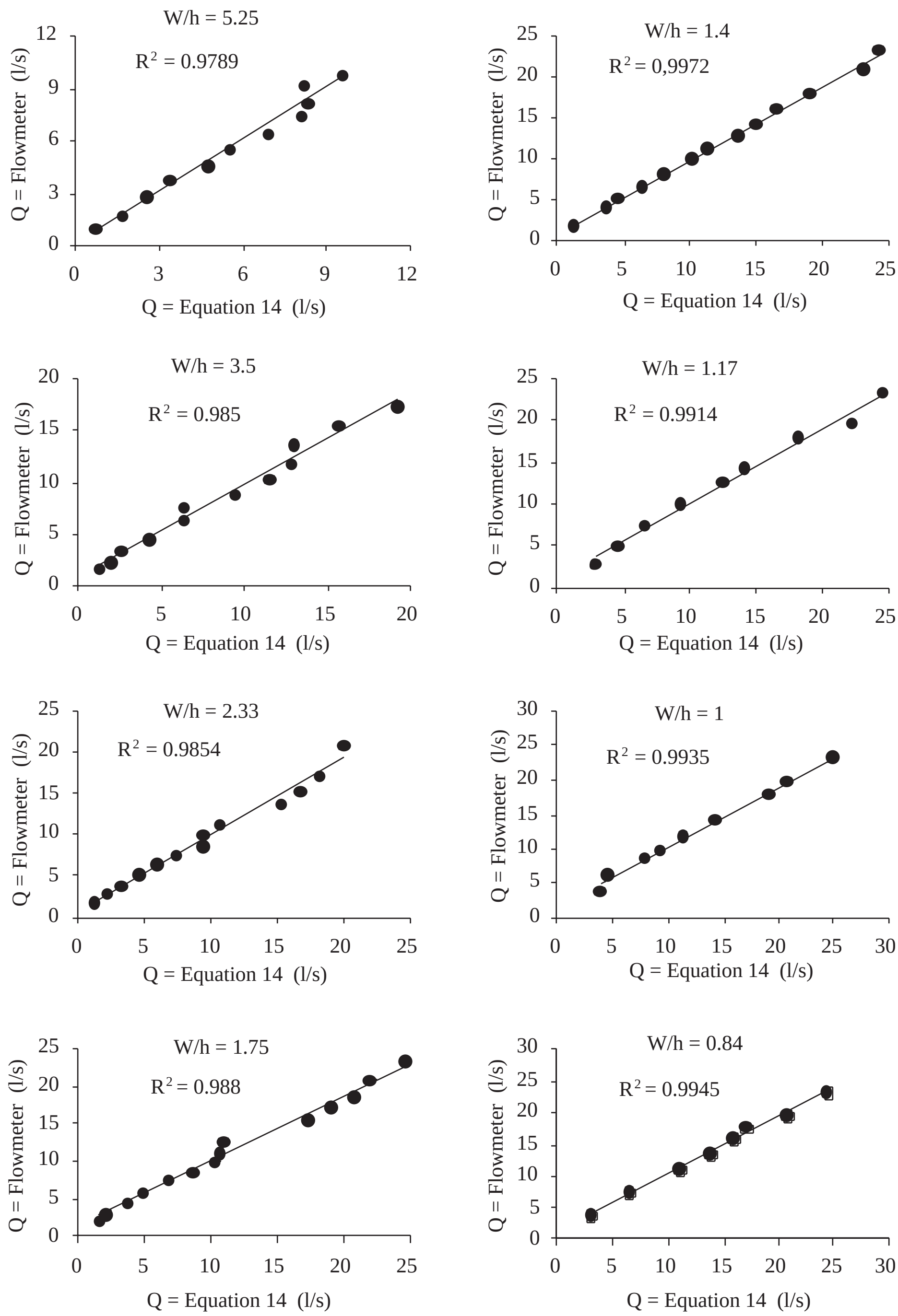

Figure 9. Comparison of discharges obtained from Eq. 14 and measured data.

Discussion

The wall shear stress is found to vanish at the corners of cross section including the point the water surface makes contact with the atmosphere. Non-uniform wall shear stress distribution was reported by several researchers, because the wall shear stress becomes zero at free surfaces (Çıray, 1970, 1995, 1999; Sarı, 1987; Alkışlar, 1993). In addition, the formation of secondary flow cells in the flows which are important for small W/h values, also affects the value of A. Therefore, the analysis of the free surfaces are usually more complex than that of pressured flows.

As seen in Figure 7, it is difficult to find a relation between W/h values and A. Rao (1969) and Myers (1982) studied the relation between W/h and the Reynolds number; however, they reached no conclusive result. The findings of Rao and Myers showed that uncertainty would begin when W/h ratios were smaller than 6 and 4 respectively. The present results show that the uncertainty may begin when the W/h ratio is smaller than 2. The experimental findings indicate that the friction loss coefficient became more complex when channel geometry was changed to square.

In this study, the roughness coefficient was studied depending on Reynolds number and W/h ratios. In previous studies, the friction coefficient was generally studied at large W/h ratios. Since the secondary flows are important at lower W/h ratios, the experiments in this study were performed at low W/h ratios. The flow was observed to be three-dimensional at low W/h ratios and, therefore, logarithmic relation was assumed to be valid in whole channel depth.

Many researchers stated that the average friction factor in open channels is nearly 8% higher than that of pipes under similar conditions. Therefore, the usage of pipe flow equations in the calculation of friction factor may lead to significant errors in channel flows (Bilgil, 1998).

There is no simple relation between the friction coefficient and Reynolds number and W/h ratios in the literature. It may be appropriate to calculate the friction coefficient in the Manning formula using Eq. (13). n represents wall shear stress, secondary flows and channel geometry. As seen in Figure 9, a high degree of correlation is evident between the discharge values calculated from Eq. (14) in which n_m is calculated from Eq. (13) and those read from the flowmeter.

$$Q = FV = F\frac{1}{n_m}R^{2/3}J^{1/2} \tag{14}$$

Conclusions

The major conclusions of this study can be summarized as follows:

- A numerical study based on experimental findings was carried out considering local friction factor and dimensionless shear stress distribution along the wetted parameter of a channel. The results obtained in this study are confirmed in the analytical model, which takes into account wall shear stress to represent velocity distribution in channel flows. It is also seen that the wall shear stress vanishes at the regions of wall corners and free surface at contact points with wall.

- A and B constants in the Prandtl logarithmic velocity distribution change with W/h ratios. The change in the value of B is not significant at different W/h ratios, with an average value of 1.7. The calculated value of B agrees very well with Prandtl's hypothesis. However, changes in the value of A are important and are influenced by friction factors. Although there is general agreement on the formation of the friction factor, a constant A value was not obtained in this study.

- The application of Eq. (13), which calculates frictional losses of uniform flow in rectangular smooth channels, may be extended to rough channels. The extension of this formulation can be sought in channels that have triangular, trapezoid, circular etc. cross-sectional areas.

Symbols

C, C'	constant
C_T	1.2958 constant
D	diameter of pipe
F	flow area
g	acceleration due to gravity
h	depth of water
J_e	hydraulic gradient
K	$\tau_w/\bar{\tau}_w$ dimensionless shear stress
$K(I)_d$	wall shear stress at side wall
$K(I)_T$	wall shear stress at the bottom
L	wetted perimeter
n	Manning frictional resistance constant
n_f	roughness coefficient from flowmeter
n_m	roughness resistance constant [Eq. (13)]
n_T	0.2744 constant
P	wetted perimeter
P_o	vatmospheric pressure
R	hydraulic radius
Re	Reynolds number
U	local velocity
U_τ	wall shear velocity
V	mean velocity
x, y, z	the spatial coordinates shown in Figure 1
y_i	distance from the wall
W	channel width
γ	specific weight of water
μ	dynamic viscosity
ν	kinematic viscosity
$\bar{\tau}_w$	mean wall shear stress

τ_w local wall shear stress
ρ fluid density

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