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# Statistical Analysis of Fracture Strength of Composite Materials Using Weibull Distribution

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**Abstract**

In this study, the fracture strength of a carbon-epoxy composite plate has been statistically analysed by Weibull distribution. Nineteen tension tests were carried out. The results obtained vary between 439 MPa and 552 MPa. Using Weibull distribution, the materials' reliability, in other words, the probability distribution according to which the material will fail was obtained.

**Key Words:** Weibull distribution, Rapture strength, Composite materials, Carbon-epoxy

## Introduction

Composite materials are not isotropic and, therefore, have different mechanical properties in different directions. In addition to this, they present varying strengths due to their internal structure, which means that there is no specific strength value to represent their mechanical behaviour. This leads to the necessity of employing statistical analyses for their safe utilisation in design and manufacturing. One of these analyses is the Weibull distribution, which has recently been used for the determination of static and dynamic mechanical properties of ceramics and metal-matrix, ceramic-matrix, and polymer-matrix composites.

Weibull distribution has the capability to model experimental data of very different characters. This is one of the reasons for its wide utilisation nowadays. In recent years, research papers and books dealing with the historical development and application of this statistical method have been published (e.g. Hallihan 1993, Dodson 1994). In his book, Dodson described the developments regarding the estimation approaches for Weibull distribution parameters. Since then, there has been considerable work on new application areas and improved estimation approaches. For example, Barbero et al. (2000) applied this analysis in modelling the mechanical properties of composite materials and suggested the Weibull distribution as a practical method in the determination of 90% and 95% reliability values used in composite material mechanics.

The variation of the fracture strength of a certain carbon-epoxy composite plate has been modelled using Weibull distribution. Nineteen tension tests were performed and using the test data, the corresponding Weibull distribution was determined. Finally, the reliability of the composite material in terms of its fracture strength was presented in graph form.

## Experiments

The composite specimens used in the experiments were prepared from carbon-epoxy sheets with $(0°)_3$ configuration, 0.89 mm thickness, and 295 g/m$^2$ weight. The mechanical properties presented in Table 1 were obtained by means of the strain gauge method. The tests were carried out according to ASTM D3039 standard (ASTM D3039, 1976) on an Instron 8516+ universal testing centre. A crosshead speed of 1.33 mm/min was used and room temperature conditions were present during the tests. The dimensions of the test specimens are shown in Figure 1 and the fracture strength values obtained are given in Table 2.

**Table 1.** Mechanical properties of the carbon-epoxy composite plate.

| $E_1$ [GPa] | $E_2$ [GPa] | $G_{12}$ [GPa] | $\nu_{12}$ | Carbon [% ] |
|---|---|---|---|---|
| 40.74 | 39.6 | 4.62 | 0.25 | 28 |

**Weibull distribution**

Weibull distribution is being used to model extreme values such as failure times and fracture strength. Two popular forms of this distribution are two- and three-parameter Weibull distributions. The (cumulative) distribution function of the three-parameter Weibull distribution is given as follows (Ghosh 1999):

$$F(x;a,b,c) = 1 - \exp\left(-(\tfrac{x-a}{b})^c\right), \quad a \geq 0,\ b \geq 0,\ c \geq 0, \tag{1}$$

where $a$, $b$, and $c$are the location, scale and shape parameters, respectively. When $a = 0$ in Eq. (1) the distribution function of the two-parameter Weibull distribution is obtained. The three-parameter Weibull distribution is suitable for situations in which an extreme value cannot take values less than $a$. In this study, the two-parameter Weibull distribution, which can be used in fracture strength studies, will be considered. The distribution function in this case can then be written as follows:

$$F(x;b,c) = 1 - \exp\left(-(\frac{x}{b})^c\right), \quad b \geq 0,\ c \geq 0 \tag{2}$$

In the context of this study, $F(x;b,c)$, represents the probability that the fracture strength is equal to or less than $x$. Using the equality $F(x;b,c) + R(x;b,c) = 1$ , the reliability $R(x; b, c)$, that is, the probability that the fracture strength is at least $x$, is defined as (Dodson 1994)

$$R(x;b,c) = \exp\left(-(\frac{x}{b})^c\right), \quad b \geq 0,\ c \geq 0. \tag{3}$$

The parameters $b$ and $c$ of the distribution function $F(x;b,c)$ are estimated from observations. The methods usually employed in the estimation of these parameters are method of linear regression, method of maximum likelihood, and method of moments (Hallinan 1993, Dodson 1994, Taljera 1981). Among these methods, use of linear regression goes back to the days when computers were not available: the linear regression line was fitted manually with the help

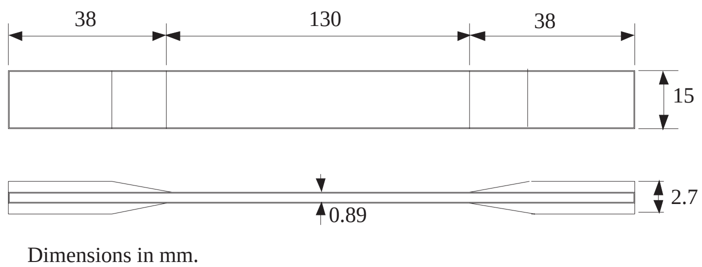


**Figure 1.** Test specimen and its dimensions.

**Table 2.** Fracture strength values from tension tests.

| Test No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Fracture strength [MPa] | 532.7 | 502.5 | 442 | 473 | 519 | 502.7 | 477 | 510 | 522 | 552 |
| Test No. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | |
| Fracture strength [MPa] | 522 | 439 | 513.6 | 497.5 | 521.6 | 450.9 | 476.5 | 507.3 | 463.5 | |

of Weibull graph papers. Linear regression is still common among practitioners, and will be used for parameter estimation in this paper. However, software programs with statistical abilities such as MS Excel$^{TM}$, SPSS$^{TM}$ and Microcal Origin$^{TM}$ have replaced the Weibull graph papers.

### Method of linear regression

This method is based on transforming Eq. (2) into $1-F(x;b,c) = \exp\left(-(\frac{x}{b})^c\right)$ and taking the double logarithms of both sides. Hence, a linear regression model in the form $Y = m\ X + r$ is obtained:

$$\ln\left[\ln\left(\frac{1}{1-F(x;b,c)}\right)\right] = c\ln(x) - c\ln(b) \qquad (4)$$

$F(x;b,c)$ is an unknown in (4) and, therefore, it is estimated from observed values: order $n$ observations from smallest to largest, and let $x_{(i)}$ denote the $i$th smallest observation ($i$=1 corresponds to the smallest and $i$= $n$ corresponds to the largest). Then a good estimator of $F(x_{(i)};b,c)$ is the median rank of $x_{(i)}$:

$$\hat{F}(x_{(i)};b,c) = \frac{i-0.3}{(n+0.4)} \qquad (5)$$

When linear regression, based on least squares minimisation, is applied to the paired values ($X$, $Y$)=$\left(\ln(x_{(i)}), \ln\left[\ln\left(\frac{1}{1-\hat{F}(x_{(i)};b,c)}\right)\right]\right)$ for the model in Eq. (4), the parameter estimates for $b$ and $c$ are obtained.

The results obtained from the experiments in the present work are given in Table 2. In order to compute $b$ and $c$, first, they are ordered from the smallest to the largest and ($X$, $Y$) values are computed. Then applying linear regression to these ($X$, $Y$) values, the linear regression model with the regression line in Figure 2 is obtained. The first point in Figure 2 does not appear to fit the line well. However, this is an expected situation in the method of linear regression; among consecutive ($Y_{(i)}$, $Y_{(i+1)}$) pairs, ($Y_{(1)}$, $Y_{(2)}$) has the largest absolute difference. The slope of the line is 17.44, which is the value of the shape parameter $c$.

A $c < 1.0$ indicates that the material has a decreasing failure rate. Similarly a $c = 0$ indicates constant failure rate and a $c > 1.0$ indicates an increasing failure rate. The $b$ value is computed as $b$ = 510.76 using the point the line intersects the $Y$ axis (= -108.77) in $b = e^{\left(-\frac{Y}{c}\right)}$. Therefore, $c$= 17.44 indicates that the material tends to fracture with higher probability for every unit increase in applied tension. The scale parameter $b$ measures the spread in the distribution of data. As a theoretical property $R(b; b, c) = 0.368$. Therefore, $R(510.76; 510.76, 17.44) = \exp\left(-(\frac{x}{b})^c\right) = 0.368$, that is 36.8% of the tested specimens have a fracture strength of at least 510.76 MPa.

The plot of $R(x;b,c)$ is shown in Figure 3. The reliability curve in Figure 3 shows that fracture strength values roughly less than or equal to 400 MPa will provide high to bereliability. For a more certain assessment, consider 0.90 and 0.95 reliability levels. When these values are put as $R(x;b,c)$ in Eq. (3) and the equation is solved for $x$, the fracture strength values 448.92 and 430.79 are obtained respectively. In other words, this material will fracture with 0.90 probability for a tension of 448.92 MPa or more, and similarly will fracture with 0.99 probability for a tension of 430.79 MPa or more.

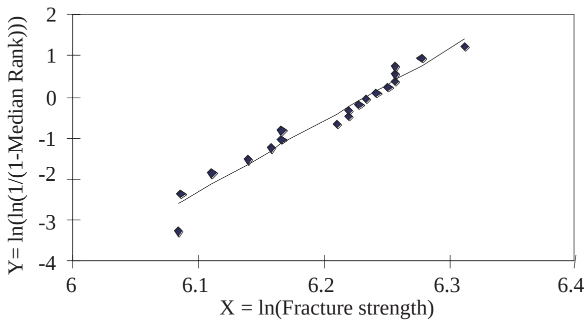


**Figure 2.** Regression line.

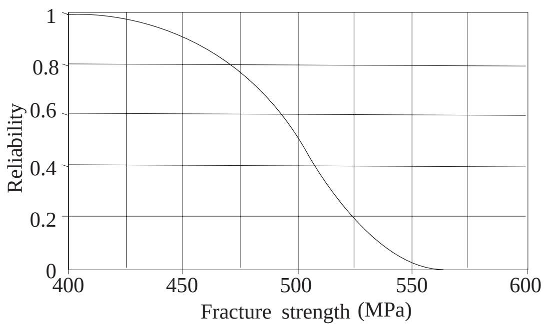


**Figure 3.** Weibull reliability distribution.

### Conclusions

Composite materials are generally used in strategically important applications. One of these is the carbon-epoxy composite and the fracture strength variation in the tension of this composite has been modelled using Weibull distribution. The study questions and then rejects the assumption that the fracture strength of composite materials be taken as an average of the experimental results. In this respect, the Weibull distribution allows researchers to describe the fracture strength of a composite material in terms of a reliability function. It also provides composite material manufacturers with a tool that will enable them to present the necessary mechanical properties with certain confidence to end users.

Lastly, the Weibull distribution was employed here to model a strength property, but it can also be used in areas with similar uncertainties as described in this study.

## List of Symbols

| | | |
|---|---|---|
| $a$ | : | location parameter |
| $b$ | : | scale parameter |
| $c$ | : | shape parameter |
| $F(x; b, c)$ | : | distribution function |
| $R(x; b, c)$ | : | reliability function |
| $n$ | : | observation number |
| $\mathrm{x}_{(i)}$ | : | $i$th order statistic |

## References

Barbero, E., Fernandez-Saez, J., Navarro, C. "Statistical Analysis of the Mechanical Properties of Composite Materials", Composites: Part B: Engineering, 31, 375-381, 2000

Dodson, B., "Weibull Analysis", American Society for Quality,1994.

Ghosh, A., "A FORTRAN Program for Fitting Weibull Distribution and Generating Samples", Computers and Geosciences, 25, 729-738, 1999.

Hallinan, A., J. Jr, "A Review of the Weibull Distribution", Journal of Quality Technology, 25 (2), 85-93, 1993.

Taljera, R, "Estimation of Weibull Parameters for Composite Material Strength and Fatigue Life Data", ASTM STP 723, 291-311, 1981.

Tensile Properties of Fiber-Resin Composites, ASTM D 3039-76, American National Standard, 1976.