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Estimation of Uncertainty of Air Buoyancy Correction for Establishment of Primary Mass Standards

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Abstract

The most accurate value of air density is obtained by a calculation based on the equation using measured atmospheric parameters approved by the International Committee for Weights and Measures in 1981. When a 1-kg steel weight was calibrated against the prototype kilogram using the international expression for determining the density of air, experimental results showed that the estimated uncertainty of the air density was $1.45x10^{-4}$ mg.cm^{-3} and the uncertainty of air buoyancy was 11.3 μg.

Key Words: air density, air buoyancy correction, primary mass standard

Primer Kütle Standartlarının Oluşturulmasında Havanın Kaldırma Kuvveti Düzeltmesi Belirsizliğinin Tahmin Edilmesi

Özet

Yüksek hassasiyette hava yoğunluğunun değeri, atmosferik parametreler ölçülerek 1981 yılında Uluslararası Ölçüler ve Ağırlıklar Konferansında kabul edilmiş olan temel denklemden hesaplanarak belirlenir. Hava yoğunluğunun belirlenmesinde uluslararası tanımı kullanarak, 1kg paslanmaz çelik ağırlık ile kilogram prototipi karşılaştırıldığında, deneysel sonuçlara göre hava yoğunluğunun tahmini belirsizliği $1.45x10^{-4}$ mg.cm^{-3} ve havanın kaldırma kuvveti belirsizliğinin de 11.3μg olduğu belirlenmiştir.

Anahtar Sözcükler: hava yoğunluğu, havanın kaldırma kuvveti düzeltmesi, primer kütle standardı

1. Introduction

When steel weights (density, 7900 - 8000 kg.m^{-3}) of high accuracy are calibrated against the prototype kilogram (density, 21500 kg.m^{-3}), the most precise balances, with a measurement precision reaching 1g, are used. In order to attain such a high level of accuracy, an air buoyancy correction with a relative accuracy of 1x10^{-5} must be made. Since the air buoyancy correction is determined by the volume difference between the weights multiplied by the air density, both of the measurements need to be determined with the same accuracy.

In the application of air buoyancy correction, the BIPM (Bureau International Des Poids Et Mesures) and most national laboratories now use an internationally recommended formula for the determination of the density of moist air, ρ_A (Giacomo, 1982). The equation of the density of moist air requires input values for air temperature, pressure, relative humidity (or dew-point temperature) and mole fraction of carbon dioxide.

The experimental method and the numerical results are reported in the sections below.

2. Theoretical background

All weights are weighed in air, and they are always affected by air buoyancy. According to Archimede's principle, buoyancy is equal to the weight of air which is displaced by the weight of the same volume. It is well known that weight is equal to the product of mass and gravitational acceleration. In the determination of the mass of a weight using an equal arm balance, the calculation formula is as follows (Xisan et al., 1995):

$$l_1(m_R g_{l_1} - V_R \rho_A g_{l_1}) = l_2(m_T g_{12} - V_T \rho_A g_{12}) \quad (1)$$

Where,

l_1, l_2;	the lengths of the left and right arms of the balance
m_R, m_T;	the masses of the reference and test bodies
V_R, V_T;	the volumes of the reference and test bodies
ρ_A;	air density during weighing

We assume that the balance has equal arms and no errors due to unequal arms exist in the balance beam; thus,

$l_1 = l_2$

$g_{11} = g_{12} =$ g is substituted into equation (1) and after simplification, we obtain

$$m_T = m_R - \underbrace{(V_T - V_R)\rho_A}_{B} \quad (2)$$

$$B = (V_T - V_R)\rho_A \quad (3)$$

B is defined as the correction of air buoyancy. In the comparison of a 1-kg steel weight with the prototype kilogram, the volumes of the steel weight and the prototype kilogram must be known. In most cases, the hydrostatically determined volumes of weights are given at 20 °C during weighing, and the volumes are obtained from the relation (Schwartz, 1991);

$$V_T = V_{20}(1 + \alpha_t(t - 20)) \quad (4)$$

where;

V_T	;	the volume of the weight at $t°C$
V_{20}	;	the volume of weight at $20°C$
α_{tt}	;	coefficient of volume expansion

The air density is determined during the weighing for the calculation of the air buoyancy correction. For the determination of the air density, in this operation a high level of accuracy ($< 5 \times 10^{-4}$) is also required.

2.1. Equation for the determination of the density of moist air

In general, the density of air, ρ, is not determined directly but is calculated by means of an equation (Giacomo, 1982) and by taking into account the experimental conditions. Consider a volume V of gas at pressure p and temperature T, containing an amount of substance n. If one first considers the gas to be perfect, one has the relation

$$pV = nRT \quad (5)$$

where R is the molar gas constant. For a real gas, it is necessary to take account of its compressibility factor Z. Then,

$$pV = nZRT \quad (6)$$

If we designate by m and M the mass of the gas and its molar mass, respectively, its density ρ is given by

$$\rho = pM(ZRT)^{-1} \quad (7)$$

One may apply this relation to moist air, which consists of a mole fraction x_v of water vapour, of molar mass M_v, and a mole fraction (1-x_v) of dry air of molar mass M_a; in these conditions one has

$$M = M_a[1 - x_v(1 - M_v/M_a)] \quad (8)$$

$$\rho_A = pM_a[1 - x_v(1 - M_v/M_a)]/ZRT \quad (9)$$

Equation (9) forms the basis for calculating the density of moist air, where p is the pressure (in Pa); T, the thermodynamic temperature (in K, according to ITS 90); x_v, the mole fraction of water vapour; M_a, the molar mass of dry air (in kg/mol); M_v, the molar mass of water ($M_v = 18.015x10^{-3}$ kg/mol); R, the molar gas constant (R = 8.31441 J $mol^{-1}.K^{-1}$); and Z is the compressibility factor which is calculated from the following equation:

$$\begin{aligned} Z = 1 - p/T[&a_0 + a_1 t + a_2 t^2 \\ &+(b_0 + b_1 t)x_v + (c_0 + c_1 t)x_v^2] \\ &+p^2/T^2(d + ex_v) \end{aligned} \quad (10)$$

where,

$a_0 = 1.62419x10^{-6}K.Pa^{-1}$ $\quad$ $b_1 = -2.589x10^{-5}Pa^{-1}$
$a_1 = -2.8969x10^{-8}Pa^{-1}$ $\quad$ $c_0 = 1.9297x10^{-4}K.Pa^{-1}$
$a_2 = 1.0880x10^{-10}K^{-1}.Pa^{-1}$ $\quad$ $c_1 = -2.285x10^{-6}Pa^{-1}$
$b_0 = 5.757x10^{-6}K.Pa^{-1}$ $\quad$ $d = 1.73x10^{-11}K^2.Pa^{-2}$
t = temperature (measured in °C) $\quad$ $e = -1.034x10^{-8}K^2.Pa^{-2}$

It is assumed that M_a is constant except for local variability of the mole fraction of carbon dioxide. The latter is further assumed to be exactly opposite to the variability of the mole fraction of oxygen, leading to an auxiliary equation for M_a:

$$M_a = [28.9635 + 12.011(X_{CO_2}0.0004)]x10^{-3}kg/mol \quad (11)$$

where X_{CO_2} is the mole fraction of carbon dioxide (in ppm). The mole fraction of water vapour x_v is not measured directly. Instead, it is derived either from a measurement of the relative humidity h or from a measurement of the dew-point temperature t_r. The relative humidity was measured to calculate the mole fraction of water vapour x_v.

$$x_v = hf(p,t)\frac{p_{sv}(t)}{p} = f(p,t_r)\frac{p_{sv}(t_r)}{p} \quad (12)$$

The enhancement factor, f, is calculated from

$$f = \alpha + \beta p + \gamma t^2 \quad (13)$$

where
$\alpha = 1.00062, \gamma = 5.6x10^{-7}K^{-2}, \beta = 3.14x10^{-8}Pa^{-1}$

The saturation vapour pressure of water, p_{sv}, is calculated from the auxiliary equation

$$p_{sv} = 1Pax\exp\left(AT^2 + BT + C + \frac{D}{T}\right) \quad (14)$$

where p_{sv} is the saturation vapour pressure of water (in Pa); T, the thermodynamic temperature; and A= 1.2811805x10$^{-5}K^{-2}$; B= -1.9509874x10$^{-2}K^{-1}$; C=34.04926034; and D = -6.3536311x10^3 K.

3. Experimental conditions

The balance used for the comparison of the prototype kilogram with a 1-kg steel weight is a single pan electromagnetically compensated mass comparator with a capacity of 1 kg (AT 1006) and readability of 1 μg. The standard deviation of the reading is 1 μg. This balance was placed into a temperature-controlled cabin made by UME (National Metrology Institute of Turkey). The pressure, temperature, humidity and carbon dioxide content of the air were recorded before each cycle of the weighing. The technical characteristics of the environmental parameters used for the determination of air density are summarised in Table 1. The double substitution method (R, Reference; T, Test; R-T-T-R) was used during the weighing.

Table 1. The technical characteristics of the environmental parameters

MEASURING INSTRUMENT	MANUFACTURER/TYPE	UNCERTAINTY ($\pm$ 1 σ)
Pressure	Desgranges et Huot/RPM1	10Pascal
Humidity	Vaisala / HMP 36 E	1%
Temperature	Thermometrics / L& N Model 8163 Platinum-resistance thermometer	10mK
CO_2 content	Hartman & Braun /URAS 10 E	20ppm

4. Experimental results

Table 2 shows the determined air density for each weighing cycle. The air density was calculated according to Eq.(5) for each cycle. As can be seen in Table 2, the following results were obtained from the measurements:

- The mean air density, ρ_A, was, 1.1728 mg.cm^{-3} (from the measurement of 18 cycles)

- The standard deviation of the air density was 9.44×10^{-5} mg.cm^{-3}
- The estimated uncertainty of the air density was 1.45×10^{-4} mg.cm^{-3} (at 1 σ level)

Table 3 shows the determined air buoyancy correction for each measurement cycle as obtained from Eq.(3). For the volume values, the volume expansion correction was made by using Eq.(4) because of temperature deviations from 20 °C. As can be seen in Table 3, the following results were obtained from the measurements;

- The mean air buoyancy correction , B, was 91.0501 mg (from the measurement of 18 cycles)
- The estimated uncertainty of the air buoyancy was the 11.3 μg (at 1 σ level)

Table 2. Measured environmental parameters and calculated air density values

Temperature (°C)	Pressure (mbar)	Humidity (%)	CO_2 concentration (ppm)	Determined air density (mg.cm^{-3})
20.770	989.350	44.30	444	1.17298
20.770	989.279	44.34	440	1.17289
20.770	989.255	44.43	424	1.17286
20.770	989.265	44.53	420	1.17287
20.775	989.250	44.57	400	1.17282
20.775	989.179	44.80	396	1.17273
20.775	989.197	44.92	400	1.17276
20.775	989.175	45.01	404	1.17273
20.775	989.197	45.07	398	1.17275
20.775	989.200	45.10	402	1.17274
20.780	989.250	45.14	422	1.17281
20.780	989.289	45.21	412	1.17283
20.780	989.279	45.21	416	1.17282
20.785	989.260	45.26	416	1.17279
20.790	989.215	45.30	412	1.17272
20.790	989.196	45.31	406	1.17268
20.795	989.170	45.33	414	1.17265
20.795	989.117	45.33	402	1.17258
The laboratory conditions:	t:20°C±0.5°C	h:45%±5		
Mean value of the air density	1.17278 mg.cm^{-3}			
Standard deviation of the air density	9.44×10^{-5} mg.cm^{-3}			
Estimated uncertainty of the air density	1.45×10^{-4} mg.cm^{-3}			

Table 3. The values of determined air buoyancy correction and mean value of air buoyancy, B, correction and estimated uncertainty value and volume values of V_R, V_T

Determined air buoyancy (mg)	Volume values of prototype kilogram (V_R) (cm^{-3})	Volume values of steel weight (V_T) (cm^{-3})
91.0656	46.41652	124.05258
91.0590	46.41652	124.05258
91.0562	46.41652	124.05258
91.0570	46.41652	124.05258
91.0531	46.41653	124.05261
91.0462	46.41653	124.05261
91.0485	46.41653	124.05261
91.0462	46.41653	124.05261
91.0478	46.41653	124.05262
91.0470	46.41654	124.05264
91.0524	46.41654	124.05264
91.0540	46.41654	124.05267
91.0532	46.41654	124.05267
91.0510	46.41655	124.05270
91.0455	46.41655	124.05270
91.0424	46.41655	124.05273
91.0401	46.41655	124.05273
91.0346	46.41655	124.05273
$\bar{B}$:91.0501 mg	$\bar{V}_R$:46.4165 cm^{-3}	V_T:124.0526 cm^{-3}
	α_{20}:25,98x$10^{-6}K^{-1}$ (vol.expansion coefficient)	α_{20}:48x$10^{-6}K^{-1}$ (vol.expansion coefficient)
Estimated uncertainty of B:11.3μg		

5. Conclusion

In this work, errors in determining the density of air were examined by measuring the pressure, temperature, humidity and concentration of carbon dioxide of air, with use of the relevant measuring instruments of high accuracy and by applying the international expression. In addition, the value of air buoyancy correction and uncertainty of air buoyancy correction in the establishment of primary mass standards were determined. As a result, when a 1-kg steel weight was calibrated against the prototype kilogram using the international expression for determining the density of air, the relative uncertainty of air density was calculated to be 1.24x10^{-4} and the estimated uncertainty of buoyancy correction was 11.3 μg at a level of 1 σ.

When experimental results are compared with obtained experimental results in the establishment of primary mass standards of the PTB (Pysikalisch-Technische Bundesanstalt which is the National Metrology Institute of Germany), we see that according to the PTB the value of relative uncertainty of the determined air density was 1.3x10^{-4} and the value of estimated uncertainty of air buoyancy correction was 12.6 μg at a level of 1 σ. (Schwartz,1995). This comparison confirms the agreement between the experimental results of the UME (Turkey) and the experimental results of the PTB (Germany).

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