Turk J Engin Environ Sci
24 (2000) , 265 – 276.

A Daily Intermittent Streamflow Simulator

Hafzullah AKSOY, Mehmetçik BAYAZIT
Istanbul Technical University, Civil Engineering Faculty,
Hydraulics Division 80626 Ayazağa, Istanbul-TURKEY
e-mail: haksoy@ucdavis.edu

Received 05.01.2000

Abstract

Stochastic models are required in order to generate synthetic values of flows statistically similar to observed ones for use in simulation studies of water related structures. Such models are extremely important for the generation of flows of streams with short record length. This study presents a model for the generation of daily flows of an intermittent stream. The model is based on Markov chains and has four steps: (i) determination of the days on which flow occurs, (ii) determination of the days on which a flow increment occurs, (iii) determination of the flow increment, and (iv) calculation of the flow decrement on days when the flow is reduced. In the first two steps, two 2-state Markov chains are used for determination of the state of the stream. In the third step of the model, it is assumed that increments occurring on the ascension curve of the hydrograph are gamma distributed. In the fourth step of the model, the recession curve of the hydrograph is calculated by an exponential decay function with two different coefficients. Parameters required for the model are estimated from daily flows for each month of the year. The model is applied to a daily series of 35 years' length. At the end of the study it was seen that the model preserves the short-term characteristics of flow values in addition to the long-term characteristics.

Key Words: Intermittent streams, daily flow, Markov chain, gamma distribution

Kuruyan Akarsularda Günlük Akım Üreticisi Bir Model

Özet

Su kaynakları projelerinde gereken uzun akım verilerinin sentetik olarak türetilmesi için stokastik modellere gereksinim duyulur. Bu tip modeller akım kayıtlarının kısa olduğu akarsular için ayrıca önemlidir. Bu çalışmada kuruyan akarsularda günlük akımların türetilmesi için geliştirilen bir model verilmektedir. Model Markov zincirlerine dayanmakta ve dört aşamadan oluşmaktadır: (1) Akım oluşan günlerin belirlenmesi, (2) Akımın bir önceki güne göre arttığı günlerin belirlenmesi, (3) Akımdaki artışın belirlenmesi, (4) Akımdaki azalmanın hesaplanması. Modelin ilk iki adımında iki adet 2-durumlu Markov zinciri kullanılmış, üçüncü adımında akımdaki artışların gama dağılımına sahip olduğu, son adımında da hidrografın çekilme eğrisinin eksponansiyel azaldığı kabul edilmiştir. Model için gereken parametreler aylık zaman aralığında günlük akım verileri kullanılarak belirlenmiştir. Modelin 35 yıl uzunluğundaki bir akım serisine yapılan uygulamasından akım değerlerinin uzun süreli karakteristiklerinin yanısıra kısa süreli karakteristiklerinin de korunduğu görülmüştür.

Anahtar Sözcükler: Kuruyan akarsu, günlük akım, Markov zinciri, gama dağılımı

Introduction

The arid and semi-arid regions of the world need water more than the humid regions. With increasing population and industry, the required amount of water has increased. Therefore a prodigious amount of water must be supplied from natural water resources such as rivers, lakes and underground sources. Supplying the water from streams through which water flows into the oceans under the effect of gravity is of great importance. In order to make water in streams useable it is necessary to build structures on the streams. For this, the hydrology of the stream in the past must be known and well defined using the hydrological data available on the stream. Synthetic flow values similar to the observed ones are required due to short observations. Therefore the mathematical structure of the observations is investigated and appropriate mathematical models are sought and developed, by which the synthetic observations are generated.

In this study it is aimed to develop a model generating daily flows of intermittent streams. The developed model is based on Markov chains and has the following steps: (i) determination of days with or without flow, (ii) determination of days with an increase or with a decrease in their flow value for days on which flow occurs, (iii) determination of the magnitude of the increment if an increment occurs, and (iv) determination of the magnitude of the decrement if a decrement occurs. The model is an extension of the model given by Sargent (1979) for a perennial stream, to an intermittent stream. A 2-state (1-0) Markov chain is used for the first step of the model and another 2-state (W-D) Markov chain for the second step. Increments occurring in the flow (corresponding to the ascension curve of the hydrograph) are assumed to be 2-parameter gamma distributed. Decrements occurring in the flow which correspond to the recession curve of the hydrograph are calculated by the well known exponential equation with two different parameters determined from the observed data for each month of the year. The model is applied to a daily flow series taken from Seytan Deresi in the Thrace region of Turkey.

Background on the Subject

Intermittent Streamflow Models

A stochastic or stochastically periodic hydrologic variable is called an intermittent variable if it sometimes takes a value greater than zero and sometimes a value of zero (Yevjevich, 1972). For instance, records taken from rain-gauges in an hourly, daily and weekly time interval always are of intermittent variables while monthly and yearly rainfall records generally are not. Monthly and yearly rainfall records may be intermittent in arid and semi-arid regions of the world. Similarly, flow series of arid and semi-arid regions become intermittent as well (Salas, 1993).

Except for the study by Yakowitz (1973), studies previously done on the flow records of intermittent streams used the monthly mean flow values. Kisiel et al. (1971) concluded that flows of arid regions had two populations, with and without flow. Clarke (1973) gave an algorithm based on the percentage of occurrence of flow to find the monthly flows of an intermittent stream. Lee (1975) developed a model to produce total flow value of the stream. Srikanthan and McMahon (1980a, b) proposed a modified fragments model well preserving monthly flow statistics. Chebaane et al. (1992, 1995) used a model which is a product of a discrete and a continuous component. Only in the study by Yakowitz (1973) was an attempt made to develop a model preserving the important features of a daily flow hydrograph taken from an arid region.

Daily Streamflow Models

Daily flow information is required in many hydrological applications, such as the development of river ecology, the design of small-scale rural water supply schemes (Smakhtin et al., 1997), the regulation, routing, and control of floods, the release of water for water quality control and fisheries during low flow period (Beard, 1967; Kavvas and Delleur, 1984), and the simulation of river regulation schemes and pumped storage schemes where short period fluctuations in flow are significant (Sargent, 1979). All available data recorded along the record length can be used when the user is interested in daily flow data. Better results are obtained with the increasing number of data used in the modelling of the hydrologic variable. When the daily data of the river are used, the model becomes more complex, calculations increase, expanded computer hardware and software and long computing time become necessary, but this is not an issue in the computer age.

Besides the investigation of daily flows of intermittent streams, many studies on daily flows of perennial streams are found in the literature, which can be classified as autoregressive (AR) models, shot noise (SN) models, and transition probabilities (TP) models. Studies by Quimpo (1967, 1968), Quimpo and Yevjevich (1967), Beard (1967), Payne et al. (1969), McGinnis and Sammons (1970), Hall and O'Connell (1972), and Kottegoda (1972) are examples of AR type models. Also Young et al. (1969) and Pentland and Cuthbert (1973) used AR type models for multi-site daily flow data generation. Aiming at reproducing the short term characteristics (ascension and recession curve) of the hydrograph in addition to the long term characteristics such as mean, variance, lag-one and higher lag correlation coefficients of flow, SN models were first developed by Weiss (1973, 1977) and later used by Cowpertwait and O'Connell (1979) and Murrone et al. (1992) in different ways. The third kind of models developed for daily streamflows, TP type models, were used by Treiber and Plate (1977), Sargent (1979), Kottegoda and Horder (1980), Kron et al. (1990), and Kottegoda et al. (1995).

Developed Model

The developed model, as stated before, consists of the following steps: (i) determination of days on which flow occurs, (ii) determination of days on which an increment in the flow occurs, (iii) determination of flow increment (ascension curve of the hydrograph), and (iv) determination of flow decrement (recession curve of the hydrograph) as given in the flow chart (Figure 1). The first two steps determine the state of the stream.

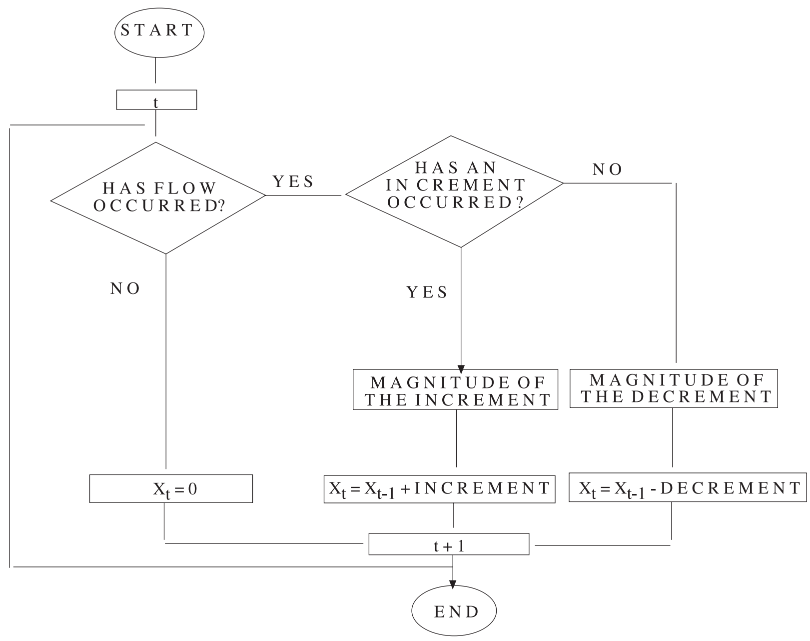

Figure 1. Flow chart of the developed model

State of the Stream

An intermittent stream may be in one of three states: increment in the flow, decrement in the flow and no flow (dried up). Using these three states, days on which flow occurs or not and, later, days on which an increment occurs in the flow (ascension curve of the hydrograph) or a decrement occurs in the flow (recession curve of the hydrograph) can be determined. We need to use two different 2-state Markov chains to mathematicaly formulate the state of the stream.

The matrix of transition probabilities of a 2-state Markov chain can be given as

$$P = \begin{bmatrix} P_{ii} & P_{ij} \\ P_{ji} & P_{jj} \end{bmatrix} \tag{1}$$

where P_{ij} shows the probability of transition from state i to state j. We first need a 2-state (1-0) Markov chain for determining the days on which flow occurs. For this, the matrix given in Eq. (1) can be written as

$$P = \begin{bmatrix} P_{11} & P_{10} \\ P_{01} & P_{00} \end{bmatrix} \quad (2)$$

where states 1 and 0 in the matrix correspond to the occurrence and non-occurrence of the flow, respectively. Once days with and without flow are determined, another 2-state ($W-D$) Markov chain is used for determining the days on which an increment or a decrement in the flow occurs in days with flow. The matrix of transition probabilities of such a Markov chain can be given as

$$P = \begin{bmatrix} P_{WW} & P_{WD} \\ P_{DW} & P_{DD} \end{bmatrix} \quad (3)$$

where states W and D correspond to a day with an increment in flow (ascension curve of the hydrograph) and a day with a decrement in flow (recession curve of the hydrograph), respectively. The same Markov chain was used by Sargent (1979) for the simulation of the daily flows of a perennial stream.

The number of parameters required is 2 for each matrix as the sum of the probabilities equals one for each row of the matrices. Parameters are determined for each month of the year due to seasonal effects resulting in 24 parameters for each matrix and 48 parameters in total. If n_{ij} is the total number of days of observation in the state j with the previous state i, the probabilities of transition from state i to state j can be calculated as

$$P_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}; \; i,j = 1,0 \text{ or } W,D \quad (4)$$

Instead of two 2-state Markov chains, a 3-state Markov chain could also be used for the determination of the state of the stream (Aksoy, 1998b; Aksoy and Bayazit, 2000).

Ascension Curve of the Hydrograph

It is known that a day with a flow greater than the flow of the previous day is on the ascension curve of the hydrograph, and a day with a flow smaller than the flow of the previous day is on the recession curve. On days when the flow increases, the magnitudes of the increments occurring in the flow are determined in this step of the model. The increment is calculated as the difference between flow values of the successive days on the ascension curve. It was seen from the data which will be defined later that differences may be zero for some days. In that case it is assumed that the ascension curve of the hydrograph continues with a magnitude of the increment equal to zero. An appropriate probability density function, which fits to the increments in the flow should be chosen for defining the ascension curve. It was found that, as in the study by Sargent (1979), the standard deviation of the increments is greater than the mean, making it necessary to search for a distribution function other than the exponential distribution. Proposed but not used by Sargent (1979) the 2-parameter gamma distribution is accepted as the distribution function of the increments in the flow on the ascension curve. The distribution has the form of

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}; \; x \geq 0 \quad (5)$$

where α and β are the shape and scale parameters, respectively and $\Gamma(.)$ denotes the incomplete gamma function. The expected value and variance of the variable are given as

$$E(x) = \alpha\beta \quad Var(x) = \alpha\beta^2 \quad (6)$$

from which the shape and scale parameters of the distribution can be estimated for each month once the mean and variance of the increments in the flow are calculated from the observed series. The total number of parameters required for this step of the model is 24. Using the estimated parameters, 2-parameter gamma distributed random variables are generated by an algorithm given in Kottegoda (1980). Generated random variables on an ascension curve are then ranked, so that the higher the increment, the closer it is to the peak of the hydrograph. Sargent (1979) did not rank the generated increments, because of the use of correlation between the increments in the flow.

Recession Curve of the Hydrograph

The recession curve starts with a peak and ends with an increment in the flow. If the flow value of a day is smaller than or equal to that of the previous day, then the current day is on the recession curve. Along the recession curve some days may have the same flow value as the previous day. In that case it is assumed that the recession curve is continuing. A technique similar to that of Sargent (1979) is used in this step of the model. The recession curve is split into two stages (upper and lower recessions) as in the study by Sargent (1979), who used a power function for the upper recession and an exponential function for the lower recession. In this study, both stages are calculated using the exponential decay function. The monthly mean flow value is accepted as a splitting criterion to avoid subjective decisions such as the ratio of 0.9 in the study of Sargent (1979). The upper recession is assumed to take the form of

$$Q_t = Q_0 e^{-b_1 t} \quad (7)$$

where b_1 is the recession coefficient for the upper part of the recession curve, t is the number of days after the peak, Q_t is flow t days after the peak, and Q_0 is the preceding peak flow value. The lower recession is assumed to take the form of

$$Q_t = Q_0^* e^{-b_2 (t-t^*)} \quad (8)$$

where t^* is the time from the start of the lower recession and Q_0^* is the initial flow in the lower part of the recession. A recession curve decays with the upper recession coefficient, b_1, until the flow takes a value smaller than the observed monthly mean flow value. If peak flow value at the beginning of a recession or flow value of a day on the recession curve is smaller than the observed monthly mean, lower recession coefficient, b_2, is used until the end of the recession curve (Aksoy, 1998a). Two recession coefficients and a monthly mean flow value have to be determined from the observed data for each month of the year, resulting in 36 parameters for the recession curve.

Data Description and Model Parameters

The model was tested using the daily data of Seytan Deresi at Babaeski in the Thrace region (European part of Turkey) for a 35-year period (1958-1992). The gauging station was 50 m above the mean sea level. The drainage area of the station was 478.4 km^2. The long term mean flow was 2.44 m^3/s and 10.6% of days had no flow during the record period.

Transition probabilities of the two 2-state Markov chains, the shape and scale parameters of the 2-parameter gamma distribution and parameters required for the recession curve were determined from the data. The transition probabilities of the chains were computed by Eq. (4) and the shape and scale parameters by Eq. (6). Recession curve parameters are determined from the observed data. For this, over one thousand recession curves were used. The transition probabilities of the 1-0 and W-D Markov chains, the shape and scale parameters of the 2-parameter gamma distribution and recession curve parameters and monthly mean flow values are given in Table 1. It is seen that both probabilities P_{11} and P_{00} are very high and close to 1. The river from which the data were taken does not dry up in the period December to May, resulting in P_{11}=1. The second 2-state Markov chain has large values for P_{DD}. The probability P_{WW} also is large except for the months April, May and July. The shape parameter of the 2-parameter gamma distribution has values smaller than 1 but the scale parameter varies from 1 to about 45. As expected, it is also seen that

Table 1. Parameters of the developed model determined from the data

Month	P_{11}	P_{00}	P_{WW}	P_{DD}	α	β	b_1	b_2	Monthly Mean Flow (m^3/s)
October	0.998	0.869	0.778	0.855	0.023	14.760	0.383	0.154	0.468
November	0.999	0.000	0.719	0.845	0.056	14.215	0.392	0.127	1.215
December	1.000	-	0.645	0.824	0.075	31.765	0.352	0.137	3.545
January	1.000	-	0.568	0.832	0.083	44.304	0.317	0.119	5.330
February	1.000	-	0.588	0.877	0.187	20.017	0.230	0.079	6.249
March	1.000	-	0.535	0.897	0.251	13.527	0.208	0.062	5.479
April	1.000	-	0.486	0.893	0.231	5.749	0.138	0.072	3.466
May	1.000	-	0.486	0.890	0.084	12.160	0.195	0.089	2.008
June	0.998	1.000	0.513	0.892	0.148	5.422	0.244	0.150	1.052
July	0.985	0.971	0.458	0.898	0.105	4.142	0.277	0.196	0.410
August	0.964	0.981	0.657	0.930	0.134	1.907	0.324	0.247	0.116
September	0.981	0.967	0.755	0.853	0.100	1.053	0.197	0.125	0.115

the recession parameters determined for the upper part are always greater than those for the lower part. The monthly mean flow value, the splitting criterion of the recession, varies between 0.12 m^3/s (September) and 6.25 m^3/s (February).

Results

It was determined whether or not the model preserves the state of the stream (days on which flow occurs or not and later days on which an increment occurs in the flow or not in a day with flow) by comparing the transition probabilities of the two 2-state Markov chains. Ten series are generated each 35 years long. Transition probabilities calculated as the average of ten simulations are given in Table 2. It is seen from comparison of the probabilities to those in Table 1 that the state of the stream is well preserved.

Checking the similarity between the generated and observed increments in the flow is done by comparison of the monthly mean, standard deviation and skewness coefficients of increments (Table 3). Mean and standard deviation of increments are seen to be well preserved, and the skewness coefficient is close to the observed value except for months with very high skew, although a 2-parameter probability distribution function is used. The number, value and height of peaks (difference between the peak flow value and flow value at the beginning of the ascension curve) are other important features of the hydrograph. In Table 4, these characteristics are compared, resulting in a very good agreement.

Table 2. Transition probabilities of the two Markov chains calculated as the average of ten simulations

Month	P_{11}	P_{00}	P_{WW}	P_{DD}
October	0.998	0.865	0.777	0.851
November	0.9995	0.000	0.727	0.848
December	1.000	-	0.642	0.821
January	1.000	-	0.542	0.830
February	1.000	-	0.595	0.882
March	1.000	-	0.550	0.896
April	1.000	-	0.449	0.889
May	1.000	-	0.486	0.889
June	0.999	1.000	0.505	0.892
July	0.986	0.979	0.471	0.899
August	0.964	0.981	0.660	0.936
September	0.984	0.964	0.746	0.865

Table 3. Characteristics of the ascension curve of the hydrograph

Month	Average (m^3/s)		Standard Deviation (m^3/s)		Skewness Coefficient	
	Obs	Sim	Obs	Sim	Obs	Sim
October	0.341	0.299	2.244	1.775	13.460	9.227
November	0.798	0.789	3.367	3.412	9.199	7.224
December	2.390	2.299	8.713	8.129	7.854	5.798
January	3.683	3.882	12.774	13.193	9.376	5.916
February	3.750	3.769	8.664	8.447	3.915	4.082
March	3.402	3.266	6.784	6.245	3.388	3.033
April	1.330	1.256	2.766	2.413	4.646	3.216
May	1.024	1.121	3.528	3.593	6.542	5.343
June	0.803	0.846	2.086	2.126	5.792	4.188
July	0.435	0.447	1.343	1.416	5.402	5.041
August	0.256	0.221	0.698	0.559	3.840	3.873
September	0.105	0.107	0.333	0.314	5.715	5.182

Table 4. Characteristics of peaks

Month	Number of Peaks		Peak Flow (m^3/s)		Peak Height (m^3/s)	
	Obs	Sim	Obs	Sim	Obs	Sim
October	85	95	1.786	1.631	1.466	1.231
November	113	106	3.434	3.793	2.763	2.832
December	125	129	9.133	8.780	6.819	6.241
January	132	134	12.33	12.51	8.521	8.499
February	98	89	14.21	14.31	9.003	9.334
March	93	93	12.42	12.57	7.287	7.564
April	93	97	5.615	5.565	2.589	2.355
May	94	98	3.793	3.992	2.030	2.221
June	94	92	2.374	2.729	1.597	1.714
July	85	79	1.146	1.349	0.835	0.879
August	36	39	0.954	0.791	0.770	0.620
September	62	58	0.527	0.512	0.375	0.365

Monthly mean flows computed from observed and generated series are given in Figure 2. Mean annual flows are compared in Figure 3. D-day (D=1, 3, 7, 10, 14, 30, 60, 90, 120, 150, 273) yearly maximum and minimum flows are calculated and compared in Tables 5 and 6, respectively. 1-day yearly maximum flow corresponding to the flood condition is given in Figure 4. It is not surprising to have obtained higher generated values for maximums and lower generated values for minimums.

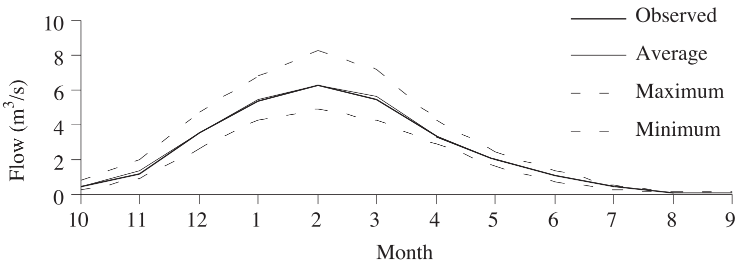

Figure 2. Monthly mean flow

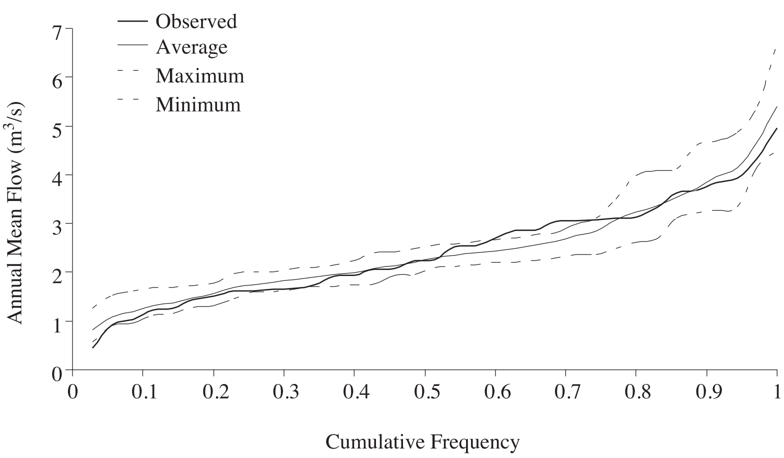

Figure 3. Annual mean flow

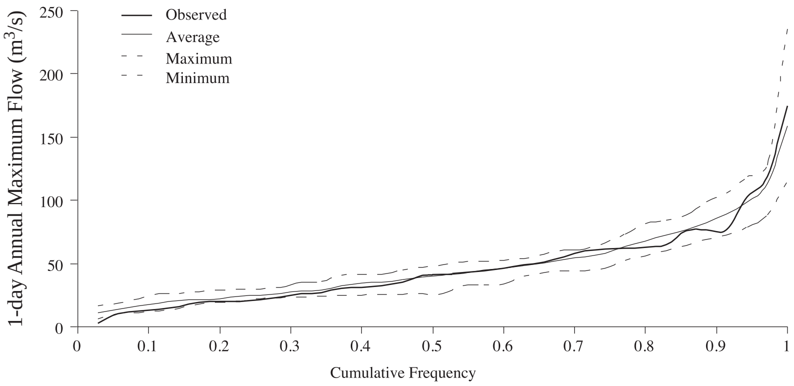

Figure 4. Annual maximum flow

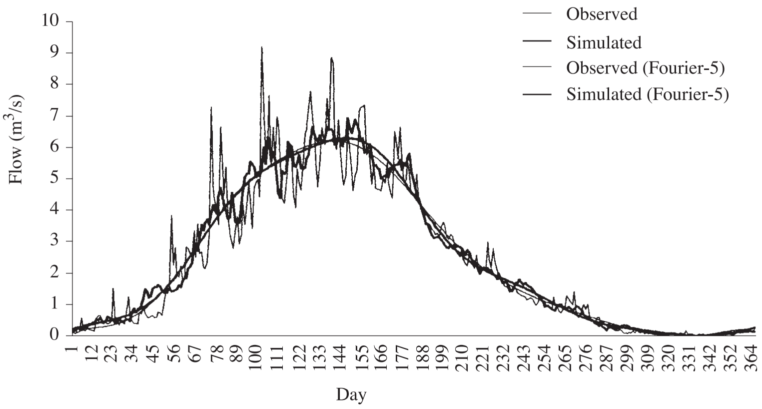

Figure 5. Daily mean flow

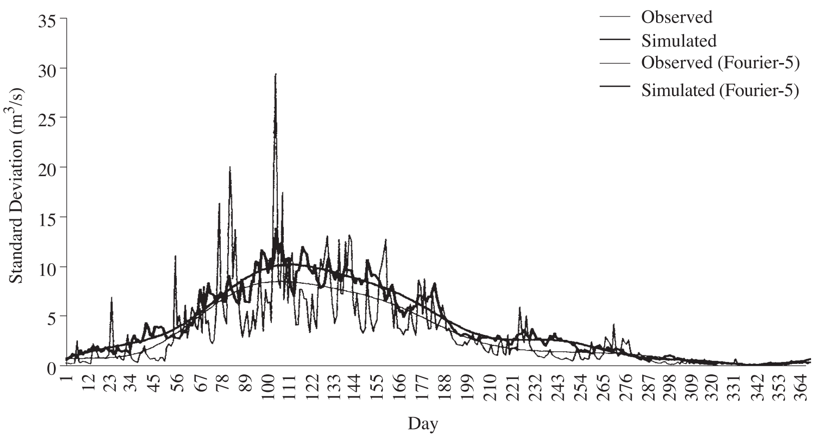

Figure 6. Standard deviation of daily mean flow

Table 5. D-day annual maximum flows

D	Average (m^3/s)		Standard Deviation (m^3/s)		Skewness Coefficient	
	Obs	Sim	Obs	Sim	Obs	Sim
1	47.154	48.516	34.476	32.324	1.797	1.681
3	32.692	37.907	23.481	24.624	1.465	1.640
7	21.971	27.271	14.120	17.126	0.966	1.510
10	18.303	22.851	11.278	14.104	0.902	1.393
14	15.475	19.026	9.117	11.660	0.739	1.355
30	11.041	12.301	5.898	7.120	0.336	1.295
60	8.027	8.493	4.125	4.623	0.369	1.258
90	6.621	6.885	3.287	3.552	0.393	1.169
120	5.764	5.904	2.830	2.909	0.481	1.061
150	5.029	5.198	2.391	2.466	0.455	1.035
273	3.214	3.251	1.392	1.412	0.369	0.981

Table 6. D-day annual minimum flows

D	Average (m^3/s)		Standard Deviation (m^3/s)		Skewness Coefficient	
	Obs	Sim	Obs	Sim	Obs	Sim
1	0.007	0.00008	0.020	0.0004	3.686	6.657
3	0.008	0.0001	0.020	0.0005	3.319	6.405
7	0.009	0.0002	0.024	0.0007	3.326	4.600
10	0.011	0.0003	0.026	0.001	3.131	3.872
14	0.015	0.0007	0.031	0.002	2.441	4.123
30	0.024	0.005	0.043	0.012	1.938	3.382
60	0.092	0.054	0.185	0.074	3.458	2.098
90	0.183	0.163	0.269	0.166	3.553	1.659
120	0.401	0.377	0.407	0.271	3.816	1.207
150	0.706	0.681	0.425	0.449	2.012	1.417
273	2.660	2.675	1.253	1.300	0.345	1.095

Daily features such as the daily mean, standard deviation, coefficient of skewness and lag-one autocorrelation coefficient and daily zero flow percentage are compared. The overall values of these characteristics obtained from the observed and simulated series are given in Table 7. Figures 5-9 are given for the mean, standard deviation, coefficients of skewness and lag-one autocorrelation and zero flow percentage, respectively, to see the variation along the year. A 5-harmonic Fourier series is also fitted to the parameters with exception of the zero flow percentage to see the similarity between the observed and generated characteristics. It is seen that the daily mean and zero flow percentage are well preserved. Standard deviation and lag-one autocorrelation are higher. The skewness coefficient is found to be close to the observed values although the overall value is lower (Table 7).

Table 7. Overall characteristics of flow series

Series	Average (m^3/s)	Standard Deviation (m^3/s)	Variation Coefficient	Skewness Coefficient	Lag-one Autocor-relation Coefficient	Zero Flow (%)
Observed	2.434	5.374	2.208	9.468	0.749	10.56
Sim. 1	2.424	6.095	2.514	7.449	0.828	10.70
Sim. 2	2.648	6.854	2.588	8.194	0.855	9.17
Sim. 3	2.537	6.118	2.411	6.890	0.846	9.09
Sim. 4	2.581	6.299	2.441	7.262	0.849	10.15
Sim. 5	2.398	6.260	2.610	8.570	0.861	11.01
Sim. 6	2.233	5.823	2.608	12.548	0.803	11.79
Sim. 7	2.594	6.362	2.453	6.904	0.852	10.33
Sim. 8	2.379	5.949	2.501	6.858	0.848	9.46
Sim. 9	2.394	6.197	2.589	7.388	0.846	10.98
Sim. 10	2.375	5.806	2.444	7.199	0.832	10.61

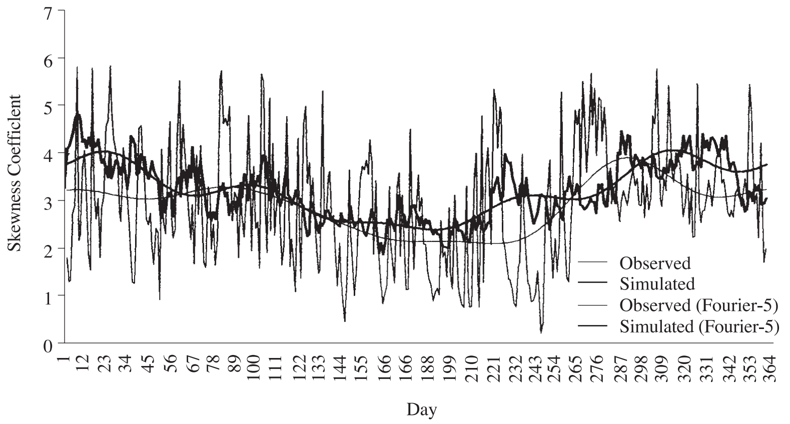

Figure 7. Skewness coefficient of daily mean flow

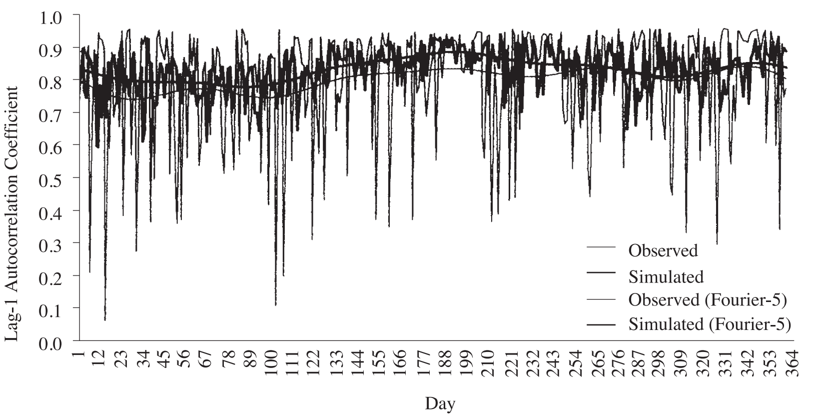

Figure 8. Lag-one autocorrelation coefficient of daily mean flow

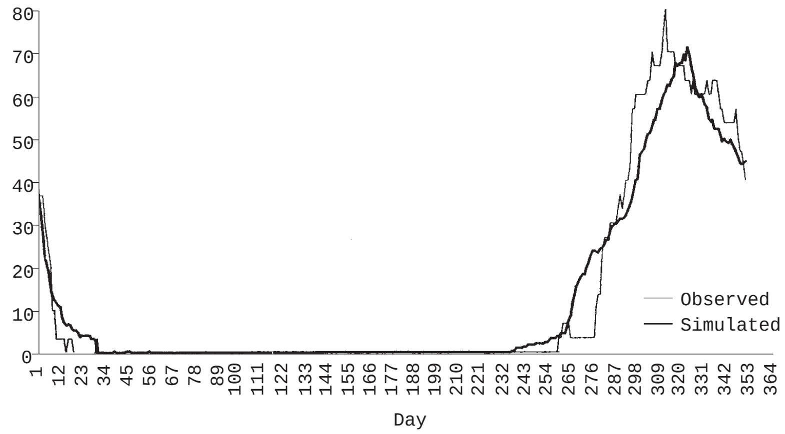

Figure 9. Zero flow percentage

Conclusions

Intermittent streams, on which a very limited amount of work has been done, play an important role in meeting water demands of arid zones of the world. In the literature only one study (Yakowitz, 1973) has been found on the modeling of daily flows of an intermittent stream.

This study gives a Markov chain-based model for generating daily flow series of intermittent streams. The model is an extension of the model given by Sargent (1979) for perennial streams to intermittent streams. The ascension curve of the hydrograph is assumed to be probabilistic while the recession curve of the hydrograph is assumed to be deterministic. Gamma distribution is fitted to the ascension curve. In calculating flow values on the recession curve of the hydrograph, an exponential decay function splitting the curve into two stages is used.

The parameters of the model are determined on a monthly basis using daily flow values of the stream. From the application it is seen that the model is capable to reproduce short-term characteristics of the hydrograph such as the ascension and recession curves in addition to long-term characteristics such as mean, standard deviation, skewness and lag-one serial correlation coefficients of flow.

Symbols

b_1, b_2	:	parameters to be determined from recession curves for the upper recession and lower recession, respectively
$E(\,)$	:	expected value
n	:	total number of days
P_{ij}	:	probability of transition from state i to state j
Q_t	:	flow at time t
Q_0	:	flow value at the beginning of recession
t	:	time
x	:	variable
$Var(\,)$	:	variance
α	:	shape parameter of the gamma distribution
β	:	scale parameter of the gamma distribution

References

Aksoy, H., "Determination of Recession Curve Parameters", Proc. Low Flows Expert Meeting, (eds. Vukmirovic, V., Radic, Z., Bulu, A.), The University of Belgrade, Belgrade, 89-96, 1998a.

Aksoy, H., Modeling of Daily Flows of Intermittent Streams, PhD Thesis, Istanbul Technical University, Institute of Science and Technology, Istanbul, 1998b.

Aksoy, H. and Bayazit, M., "A Model for Daily Flows of Intermittent Streams", Hydrological Processes, (in press), 2000.

Beard, L.R., "Simulation of Daily Streamflow", Proc. International Hydrology Symposium, Fort Collins, Colorado, 624-632, 1967.

Chebaane, M., Salas, J.D. and Boes, D.C., Modeling of Intermittent Monthly Streamflow Processes, Water Resour. Pap. 105, Colo. State Univ., Fort Collins, 1992.

Chebaane, M., Salas, J.D. and Boes, D.C., "Product Periodic Autoregressive Processes for Modeling Intermittent Monthly Streamflows", Water Resour. Res., 31(6), 1513- 1518, 1995.

Clarke, R.T., Mathematical Models in Hydrology, Irrigation and Drainage, Pap. 19, U.N. Food and Agric. Org., Rome, 1973.

Cowpertwait, P.S.P., and O'Connell, P.E., "A Neyman-Scott Shot Noise Model for the Generation of Daily Streamflow Time Series", Advances in Theoretical Hydrology - A Tribute to James Dooge, (ed. O'Kane, J.P.), Elsevier, Amsterdam, 75-94, 1992.

Hall, M.J., and O'Connell, P.E., "Time-Series Analysis of Mean Daily River Flows", Water and Water Engin., 125-133, 1972.

Kavvas, M.L., and Delleur, J.W., "A Statistical Analysis of the Daily Streamflow Hydrograph", J. Hydrol., 71, 253-275, 1984.

Kisiel, C.C., Duckstein, L. and Fogel, M., "Analysis of Ephemeral Flows in Arid Lands", ASCE, J. Hydraul. Div., 97 (HY10), 1699-1717, 1971.

Kottegoda, N.T., "Stochastic Five Daily Stream Flow Model", ASCE, J. Hydraul. Div., 98 (HY9), 1469-1485, 1972.

Kottegoda, N.T., Stochastic Water Resources Technology, John Wiley & Sons, New York, 1980.

Kottegoda, N.T., and Horder, M.A., "Daily Flow Model Based on Rainfall Occurrences Using Pulses and a Transfer Function", J. Hydrol., 47, 215-234, 1980.

Kottegoda, N.T., Natale, L., Raiteri, E. and Saccardo, I., "A Stochastic Model of Daily Flows for Simulating Low Flows in a Highly Developed Basin", Proc. Int. Conf. Statistical and Bayesian Methods in Hydrological Sciences in Honor of Jacques Bernier, UNESCO, Paris, 1995.

Kron, W., Plate, E.J. and Ihringer, J., "A Model for the Generation of Simultaneous Daily Discharges of Two Rivers at their Point of Confluence", Stochastic Hydrol. and Hydraul., 4, 255-276, 1990.

Lee, S., "Stochastic Generation of Synthetic Streamflow Sequences in Ephemeral Streams", IAHS Publ., 117, 691-701, 1975.

McGinnis, D.F., and Sammons, W.H., Discussion of "Daily Streamflow Simulation" by Payne, K., Neumann, W.R. and Kerri, K.D., ASCE, J. Hydraul. Div., 96 (HY5), 1201-1206, 1970.

Murrone, F., Rossi, F., and Claps, P., "A Conceptually-Based Multiple Shot Noise Model for Daily Streamflow", Stochastic Hydraulics'92, (eds. Kuo, J.T. and Lin, G.F.), Water Resources Publication, Littleton, Colorado, 857-864, 1992.

Payne, K., Neumann, W.R. and Kerri, K.D., "Daily Streamflow Simulation", ASCE, J. Hydraul. Div., 95 (HY4), 1163-1179, 1969.

Pentland, R.L., and Cuthbert, D.R., "Multisite Daily Streamflow Simulator", Water Resour. Res., 9 (2), 470-473, 1973.

Quimpo, R.G., Stochastic Model of Daily River Flow Sequences, Hydrol. Pap. 18, Colo. State Univ., Fort Collins, 1967.

Quimpo, R.G., "Stochastic Analysis of Daily River Flows", ASCE, J. Hydraul. Div., 94 (HY1), 43-57, 1968.

Quimpo, R.G., and Yevjevich, V., "Stochastic Description of Daily River Flows", Proc. Int. Hydrol. Symp., Fort Collins, Colorado, 291-297, 1967.

Salas, J.D., "Analysis and Modeling of Hydrologic Time Series", Chapter 19 in Handbook of Hydrology, (ed. Maidment, D.R.), McGraw-Hill, New York, 1993.

Sargent, D.M., "A Simplified Model for the Generation of Daily Streamflows", Hydrol. Sci. Bull., 24 (4), 509-527, 1979.

Smakhtin, V.Y., Hughes, D.A. and Creuse-Naudin, E., "Regionalization of Daily Flow Characteristics in Part of the Eastern Cape, South Africa", Hydrol. Sci. Journal, 42 (6), 919-936, 1997.

Srikanthan, R., and T.A. McMahon, "Stochastic Generation of Monthly Flows for Ephemeral Streams", J. Hydrol., 47, 19-40, 1980a.

Srikanthan, R., and T.A. McMahon,"Stochastic Time Series Modelling of Arid Zone Streamflows", Hydrol. Sci. Bull., 25, 4, 423-434, 1980b.

Treiber, B., and Plate, E.J., "A Stochastic Model for the Simulation of Daily Flows", Hydrol. Sci. Bull., 22 (1), 175-192, 1977.

Weiss, G., "Shot Noise Models for Synthetic Generation of Multisite Daily Streamflow Data", IAHS Publ., 108, 457-467, 1973.

Weiss, G., "Shot Noise Models for the Generation of Synthetic Streamflow Data", Water Resour. Res., 13 (1), 101-108, 1977.

Yakowitz, S.J., "A Stochastic Model for Daily River Flows in an Arid Region", Water Resour. Res., 9 (5), 1271-1285, 1973.

Yevjevich, V., Stochastic Processes in Hydrology, Water Resources Publ., Fort Collins, Colorado, 1972.

Young, G.K., Somers, W.P., Pisano, W.C. and Fitch, W.N., "Assessing Upland Reservoirs using a Daily Flow Model", Water Resour. Res., 5 (2), 362-379, 1969.