

# Optimum Operation Management of the Istanbul Water Supply System

İsmail DURANYILDIZ, Mehmetçik BAYAZIT,  
Bihrat ÖNÖZ, İlhan AVCI & Beyhan OĞUZ  
*Istanbul Technical University, Civil Engineering Faculty,  
Hydraulics Dept., Ayazaga, 80626 Istanbul,-TURKEY  
e-mail: duranyil@itu.edu.tr*

Received 24.06.1999

## Abstract

Water for cities containing millions of citizens is at present being supplied mostly from surface resources. These resources are generally in different river basins, far from the city center, and are developed by means of multireservoirs planned for the single purpose of water supply. They must be investigated with the system concept both in planning and operation phases in order to provide an integrated management. This study emphasizes the operation problem of the multireservoir water supply systems. The water supply system of Istanbul is introduced. In order to find optimum operating rules for the system, dynamic programming is used and operation results with these rules are given.

**Key Words:** Water resources, reservoir, optimization, operating rule, dynamic programming.

## İstanbul Su Temini Sisteminin Optimum İşletmesi

### Özet

Halen milyonlarca kişiyi barındıran büyük şehirlerin suyu çoklukla yüzeysel kaynaklardan temin edilmektedir. Bu kaynaklar genellikle şehir merkezinden uzak değişik akarsu havzalarında olup, her bir haznenin tek amacı su temini olan çok hazneli bir sistemle geliştirilmektedirler. Bu haznelerin entegre olarak yönetilebilmeleri için, hem planlama hem de işletme aşamalarında bir sistem olarak ele alınıp incelenmeleri gereklidir. Bu çalışma çok hazneli su temini sistemlerinin işletme problemine ağırlık vermektedir. İstanbul su temini sistemi tanımlanmaktadır. Sistemin en iyi işletme kurallarını bulmak için dinamik programlama kullanılmış, bu kurullarla işletme sonuçları verilmiştir.

**Anahtar Sözcükler:** Su kaynakları, Hazne, Eniyileme, İşletme kuralı, Dinamik programlama

### Introduction

Water supply has emerged as a critical issue for the 1990 s and beyond. Water resources, until recently considered cheap and plentiful, are now recognized to be scarce and valuable. More than 230 million people live in 26 countries classified as “wa-

ter deficient”. The number of countries facing severe water shortage is likely to increase dramatically in the next decade.

Demand for water is escalating, contributing to intensified competition among users. In many areas, giving water to one user means denying it to

another. In a world where available fresh surface water resources are extremely unevenly distributed, this has the potential to provoke national, regional and international disputes.

The lifestyle of nations reached today depends heavily upon having plenty of cheap, clean water available as well as an inexpensive safe way to dispose of it after use. Nature has limited the supply of water for our use. Although there seems to be plenty of water on earth, only 0.3% of it is available and not always in the right place at the right time. Also chemicals, wastes improperly discarded yesterday, are showing up in our water resources today. In most areas man faces uncertain supplies for high rate consumptions and the health and economic effects of shortage of clean water are matters of great concern.

The idea that the water is a finite resource to be conserved and protected is not yet universally perceived. Generally fresh water is respected as a giver of life only in regions of chronic shortage. Thus the effective use of water resources has become a vital problem for all countries. Since supplies and needs do not match in space and in time because of their conflicting behavior, the required regulation can be carried out with water resources development studies. One of the most important components in these regulations is the storage reservoir built to increase the reliability of local supplies. The planning as well as the operation of it constitute the important part of these studies. Recently the development of many major water resources systems has been completed. The easy operation of simple systems in the beginning becomes more complex from day to day as a result of the multiplicity of the objectives and the complexity of the interconnections.

The resources for specific purposes and the targets are known in most water resources development projects. In an interconnected reservoirs system, the water available for specific purposes can be less than or equal to the water stored during that time period. In periods of excess water or drought the manager has to know the distribution of releases from reservoirs or the shortages to users. Thus, the operation policy must be determined in order to meet the targets for inyear or multiyear regulations.

The increasing number both of objectives and of reservoirs, makes the system more complex and this results in a high number of operation alternatives. The cost and reliability of each alternative is different. Thus, the system has a unique operation policy for each state and target and, as a result, there is a

unique optimum operation policy corresponding to each system planning.

Planning, construction and operation of water resources systems are carried out for a firm yield value. This value is determined as the target in the planning phase and is guaranteed for the whole operation period, whatever the hydrologic conditions are. When the need exceeds this value as the result of developments, planning new resources is the best solution.

Also, in many important water resources systems in operation, various new objectives, not present in the planning period, must be taken into consideration now and this requires an optimization of the operation policies.

### **Determination of the Optimum Operation Policy of Multireservoir Systems**

Operation of water resources is a large-scale storage control problem. When the system is considered as a whole with all components, a high number of feasible decision alternatives makes the problem very complex. In this problem, the system is characterized by a mathematical model and related to some variables. Some of them indicate the state of the system at a specific time. Others allow decisions about some controllable inputs. The set of values of these inputs subject to some constraints, in determined decision stages, constitutes a policy. The methods below can be used to obtain the optimum among these policies.

- Empirical Methods: These generally depend on the engineering practice of the user and can be used in the resolution of some simple subsystems.

- Simulation Method: In this method the dynamic behavior of the system is simulated on a computer and the results of different alternatives can be obtained without simplifications in the model. Although complex systems like water resources problems can be investigated, it is not guaranteed that the result is optimal since this is a trial and error based method.

- Mathematical Programming Methods: The optimal result can be obtained directly by analytical optimization or search methods, but it is difficult in practice to apply them to complex problems like in water resources. Most widely used are Dynamic Programming and Linear Programming methods. In Dynamic Programming a model has to be rebuilt for each problem. Linear Programming can be applied easily using ready-to-use solution packages, but difficulties arise where objective function and constraints

are not linear as in water resources problems. In these methods the random nature of the system inputs can also be considered.

Once the mathematical model of the system is built, solutions to many problems related to the system can easily be obtained. Thus, this allows investigation of many alternatives, which is important especially in the planning phase.

Studies of reservoir planning and operation are based on Rippl's (1883) graphical method and have been developed in the last 20-30 years. This method has been evolved by Thomas and Burden (1963) to sequent peak algorithm. The first researcher who used Linear Programming in this field was Dorfmann (1962). In the state-of-the-art review of Yeh (1985), DP and LP appear as most preferred methods for the solution of this problem. Yeh (1981) used LP for the short term decisions and DP for the long term decisions. In addition, some attempts at aggregation/disaggregation methods have been made (Valdes et al., 1992). Although the aggregation of the reservoirs can be done successfully, the same success cannot be achieved with the disaggregation of the results to the reservoirs. In the study by Bayazit and Duranyıldız (1987), a method based on successive approximations and linearly dependent on the number of reservoirs, which overcomes the problem of dimensionality was developed and it was shown that the global optimum is reached. A continuous hedging policy was used in the work of Shih and ReVelle (1994) for a single reservoir system for water supply. Loucks (1992) described the information need and performances of different application packages in use in the field. Duranyıldız et al. (1998) developed a chance-constrained LP model which takes into consideration the stochasticity of the input, for short term operation of reservoirs.

Unfortunately the use of these methods and application packages developed for water resources development is not so frequent in practice. A gap between model developers, planners and operators is seen in every country of the world (Simonovic, 1992).

### **Investigation of the Problem With Regard to Water Resources of Megacities**

Most cities meet their need for water by withdrawing it from the nearest river, lake or reservoir. Such systems are highly affected by drought periods. Also, increasing population and unbalanced settling

increase this effect.

Although recent advances in computer technology have speeded up the use of optimization methods, institutions are slow to change, in the face of technological and social evolution, usually lagging far behind the need for more appropriate policies. This is particularly true in water resources management where policy making and administrative processes are subject to the inertia of the historical status quo of special interests. This results in inefficient use and degradation of the available water by pollution. Thus, inadequate freshwater management, lack of water resources planning and unsustainable development of freshwater resources lead to water shortages. The planning of new resources is indispensable to overcome this shortage problem but it is a high priced and long term solution. New resources can easily become insufficient on the opening ceremony day, because in general the speed of the development of the needs and environmental states is higher than that of the construction of water resources systems.

The solution to this problem can be found, from the country's economic and social points of view, in the effective, sustainable and integrated management of water resources. Thus, in order to find the equilibrium in water usage between the needs and the resources, in addition to the investments for the storage and balanced freshwater distribution systems in the region to supply future needs, optimal management of the water quantity and quality is strongly required.

Deciding how much water to release and how much to store depends upon the time of year, flow prediction for the next several months and the needs of the cities. Decisions must be coordinated for all reservoirs in the system. Otherwise, independent decisions in each reservoir, mostly cause the resource to become exhausted before the most necessary use.

### **Water Supply System of Istanbul**

Like many large cities, water for Istanbul is supplied mainly from surface resources. The water supply system with six existing and three planned reservoirs is shown in Figure 1. Some of the reservoirs are linked and the water distribution system is complex. The European and Asian sides are linked via a sub-Bosphorus pipeline. Table 1 shows the characteristics of the components in the present system (ISKI, 1994).

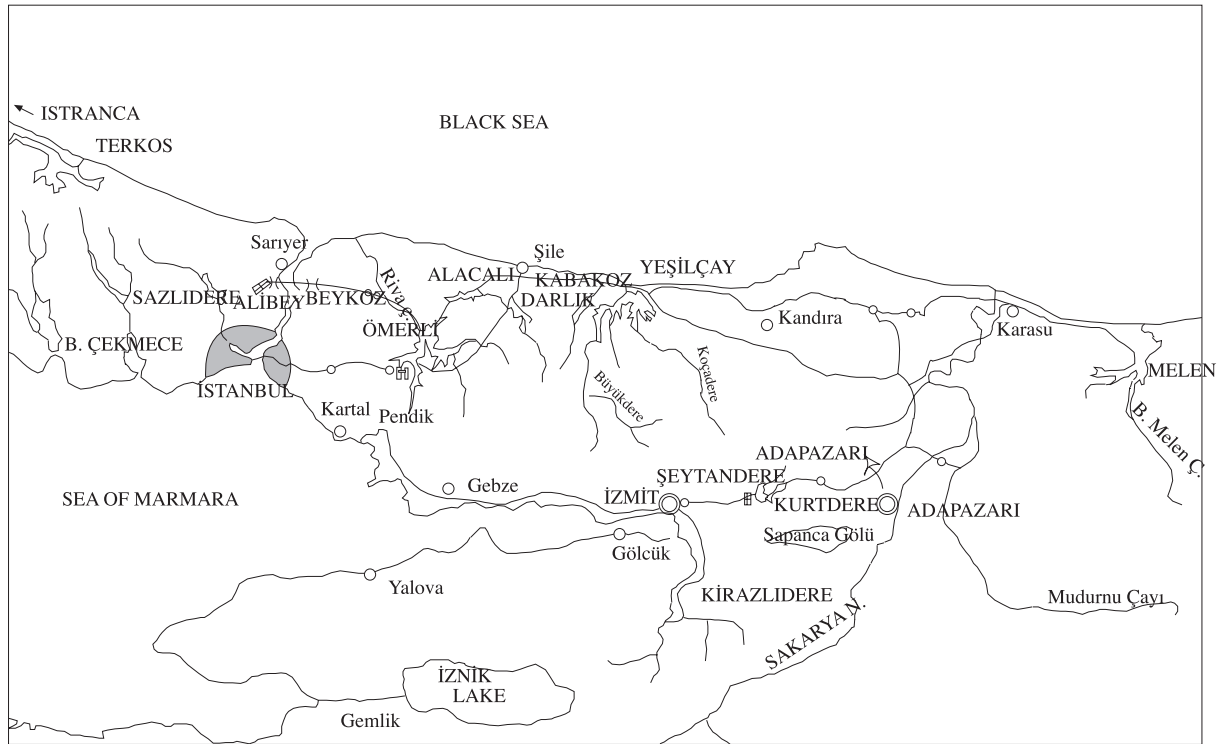


Figure 1. Water Supply System of Istanbul

Table 1. Characteristics of the Existing Reservoirs in the Istanbul Water Supply System

Reservoir	Reservoir Characteristics			Catchment Characteristics				Percentage in the System
	Max. Vol. (10 <sup>6</sup> m <sup>3</sup> )	Mix. Vol. (10 <sup>6</sup> m <sup>3</sup> )	Active Vol. (10 <sup>6</sup> m <sup>3</sup> )	Catchment area (km <sup>2</sup> )	Mean Ann. Prec. (mm)	Mean Ann. Flow (mm)	Flow Coeff. (%)	
Ömerli	357	122	235	600	850	236	45	33
Darlık	113	6	107	207	850	108	59	16
Elmalı	12	2	10	76	620	32	51	2
Istanbul, Asian Side			352	883	835	376	51	51
Terkos	167	42	145	619	750	163	35	21
B.Çekmece	162	20	162	621	700	219	50	23
Alibeyköy	36	1	35	160	830	54	40	5
Istanbul, European Side			342	1400	760	436	40	49
Istanbul Water Supply System			694	2283	800	812	45	100

It can be seen that the reservoir capacities are in accordance with the mean flows of the corresponding catchments, but their firm yields vary because of the random nature of the precipitation and of the natural flows. An investigation of the drought periods of the last two decades. (Table 2) shows some serious water crises (Bayazit et. al., 1992). Especially in the last

two periods inhabitants suffered shortages because of the crisis caused mainly by the high urbanization, old and insufficient water distribution system, unplanned settlements and the absence of an “optimal dynamic program” for the reservoir operation and water distribution. Since, in general, a period of apathy comes after the crisis and all research is then

disregarded, this problem seems to persist for Istanbul, although recent years have in general been wet

above normal.

**Table 2.** Drought Periods in Istanbul With Return Periods Larger than 20 Years and Deficits

	Long term averages	Averages during dorught periods			
		1971-1977	1982-1984	1988-1990	1992-1994
Yearly mean precipitation (mm)	800	718	726	719	652
Yearly mean flow ( $10^6m^3$ )	812	738	746	739	670
Deficit in precipitation (mm)	Sum	574	222	242	445
	Yearly mean	82	74	81	148*
Deficit in flow ( $10^6m^3$ )	Sum	590	228	249	457
	Yearly mean	84	76	83	152

(\*) Deficit in precipitation in 1993 is 350 mm corresponding to a return period of 100 years.

**Problem Formulation**

The most important component of the management of a water supply system is its operating rule. In order to determine the optimal operating rule of the Istanbul Water Supply System, Dynamic Programming (DP) has been used. The objective function of the problem is

$$V_f = \min_t \left[ \sum_{j=1}^N \right] ; \quad t = 1, 2, \dots, T \quad (1)$$

where  $V_f$  is the monthly firm yield of the system of  $N$  reservoirs,  $Y_{j,t}$  is the release from reservoir  $j$  in month  $t$ ,  $T$  is the operating horizon in months. The objective is to find the values of the decision variables  $Y_{j,t}$  that maximize this firm yield value (max  $V_f$ ) which is a max-min problem subject to reservoir capacity constraints

$$S_{min,j} \leq S_{j,t} \leq S_{max,j}; \quad j = 1, 2 \dots N, t = 1, 2, \dots T \quad (2)$$

and to continuity constraints

$$S_{j,t+1} = S_{j,t} + X_{j,t} - Y_{j,t}; \quad j = 1, 2, \dots N, t = 1, 2, \dots T \quad (3)$$

where  $S_{min,j}$ ,  $S_{max,j}$  denote minimum and maximum volumes of the reservoir respectively,  $S_{j,t}$  and  $X_{j,t}$  are the storage and the net input in time period  $t$  for reservoir  $j$ . The net input to the reservoir can be calculated as

$$X_{j,t} = Q_{j,t} + P_{j,t} - E_{j,t} + V_{k,t}; \quad k = 1, 2, \dots, K, \quad k \neq j, \text{ and } K < N \quad (4)$$

where  $Q_{j,t}$ ,  $P_{j,t}$  and  $E_{j,t}$  denote natural inflow, precipitation and evaporation of reservoir  $j$  in time period  $t$ .  $V_{j,t}$  is the derivation from (an)other reservoir(s) to reservoir  $j$ , if any, in the same time period.

**Operating Rule**

In the operation of the reservoirs, deciding the value of the release depends on the stored water in the system, and the flow predictions for the next several months. In this context an operating rule, for each month of the year  $i$ , of the form

$$Y_{j,t} = f_i(Z_{j,t}); \quad t = 1, 2, \dots, T, \quad j = 1, 2, \dots, N, i = 1, 2, \dots, 12 \quad (5)$$

can be derived from the optimal policy maximizing the firm yield, found as a result of the optimization studies with DP. Here  $Y_{j,t}$  and  $Z_{j,t}$  are the release from and the available water in reservoir  $j$  in month  $t$ ,  $f_i$  is the release function (operating rule) for month  $i$  of the year which is taken in a single or multi-step linearized form. Available water can be calculated as the sum of the stored water ( $S_{j,t}$ ), the net input ( $X_{j,t}$ ) and the net input prediction for the next month ( $X_{j,t+1}$ ).

$$Z_{j,t} = S_{j,t} + (X_{j,t} + X_{j,t+1}) \quad (6)$$

In the operation phase the flow of the next month is estimated, based on the observed flow of the present month, preserving the monthly mean ( $\bar{X}$ ),

standard deviation ( $\sigma$ ) and autocorrelation coefficients  $\rho$  between successive months as

$$X_{j,t+1} = \sigma_{j,i} \rho_{j,i,i-1} \{ (X_{j,t} - \bar{X}_{j,i-1}) / \sigma_{j,i-1} \} + \bar{X}_{j,i}$$

$$j = 1, 2N, i = 1, 2..12, t = 1, 2..T \quad (7)$$

**Application and Results**

The Istanbul water supply system allows the disaggregation of the whole system into subsystems of Büyükçekmece as a single reservoir, Terkos plus Alibeyköy on the European side, and Ömerli plus Darlık on the Asian side as double-reservoir subsystems.

**Büyükçekmece reservoir:**

*Operation Optimization*

- i) The first subsystem investigated is Büyükçekmece reservoir which was completely emptied in 1990 and 1994. The operation optimization of the system is done for the whole period of 20 years of observed inflows. In this optimization the objective is to maximize the monthly firm yield. Precipitations and evaporations are taken as the monthly mean values. In the optimization, in order to investigate the effect of the initial storage, the reservoir is started full and empty for two runs and 7.1 and 6.7x10<sup>6</sup>m<sup>3</sup>/month firm waters corresponding both to a mean monthly supply of 7.53x10<sup>6</sup>m<sup>3</sup>/month are obtained respectively. In subsequent studies reservoir is assumed to be empty initially because this reservoir is frequently emptied.
- ii) In the next step, the optimal operation of the system has been determined in order to maximize the total water supplied. The system is subject to the firm water value constraint (V<sub>f</sub>) (8). This minimum water supply requirement is increased parametrically and the risks of not obtaining the firm yield are determined (Table 3).

$$V_m = \max \left( \sum_t V_t \right) ; st : V_t \geq \min \geq V_f ,$$

$$t = 1, 2, \dots T \quad (8)$$

Where  $V_m$  is the total (or mean) water supplied over the operation horizon,  $V_t$  is the water

supplied at month  $t$ , and  $V_{min}$  is the minimum water supply requirement.

- iii) Another objective investigated is to supply a constant target ( $V_c$ ) volume of water during the operation period  $T$ . In this operation, the supply is subject to the firm water ( $V_f$ ) constraint. The objective function considered in this case is in the quadratic form as:

$$V_{min} = \min \left\{ \sum (V_t - V_c)^2 \right\} ; st : V_t \geq V_f$$

$$t = 1, 2, \dots T \quad (9)$$

in which  $V_c \geq V_j$ . This optimization yields a mean monthly supply of 7.66 × 10<sup>6</sup>m<sup>3</sup> for a monthly target of 9 × 10<sup>6</sup>m<sup>3</sup>.

**Table 3.** Maximization of Total Water Supplied (Büyükçekmece)

Min Water ( $V_{min}$ ) (10 <sup>6</sup> m <sup>3</sup> /m)	Total/Mean Water ( $V_m/V_t$ ) (10 <sup>6</sup> m <sup>3</sup> /10 <sup>6</sup> m <sup>3</sup> /m)	Risk (%)
6.7	1800/7.53	0
7.5	1808/7.56	5
8.0	1811/7.58	8
9.0	1830/7.66	17
9.5	1839/7.69	22

*Operating Rules*

In the second step an operating rule (10) for the system, between the release ( $Y$ ) from the reservoir and the available water in the reservoir ( $Z$ ) based on the optimum operation policy derived as explained above, is determined for each month of the year as:

$$Y = a_i + b_i Z ; i = 1, 2, \dots 12 \quad (10)$$

which is a  $Y - Z$  regression relationship, where  $Z$  is the available water:

$$Z = S_t + (Q_t + P_t - E_t) = S_t + X_t ; t = 1, 2, \dots T \quad (11)$$

In (11), the net input for the next month and the derivation from other reservoirs are considered zero,  $a_i$  and  $b_i$  are regression coefficients. The correlation coefficients ( $r_i$ ) for (10) are found close to zero indicating weak relationships between  $Y$  and  $Z$ . This can be explained by the behavior of the Dynamic

Programming. In this method, the whole operating period is taken into account, which means the model knows (can predict) the inflows of the whole operating horizon and behaves conservatively. This behavior is far from the real world point of view and decreases the correlations between successive months. To overcome this deficiency it is assumed that the model can predict only a limited horizon of the operating period. In the model this projection is taken as 12 months. This yielded a monthly firm water value of  $4.94 \times 10^6 \text{m}^3$ , which corresponds to a mean monthly water supply of  $7.69 \times 10^6 \text{m}^3$ . The correlation coefficients for relationship (10) are then increased to a value greater than 0.70 for all months except for November to February with coefficients between 0.50 and 0.65. The investigation of regression relationships for different months showed that two-step linearized relationships are required for March to September and for November to February. Also a new term is added to (11) to take into account the net input of the next month ( $t + 1$ ) as:

$$\begin{aligned} Z &= S_t + (X_t + P_t - E_t) + (Q_{t+1} + P_{t+1} - E_{t+1}) \\ &= S_t + X_t + X_{t+1} ; t = 1, 2, \dots T \end{aligned} \quad (12)$$

The operation of the system for 20 years, with these derived rules using observed inflows for month  $t$  and estimated inflows for month  $t + 1$  using (7) yields a failure of the firm water in three periods lasting 1, 3 and 2 months where water volumes of (4.09), (4.56,3.57,4.22) and (2.71,1.67)  $10^6 \text{m}^3$  are supplied, which correspond to 17%, 17% and 56% deficits of firm water, respectively, where the deficit is calculated as the difference between the supply and the firm value. In this operation a total water volume of  $1816 \times 10^6 \text{m}^3$  is supplied for 20 years, which corresponds to a monthly mean volume of  $7.60 \times 10^6 \text{m}^3$  and the risk can be calculated as 2.5%. In the real world operation, this reservoir supplied  $4.44 \times 10^6 \text{m}^3$  and  $3.73 \times 10^6 \text{m}^3$  on average in 1989 and 1990 but during a period of 6 months in the first year and a period of 11 months in the second, mean deficits of 32% and 30% were observed. For the same period in the operation with the obtained rules,  $8.8 \times 10^6 \text{m}^3$  and  $7.9 \times 10^6 \text{m}^3$  monthly averages have been supplied.

**-Terkos and Alibeykoy reservoirs:**

The second subsystem investigated is the Terkos and Alibeykoy two reservoirs sub-system. The operation optimization of this system is carried out for

the same period of time with the observed inflows. The reservoirs are in series. The firm water value for this system as a result of the optimization is determined as  $V_f = 13.7 \times 10^6 \text{m}^3/\text{m}$ , which corresponds to a mean monthly supply of  $15 \times 10^6 \text{m}^3$ . When one investigates the operational behaviors of the reservoirs in the system (Fig. 1) it is seen that Alibeykoy acts only as a derivation reservoir without an important contribution. Therefore Terkos reservoir is taken as a single reservoir and the optimal operation is determined. The firm water of this reservoir is found as 12.3 and  $8.3 \times 10^6 \text{m}^3$  for full length and one year projections, respectively, corresponding to a mean monthly supply of  $13.4 \times 10^6 \text{m}^3$  each.

The operating rules of this reservoir are determined based on (10) and the reservoir is operated for 20 years according to (12) and (7), like the previous system. The reservoir emptied at 8% of the time and failed the firm water value  $V_1 = 8.3 \times 10^6 \text{m}^3$  at 8.3% of time in 8 periods lasting 2 months on average. In these periods the deficit from  $V_f$  averaged to  $7 \times 10^6 \text{m}^3$  (42%).

**-Darlık and Omerli reservoirs:**

In this subsystem reservoirs are also serially connected. The optimization of the operation results in a firm water value for the system of  $V_f = 19.7 \times 10^6 \text{m}^3$  corresponding to a monthly average of  $27 \times 10^6 \text{m}^3$ . The operating rules for each reservoir is determined according to (10) and the system is operated for the same 20 years period as in the previous systems. The operational results can be summarized as follows. The system failed the firm water value in 6 periods (totally 6% of the time) with a total deficit of 51%. The operational released water from the system compared with the DP results and real world supply is given in Table 4.

**Table 4.** Released Water From Darlık + Ömerli System in Real World and Operation

Year	Released Water ( $10^6 \text{m}^3$ )		
	DP	Ruled Operation	Real Time Operation
1987	353.5	334.9	249.5
1988	346.2	343.0	305.8
1989	303.2	329.5	285.6
1990	241.9	238.6	254.6

## Conclusion

Istanbul water supply system is currently being operated by the Istanbul Water and Sewage Authority (ISKI) with empirical methods. These trial and error methods based on past experiences cannot solve the long term operation problem of the system as was seen in the drought periods of the last two decades where the inhabitants suffered serious water shortages. Planning new resources as soon as a similar crisis arises is generally a high priced, non-feasible and premature decision. Primarily the operation policy of the system must be optimized for the present conditions. It is seen from the results of

this study that an operating rule based on the system concept can provide a considerable increase in the yield of the water supply system. It can be concluded that if the Istanbul water supply system had an optimum dynamic operating rule, the crisis would not be so terrifying.

## Acknowledgement

The study of "Determining the Optimal Operating Rule of a Reservoir System with Water Supply Purpose" cited here is a project supported by ITU Research Foundation directed by the first author.

## References

- Bayazit, M. and Duranyıldız, İ., "An iterative method to optimize the operation of reservoir systems", *Wat. Res. Man. (I)*, pp.255-266, 1987.
- Bayazit, M. Oğuz, B., Duranyıldız, İ., "Statistical Analysis of Precipitations in the Istanbul Region", ITU Research Foundation Project Report, (in Turkish), 1992.
- Dorfmann, R., "Mathematical Models: The multi structure approach", *Design of Wat. Res. Syst.* Harvard Univ., Cam., Mass, 1962.
- Duranyıldız, İ., Önöz, B., Bayazit, M., "A chance constrained LP model for short term reservoir operation optimization", *J. Sci. Env. Eng.*, 1998.
- ISKI (Istanbul Water and Sewage Authority), Istanbul Water Supply, Sewage and Drainage Master Plan Feasibility Report, IMC, 1994.
- Loucks, D.P., "Water resources systems models: Their role in planning", *J. Wat. Res. Plan. & Man.*, 118(3), 214-223, 1992.
- Rippl, W., "The capacity of storage reservoirs for water supply", *Proc. Inst. Civ. Eng.*, (71), 270-278, 1883.
- Shih, J.S. and ReVelle, C., "Water-supply operations during drought: Continuous hedging rule", *J. Wat. Res. Plan. & Man.*, 120(5), 613-629, 1994.
- Simonovic, S.P., "Reservoir systems analysis: Closing gap between theory and practice", *J. Wat. Res. Man.*, 118(3), 262-280, 1992.
- Thomas, H.A. and Burden, R.P., "Operations Research in Water Quality Management", Harvard Univ. Cambridge, Mass, 1963.
- Valds, J.B., Di Filippo, J.M., Strzepek, K.M., Restrepo, P.J., "Aggregation disaggregation approach to multi reservoir operation", *J. of Wat. Res. Plan. and Man.*, 118(4), 423-444, 1992.
- Yeh, W.W-G., "Real time reservoir operations: The California Central Valley Project case study". *Proc. Nat. Workshop on Res. Syst. Op.* ASCE, NY, 1981.
- Yeh, W.W-G., "Reservoir management and operations model: A state-of-the-art review" *WRR*, 21(12), 1797-1818, 1985.