

Chaos in metaheuristic based artificial intelligence algorithms: a short review

Gökhan ATALI¹, İhsan PEHLİVAN^{2,*}, Bilal GÜREVİN³, Halil İbrahim ŞEKER⁴

¹Mechatronics Engineering Department, Faculty of Technology, Sakarya University of Applied Sciences, Sakarya, Turkey

²Electrical and Electronics Engineering Department, Faculty of Technology, Sakarya University of Applied Sciences, Sakarya, Turkey

³Mechatronics Engineering Department, Institute of Graduate Studies, Sakarya University of Applied Sciences, Sakarya, Turkey

⁴Electrical-Electronics Engineering Department, Institute of Graduate Studies, Sakarya University of Applied Sciences, Sakarya, Turkey

Received: 01.02.2021

Accepted/Published Online: 27.05.2021

Final Version: 31.05.2021

Abstract: Metaheuristic based artificial intelligence algorithms are commonly used in the solution of optimization problems. Another area -besides engineering systems- where chaos theory is widely employed is optimization problems. Being applied easily and not trapping in local optima, chaos-based search algorithms have attracted great attention. For example, it has been reported that when random number sequences generated from different chaotic systems are replaced with parameter values in bioinspired and swarm intelligence algorithms, an increase in the performance of metaheuristic algorithms is observed. Many scientific studies on developing hybrid algorithms in which metaheuristic algorithms and chaos theory are used together are already in process.

In this article, scientific studies that cover the most popular metaheuristic algorithms in the literature in recent years and chaos theory subtopics together are examined. Great number of studies on metaheuristic algorithms and chaos issues exist in scientific literature. This article, hitherto, had to be limited to the most common meta-heuristic algorithms. In chaos-based metaheuristic algorithms, some advantages such as easy implementation, short application time and search acceleration have been addressed.

This article is believed to contribute to two groups of researchers: The first group includes researchers already using meta-heuristic algorithms and who will come to understand that they can even improve their current techniques with chaos theory subarguments. The second group includes those who currently utilize chaotic analysis and methods in areas like nonlinear prediction modeling design and who will realize that they can make their existing methods even smarter with metaheuristic methods.

Some metaheuristic algorithms use chaotic maps to solve problems such as trapping in local optimal solutions and premature convergence. In this study, such algorithms are examined through the benchmark functions they use. In addition, PSO, FA, ABC, WO algorithms are compared in terms of their common features, and algorithms with the best success rate are presented.

Key words: Chaos, artificial intelligence, metaheuristic algorithm, chaotic map, swarm, bioinspired, nature-inspired, evolutionary algorithm, physics-based algorithm, benchmark function

*Correspondence: ipehlivan@subu.edu.tr

1. Introduction

As a result of developing science and technology, the complexity of optimization problems also increases exponentially. It has always been one of the most significant objectives to maximize profits or minimize losses with the use of optimization methods in engineering as well as other technological problems. Algorithms seeking solutions to optimization problems have been developed with a metaheuristic approach as -in order to accelerate the solution of the problem- they have been commonly inspired by creatures living in nature, physics events, herd behaviors, etc. Metaheuristic algorithms are heuristic method that can provide a good enough solution to an optimization problem with an incomplete or limited computational capacity that used a higher-level procedure in computer science and mathematical optimization. These algorithms often have fast convergence to an optimum, are simple to calculate and easy to implement.

Although various optimization algorithms are available in the literature, there is no such thing as a main algorithm to be accepted as the best for any situation [1–5] and algorithm developers keep developing stronger and more efficient algorithms every day [6]. Metaheuristic based algorithms have generally been classified as bioinspired [7–11], swarm intelligence [12–15], evolutionary and nature-inspired algorithms [16–19] and physics-based [20–25].

Some metaheuristic algorithms have their disadvantages. These disadvantages are that they are sometimes easily trapped in local optimum; the convergence rate decreases significantly in the later period of the cycle; optimization finish when a near-optimum solution is found; and the accuracy that the algorithm can provide is limited. In addition, metaheuristic algorithms are not able to seek the global optimal solution while optimizing particularly complex high dimensional functions. Hybrid methods are needed to overcome such deficiencies. Experimental studies have suggested that using chaotic signals instead of random signals could be more practical.

Deterministic chaos can be defined as a semirandom behavior generated by nonlinear deterministic systems. A chaotic movement, thus, can travel through all situations within a given range without any repetition. A lot of scientific research has already been conducted on the development of smarter hybrid algorithms and solution technologies in which nonlinear science arguments such as chaos theory are used together with metaheuristic algorithms, which are one of the most basic tools of artificial intelligence. Thus, it is clear that such a trend will also exist in the future.

Some limitations had to be observed in the literature review presented in this study. For database, Web of Science (WOS), which is widely used and reliable, was preferred. Only the articles of the last 6 years were discussed in the detailed examination and comparison tables. Given the huge amount of research in the area, international conference papers, books, scientific reports, etc. could not be included. Instead, only the Science Citation Index-Expanded (SCI-Exp) articles on international scientific journals were studied. In order to restrict research areas on WOS, categories of scientific study, in the same manner, were limited to these: "computer science: information systems, artificial intelligence, interdisciplinary applications, software engineering", "electrical-electronics engineering", "engineering multidisciplinary", "robotics", "automation control systems", "physics applied", "mathematics interdisciplinary applications".

2. Metaheuristic based algorithms and chaos

In this article, given the limitations stated in Section 1, studies in the literature that cover chaos and metaheuristic algorithms were examined and presented in four sections. Detailed examinations of the optimization algorithms containing chaos are included in the section ranking based on the artificial intelligence subdomain classification of metaheuristic algorithms.

2.1. Swarm-based algorithms and chaos

Duarte and Carvalho proposed a new hybrid particle swarm optimization with a spiral-shaped mechanism (HPSO-SSM) algorithm to increase the success of the search for global optimal in regular PSO algorithm. These researchers used logistic map for the development of the proposed HPSO-SSM algorithm and they used 24 benchmark functions so as to measure the performance of their algorithm and also to make comparisons between algorithms [26].

Chen et al. proposed a new chaotic dynamic weight particle swarm optimization (CDW-PSO) algorithm to overcome deficiencies like premature convergence and easy trapping in local optimum solutions. Sine map was used for the development of the proposed CDW-PSO algorithm. Furthermore, 17 benchmark functions were used to measure the performance of the CDW-PSO algorithm and to make comparisons between algorithms [27].

Chen et al. proposed a new chaotic grouping PSO with a dynamic regrouping strategy (CGPSO-DRS) algorithm by using different chaotic maps to overcome the lack of diversity in the population during the search process of the PSO algorithm. For the performance measurement of this unique CGPSO-DRS algorithm, developed with the help of 14 chaotic maps, 41 benchmark functions were used [28].

Tian et al. proposed a sigmoid-based chaotic particle swarm optimization to prevent trapping in local optimum solutions and to develop a solution for the problem of premature convergence. In this study, the researchers used logistic map to develop chaotic particle swarm optimization with sigmoid-based acceleration coefficients (CPSOS) algorithm and they used 28 benchmark functions to measure the performance of the algorithm as well as to make comparison [29].

Koyuncu examined 10 chaotic map based PSOs to determine the essentiality of chaotic maps for PSO and also to determine the most appropriate one. In the first experiment, he used 13 benchmark functions to achieve a detailed evaluation. Chaotic PSO (CPSO) methods were tested in global function optimization. As a result of all function evaluations and comparison with the latest technological methods, Gauss map based chaotic particle swarm optimization (GM-CPSO) came to the fore with its promising conformity values [30].

Liu et al. proposed modified particle swarm optimization (MPSO) to avoid premature convergence and poor balance disadvantages of global exploration and local exploitation in the original PSO. This MPSO was developed via a chaos-based nonlinear inertia weight. These researchers ran 30 benchmark functions to measure the convergence performance of MPSO in optimization problems [31].

Nie et al. proposed an adaptive chaos particle swarm optimization (ACPSO) to adjust the parameters of the proportional-integral-derivative (PID) controller. It was combined with particle swarm optimization to avoid local minima and improve the capability of the proposed algorithm with chaotic search. To verify the performance, 6 benchmark functions were used. It was concluded that ACPSO could provide a better-quality solution for overcoming global optimization problems and avoiding premature convergence. The proposed algorithm was used to adjust the parameters of the PID control device [32].

Demir et al. proposed 1-D hybrid chaotic map-based novel swarm optimization method to achieve higher numerical results. Logistic-sine map has good statistical result, and this advantage was used directly to calculate global optimum value in their study. In order to test the success rate of the proposed hybrid chaotic map-based optimization method, 16 benchmark functions were used [33].

Table 1 exhibits the comparison of seven different PSO algorithms containing chaotic map through common features. The comparison is based on common benchmark functions and chaotic maps. Rows of

the table include logistic, circle, tent, sine, iterative, Gauss, logistic-sine chaotic maps used for improving the performance.

The columns titled “Rosenbrock”, “Rastrigin” and “Griewank” on the Tables 1– 4 are the benchmark functions used for comparison in the studies listed below them. Performance evaluation results, also known as success rate, presented on Tables 1– 4 are the mean value and standard deviation values. For example, in Table 1, the mean value and standard deviation values of the optimization system in [26], whose performance was improved using the logistic map, were $2.90 \times 10^1 \pm 1.53 \times 10^{-2}$ in Rosenbrock, and 0 in Rastrigin and Griewank.

Alatas used 7 chaotic maps to improve the convergence characteristics of the artificial bee colony (ABC) algorithm and to prevent getting trapped in local solutions and he proposed the algorithm named CABAC. Alatas also used 3 benchmark functions to measure the performance of chaotic artificial bee colony (CABAC) algorithm as well as to make comparison between algorithms [34].

Li et al. proposed a system using random perturbation to solve the deficiency of the ABC algorithm to balance between exploration and exploitation and to increase the diversity of the population. Twenty-seven benchmark functions were used to prove that the algorithm in the study, proposed for solving high-dimensional and complex optimization problems, has a higher convergence speed and search sensitivity than other ABC algorithms [35]. Table 2 shows the comparison of two different ABC algorithms containing chaotic maps with common features. The comparison is based on common benchmark functions and chaotic maps. Logistic, circle, tent, sine, iterative, Gauss chaotic maps in the rows show the chaotic maps used for performance improvement. The columns titled Rosenbrock, Rastrigin and Griewank show the benchmark functions used for the comparison.

The success rate evaluation results presented on Table 2 are the mean value and standard deviation values. For instance, the average value and standard deviation values of the optimization system in [35], whose performance was improved using Logistic map, were obtained as $1.55 \times 10^{-25} \pm 2.51 \times 10^{-26}$ in Rosenbrock and 0 in Rastrigin and Griewank.

Anand and Arora developed chaos based selfish herd optimizer (CSHO) algorithm in order to improve some disadvantages of selfish herd optimization metaheuristic algorithm, which is inspired by predatory interactions of herd and predators, such as getting easily trapped in local optimal solutions, low precision and slow convergence speeds. For the development of this novel CSHO algorithm, 10 chaotic maps were employed. In addition, 13 benchmark functions were used to measure the performance of CSHO algorithm as well as to make comparison between algorithms [36].

Altay and Alatas developed new optimization algorithms by integrating chaotic maps to eliminate the disadvantages of bird swarm optimization such as early convergence and trapping in local optima. Ten chaotic maps were used in these algorithms. Furthermore, 5 benchmark functions were used to test the performance and to make comparisons between methods [37].

Saxena et al. added the β -chaotic map to the grey wolf optimization for better exploration and exploitation qualities. In order to measure the performance of this novel grey wolf optimizer (β -GWO) algorithm, the researchers used 41 benchmark functions in total for 2 different tests [38].

Mitic et al. presented a novel approach integrating learning from demonstrations methodology and chaotic bioinspired optimization algorithms to reproduce required motion trajectories. Four chaotic methods namely chaotic bat algorithm, chaotic firefly algorithm, chaotic accelerated particle swarm optimization, and newly developed chaotic grey wolf optimizer (CGWO) were applied. The algorithms mentioned were compared in reproduction of two complex motion trajectories which had different length and shape. Tests on a mobile

Table 1. Benchmark success rate of particle swarm optimization.

Rosenbrock							
Chaotic maps	[26] mean \pm std	[27] mean \pm std	[33] mean \pm std	[28] mean \pm std	[29] mean \pm std	[30] mean	[31] mean \pm std
Logistic	2.90×10^1 $\pm 1.53 \times 10^{-2}$	***	***	3.03×10^1 $\pm 2.06 \times 10^1$	2.33×10^1 $\pm 3.61 \times 10^{-1}$	***	5.04×10^2 $\pm 2.56 \times 10^1$
Circle	***	***	***	2.90×10^{-1} $\pm 1.97 \times 10^1$	***	***	***
Tent	***	***	***	3.35×10^1 $\pm 2.34 \times 10^1$	***	***	***
Sine	***	2.81×10^1 $\pm 1.88 \times 10^{-1}$	***	3.59×10^1 $\pm 2.56 \times 10^1$	***	***	***
Iterative	***	***	***	2.99×10^1 $\pm 2.71 \times 10^1$	***	***	***
Gauss	***	***	***	***	***	***	***
Logistic-sine	***	***	0 ± 0	***	***	***	***
Rastrigin							
Chaotic maps	[26] mean \pm std	[27] mean \pm std	[33] mean \pm std	[28] mean \pm std	[29] mean \pm std	[30] mean	[31] mean \pm std
Logistic	0 ± 0	***	***	2.90×10^1 $\pm 7.38 \times 10^1$	6.92×10^{-5} $\pm 8.05 \times 10^{-5}$	6.63×10^1	5.70×10^2 $\pm 1.75 \times 10^1$
Circle	***	***	***	4.38×10^1 $\pm 1.45 \times 10^1$	***	9.36×10^1	***
Tent	***	***	***	3.09×10^1 $\pm 1.04 \times 10^1$	***	1.34×10^1	***
Sine	***	0 ± 0	***	3.03×10^1 $\pm 9.48 \times 10^0$	***	6.84×10^1	***
Iterative	***	***	***	3.01×10^1 $\pm 1.01 \times 10^1$	***	8.84×10^1	***
Gauss	***	***	***	***	***	0	***
Logistic-sine	***	***	0 ± 0	***	***	***	***
Griewank							
Chaotic maps	[26] mean \pm std	[27] mean \pm std	[33] mean \pm std	[28] mean \pm std	[29] mean \pm std	[30] mean	[31] mean \pm std
Logistic	0 ± 0	***	***	7.22×10^{-2} $\pm 8.35 \times 10^{-3}$	7.26×10^{-1} $\pm 4.68 \times 10^{-2}$	1.08×10^0	3.40×10^3 $\pm 2.40 \times 10^3$
Circle	***	***	***	4.84×10^{-3} $\pm 8.10 \times 10^{-3}$	***	2.02×10^0	***
Tent	***	***	***	5.17×10^{-3} $\pm 6.64 \times 10^{-3}$	***	3.79×10^0	***
Sine	***	0 ± 0	***	5.42×10^{-3} $\pm 8.29 \times 10^{-3}$	***	1.17×10^0	***
Iterative	***	***	***	2.06×10^{-3} $\pm 9.00 \times 10^{-3}$	***	1.31×10^0	***
Gauss	***	***	***	***	***	0	***
Logistic-sine	***	***	1.40×10^{-2} $\pm 2.83 \times 10^{-2}$	***	***	***	***

Table 2. Benchmark success rate of artificial bee colony optimization.

	Rosenbrock		Rastrigin		Griewank	
Chaotic maps	[34] mean ± std	[35] mean ± std	[34] mean ± std	[35] mean ± std	[34] mean ± std	[35] mean ± std
Logistic	6×10^{-5} $\pm 4 \times 10^{-6}$	1.55×10^{-25} $\pm 2.51 \times 10^{-26}$	89×10^{-5} $\pm 69 \times 10^{-6}$	0×10^0 $\pm 0 \times 10^0$	25×10^{-5} $\pm 22 \times 10^{-6}$	0×10^0 $\pm 0 \times 10^0$
Circle	4×10^{-5} $\pm 4 \times 10^{-6}$	***	88×10^{-5} $\pm 81 \times 10^{-6}$	***	17×10^{-5} $\pm 17 \times 10^{-6}$	***
Tent	6×10^{-5} $\pm 5 \times 10^{-6}$	***	87×10^{-5} $\pm 79 \times 10^{-6}$	***	23×10^{-5} $\pm 16 \times 10^{-6}$	***
Sine	***	***	***	***	***	***
Iterative	***	***	***	***	***	***
Gauss	7×10^{-5} $\pm 6 \times 10^{-6}$	***	91×10^{-5} $\pm 82 \times 10^{-6}$	***	23×10^{-5} $\pm 21 \times 10^{-6}$	***

robot exhibited the applicability of the proposed approach. To test the performance and to determine the best map for CGWO, this algorithm was tested on ten benchmark problems using ten well-known chaotic maps [39].

Gaidhane and Nigam proposed a novel hybrid algorithm based on the GWO and ABC algorithm. In the GWO-ABC algorithm, wolves adopt the information sharing strategy of bees to improve their exploration abilities, while maintaining original hunting strategies to preserve their exploitability. Additionally, a new method based on chaotic mapping and opposition-based learning was proposed to initialize the population. Through this method, it is aimed to help the algorithm to avoid premature convergence and to direct the search towards the potential search region in a quicker manner. For performance evaluation of GWO-ABC, it was tested with 27 comparison functions with different properties. Furthermore, GWO-ABC based FOPID controller was designed for 2 degree-of-freedom (DOF) robotic manipulator. All the design requirements such as low overshoot, better rise time, faster settling time, minimum steady state error and performance index were evaluated and thus GWO-ABC was found to be efficient in optimizing the parameters of FOPID controllers [40].

2.2. Chaos in nature-inspired and evolutionary algorithms

Arora and Singh proposed a novel chaotic butterfly optimization algorithm (BOA) by using different chaotic maps to increase the convergence speed of the original BOA and avoid local optima. Ten chaotic maps were used for the development of the CBOA proposed by these researchers. Arora and Singh employed 3 benchmark functions to test the performance of chaotic butterfly optimization algorithm (CBOA) and to make comparisons between algorithms [41].

Zhang et al. proposed a new variation of the firefly algorithm (FA) used for feature selection problems. They made use of simulated annealing mechanism to avoid local optimum and lessen the premature convergence problem. Logistics chaotic map was used in swarm initialization to provide more swarm diversity. For the acceleration of convergence, they utilized logistic, Gauss, sinusoidal, tent and kent maps as local and global attractiveness coefficients. The proposed FA model was evaluated with a total of 40 feature selection problems and 8 benchmark functions [42].

Brajevic and Stanimirovic proposed a new developed chaotic FA using Gauss map to solve the global optimization problems of the original FA, and they used 19 benchmark functions to assess the performance of the ICFA (improved chaotic firefly algorithm) [43].

Brajevic and Ignjatovic proposed upgraded firefly algorithm (UFA) to promote the problem-solving performance of the original FA and to avoid getting trapped in local optimum. A chaotic map was used for the development of this new firefly algorithm. Twenty-four benchmark functions were employed to measure the performance of the proposed chaotic map-based UFA method [44].

Table 3 shows the comparison of two different FAs with chaotic maps in common features. Comparison was based on common benchmark functions and chaotic maps. Logistic, circle, tent, sine, iterative and Gauss in the rows are the chaotic maps used for performance improvement. On Table 3, for example, the mean value and standard deviation values of the optimization system [42], whose performance was improved using logistic map, were obtained as $1.53 \times 10^{-5} \pm 5.96 \times 10^{-6}$ in the Griewank benchmark function.

Table 3. Benchmark success rate of firefly optimization.

	Rosenbrock		Rastrigin		Griewank	
Chaotic maps	[43] mean ± std	[42] mean ± std	[43] mean ± std	[42] mean ± std	[43] mean ± std	[42] mean ± std
Logistic	***	***	***	***	***	$1.53 \times 10^{-5} \pm 5.96 \times 10^{-6}$
Circle	***	***	***	***	***	***
Tent	***	***	***	***	***	$1.53 \times 10^{-5} \pm 5.96 \times 10^{-6}$
Sine	***	***	***	***	***	***
Iterative	***	***	***	***	***	***
Gauss	$8.81 \times 10^{-6} \pm 1.30 \times 10^{-5}$	***	$5.92 \times 10^{-17} \pm 3.19 \times 10^{-16}$	***	$3.70 \times 10^{-18} \pm 1.99 \times 10^{-17}$	$1.53 \times 10^{-5} \pm 5.96 \times 10^{-6}$

Kaur and Arora proposed a new chaotic whale optimization algorithm (CWOA) using 10 chaotic maps to improve the convergence property of whale optimization algorithm (WOA). These researchers used 20 benchmark functions to measure the performance of the CWOA and to make comparisons between algorithms [45].

Ding et al. proposed three developed versions of WOA with chaos initialization, nonlinear convergence factor and chaotic inertia since WOA, based on social predatory behavior of humpback whales, has some deficiencies like easily trapping in a local optimum while solving complex problems and slow convergence speed. 19 benchmark functions were utilized to measure and test the performance of the proposed versions of WOA. It was also compared with two recently proposed hybrid WOAs. Experimental results proved the proposed algorithms to perform better with regard to complexity and convergence speed [46].

Table 4 shows the comparison of two different WO algorithms including chaotic maps in common features. The comparison was based on common benchmark functions and chaotic maps. Logistic, circle, tent, sine, iterative, cubic chaotic maps on the rows are the chaotic maps used for performance improvement. For example, the average value and standard deviation values, on Table 4, of the optimization system [45], whose performance was developed using the sine map, were obtained as $2.88 \times 10^1 \pm 2.48 \times 10^{-2}$ in Rosenbrock, while they were obtained as 0 in Rastrigin and Griewank.

Table 4. Benchmark success rate of whale optimization.

	Rosenbrock		Rastrigin		Griewank	
Chaotic maps	[45] mean \pm std	[46] mean \pm std	[45] mean \pm std	[46] mean \pm std	[45] mean \pm std	[46] mean \pm std
Logistic	2.88×10^1 $\pm 4.59 \times 10^{-2}$	***	0.00×10^0 $\pm 0.00 \times 10^0$	***	0.00×10^0 $\pm 0.00 \times 10^0$	***
Circle	4.61×10^5 $\pm 5.17 \times 10^5$	***	2.54×10^2 $\pm 4.56 \times 10^1$	***	1.41×10^2 $\pm 6.63 \times 10^1$	***
Tent	2.88×10^1 $\pm 3.78 \times 10^{-2}$	***	0.00×10^0 $\pm 0.00 \times 10^0$	***	0.00×10^0 $\pm 0.00 \times 10^0$	***
Sine	2.88×10^1 $\pm 2.48 \times 10^{-2}$	***	0.00×10^0 $\pm 0.00 \times 10^0$	***	0.00×10^0 $\pm 0.00 \times 10^0$	***
Iterative	1.95×10^5 $\pm 2.18 \times 10^5$	***	2.55×10^2 $\pm 4.71 \times 10^1$	***	1.04×10^2 $\pm 4.36 \times 10^1$	***
Cubic	***	0.00×10^0 $\pm 0.00 \times 10^0$	***	0.00×10^0 $\pm 0.00 \times 10^0$	***	0.00×10^0 $\pm 0.00 \times 10^0$

Wei and Yu proposed a novel adaptive cuckoo search (CSAPC) algorithm to increase the optimization performance of the original cuckoo search (CS). Employing Rössler chaotic system in CSAPC algorithm, Wei and Yu used 48 benchmark functions to evaluate the performance of the logistic sine map-based optimization algorithm [47].

Sawant and Manoharan proposed a new hybrid global optimization algorithm, based on wind driven optimization (WDO) and CS, to overcome band selection problems in hyperspectral image classification. The approach proposed in their study used the Chebyshev chaotic map to initialize the population at initial step. Three benchmark functions were used to measure the performance of WDO-modified cuckoo search (WDO-MCS) algorithm and to make comparisons between algorithms [48].

Toz proposed chaotic vortex search (CVS) method to decrease the maximum number of iterations (NOI) and solve optimization problems with VS algorithm. For the development of the chaos-based VS algorithm proposed in the study, 10 logistic maps were used. Fifty benchmark functions were used to measure the performance of CVS algorithm and to make comparisons between algorithms [49].

Qu et al. proposed a new optimization algorithm based on information exchange to optimize the application of Harris Hawks optimization algorithm to engineering problems. These researchers designed a nonlinear escaping energy factor with chaos disturbance to balance the local searching and the global searching of the algorithm in a better manner. In addition, they conducted test with four benchmark test functions and five CEC-2017 real-parameter numerical optimization problems. It was concluded that the proposed algorithm performed better than other intelligence optimization algorithms regarding the convergence rate, solution accuracy and robustness [50].

Hongwei et al. proposed an advanced chaos-based moth-flame optimization (CMFO) algorithm to prevent trapping in local optimum and to solve the slow convergence speed problem, like in other meta-heuristic algorithms. They utilized 10 logistic maps during the development phase of the algorithm. They also used 18 benchmark functions to measure its performance and to make comparisons between algorithms [51].

Huang and Lei proposed a new damage detection method based on sensitivity analysis and chaotic moth-flame-invasive weed optimization (CMF-IWO), used to concurrently detect damage to structural elements and bearings. The optimization effectiveness of the hybrid algorithm was tested and verified via five benchmark functions and damage identification numerical example of a simply supported beam. The results revealed its excellent global search ability and better convergence effectiveness [52].

Yu et al. proposed a new advanced chaotic bat algorithm (BA) using logistic map to solve the global optimization problems of the original BA. For the comparison of the performance of the proposed chaotic mapping enhanced bat algorithm (CEBA) with 18 metaheuristic algorithms, they used 30 benchmark functions [53].

Wangkhamhan proposed a new adaptive chaotic satin bowerbird optimization (AC-SBO) algorithm to increase the global convergence speed of the satin bowerbird optimization (SBO) algorithm and achieve higher performance. Ten benchmark functions were used to measure the performance of this new SBO algorithm [54].

Li et al. proposed a novel algorithm including triple distinct search dynamics (TDS), namely spherical search, hypercube search and chaotic local search, to create an algorithm with better exploration and exploitation abilities in solving complex optimization problems. It was verified via 30 CEC2017 benchmark functions and three real-world optimization problems [55].

Misaghi and Yaghoobi proposed a new chaotic-based invasive weed optimization (IWO) using Logistic map to improve the performance of IWO standard deviation algorithm. To measure the performance of chaotic invasive weed optimization (CIWO), which is used to optimally adjust the PID controller parameters, the researchers used 5 benchmark functions [56].

2.3. Bioinspired algorithms and chaos

Yousri et al. proposed a new chaotic FPA using 3 chaotic maps and their fractional-order versions for the acceleration of the convergence speed of the flower pollination algorithm (FPA). The performance of the proposed chaotic flower pollination algorithm (CFPA) was verified on 4 benchmark functions [57].

Saha and Mukherjee presented a new optimization technique for the optimal allocation of distributed generation (DG) units in the radial distribution system (RDS). For the solution of this optimal DG allocation, a new metaheuristic, i.e. multiobjective modified symbiotic organisms search (MOMSOS) algorithm was proposed. To enhance diversity in the population, a chaos-based crossover operator was utilized in the parasitism phase of the proposed MOMSOS. The performance of the proposed MOMSOS algorithm was verified on ten test problems from CEC 2009 multiobjective benchmark test suite [58].

Zhang et al. included the Gauss mutation operator in fruit fly optimization algorithm (FOA) to prevent its global optimal premature convergence. These researchers utilized logistic map to promote the local searching capability of the agent swarm. They also used 23 benchmark functions to measure the performance of the chaotic fruit fly optimization algorithm (CFOA) [59].

Sayed et al. presented a new algorithm, namely chaotic optimal foraging algorithm (COFA), combining chaos with optimal foraging algorithm (OFA) to improve the search history and population fitness and increase the convergence rate. In this article, 10 optimal food search algorithms were presented using 10 chaotic maps. Thirteen benchmark functions were used to measure the performance of the proposed COFA as well as the performance of exploration, exploitation, food search history, population fitness and also to make comparison between algorithms [60].

2.4. Physics-based algorithms and chaos

Ouertani et al. attempted to improve lightning search algorithm (LSA) and proposed a new CLSA. Eleven chaotic maps were utilized in the development of the CLSA algorithm while 7 benchmark functions were used for its performance measurement [61]. Xu and Mei proposed an advanced DC-WCA algorithm to help water cycle algorithm (WCA) escape from local optima and discover global optimal solutions. A logistic chaotic map was used in the development of this new DC-WCA algorithm. Additionally, 6 benchmark functions were utilized to measure the performance of the DC-WCA method and to make comparisons between algorithms [62].

Graphs expressing the convergence speeds of 4 different metaheuristic algorithms that are commonly used are given in Figure. Looking at the WOA algorithm, it is understood that it has the fastest convergence. Likewise, it is seen that Ant Lion Optimization (ALO) has the slowest convergence speed.

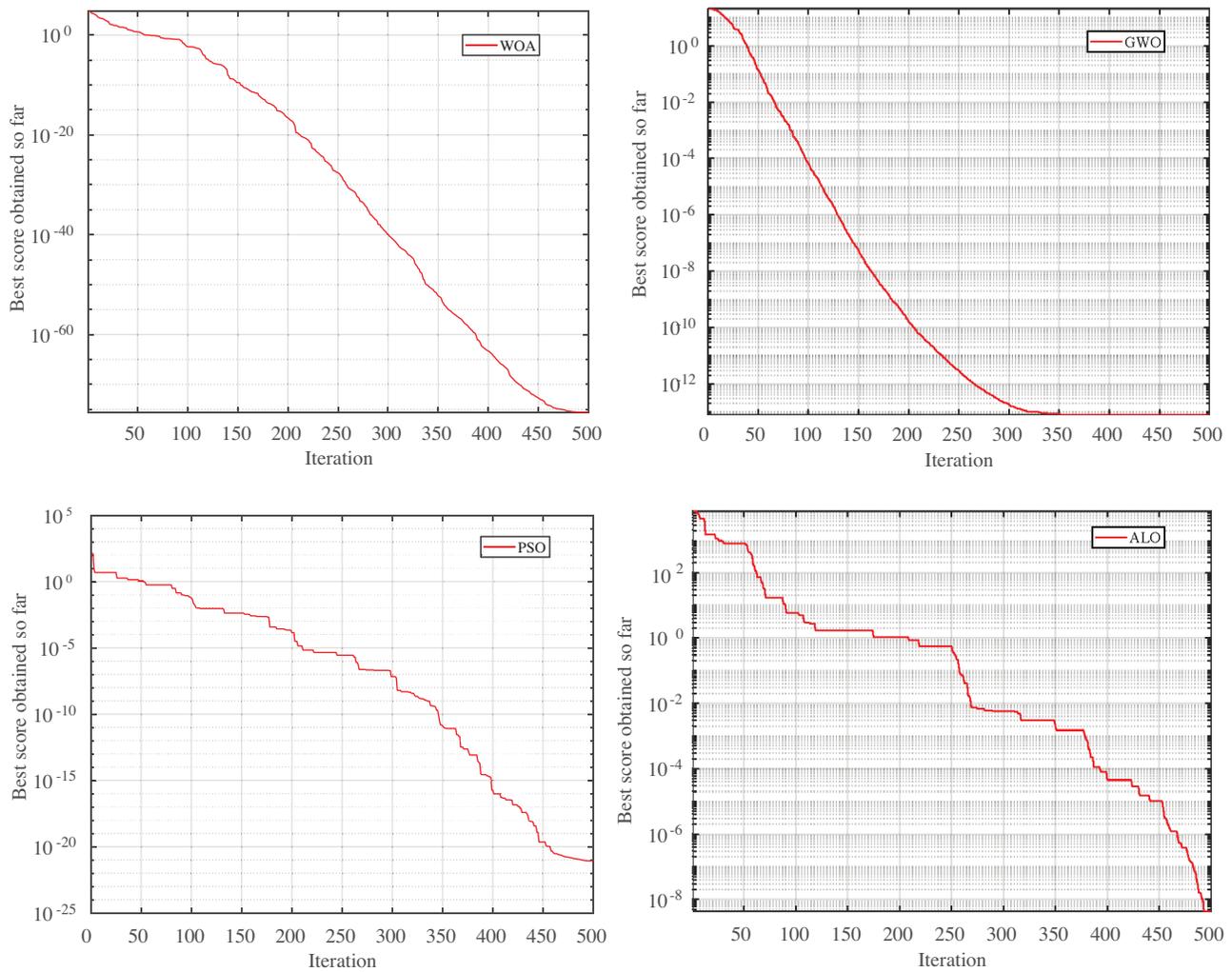


Figure. Convergence graphs of benchmark functions.

3. Result and discussion

It is clear that the artificial intelligence and the chaos theory, in recent years, have come to be utilized in a huge variety of areas as the followings: logistic map-based image recognition for IoT devices; determination of mobile robot trajectory using chaotic bioinspired optimization algorithm; PID controller design for automatic voltage regulator system with CPSO algorithm; RGB image encoding using chaotic system, DNA technology and 15-puzzle artificial intelligence problem; chaos and ant colony optimization based traveling salesman problem; planning of power consumption, cost and production time in cloud computing environment with chaos based GWO algorithm; real-time detection of energy consumption of IoT network nodes based on chaotic ant colony; chaotic based watermark method for data security; parameter identification of photovoltaic modules using chaotic based Harris Hawks algorithm.

Detailed examination of the studies in the literature has revealed that performance enhancements for solving problems like trapping in local optimal solutions and premature convergence of metaheuristic algorithms are generally performed with chaotic maps. Section 2 includes tables that present the success rates of PSO, FA, ABC, WO algorithms with regard to common chaotic maps and benchmark functions.

Of the studies with PSO algorithm on Table 1: the study by Demir et al. [33] using logistic-sine map observes the highest success rate result, in comparison with Rosenbrock benchmark function; the studies by Duarte and Carvalho [26] using llogistic map, Chen et al. [27] using sin map, Demir et al. [33] using logistic-sine map and Koyuncu [30] using Gauss map observe the highest success rate results, in comparison with Rastrigin benchmark function; the studies by Duarte and Carvalho [26] using logistic map, Chen et al. [27] using sin map and Koyuncu [30] using Gauss map observe the highest success rate results, in comparison with Griewank benchmark function.

Of the studies with ABC algorithm on Table 2: the highest success rate result, for comparison with Rosenbrock, Rastrigin, Griewank benchmark functions, is observed in the study by Li et al. [35] using logistic map.

Of the studies with FA on Table 3: the highest success rate result, for comparison with Rosenbrock, Rastrigin, Griewank benchmark functions, is observed in the study by Brajevic and Stanimirovic [43] using Gauss map.

Of the studies with WO algorithm on Table 4: the study by Ding et al. [46] using cubic map observes the highest success rate result, in comparison with Rosenbrock benchmark function; the study by Kaur and Arora [45] using logistic, tent and sine map observes the highest success rate result, in comparison with Rastrigin benchmark function; the studies by Kaur and Arora [45] using logistic, tent and sine map and Ding et al. [46] using cubic map observe the highest success rate results, in comparison with Griewank benchmark function.

It seems inevitable that chaos theory arguments with high predictive capacity and nonlinear modelling will be used with metaheuristic methods for progress in areas, including complex problems, such as “cyber security”, “energy technologies”, “advanced robotics”, “drone technologies”, “aviation and space technologies”, “software technologies”, “Internet of Things”, “nanotechnology and materials science”, “smart farming” all of which will be of even greater importance in the near future.

It is believed that the comparative review of studies including “chaos” arguments, in the future, in areas of the artificial intelligence such as deep learning, machine learning, genetic algorithm, artificial neural network, cellular neural network, convolutional neural network, simulated annealing, adaptive-network-based fuzzy inference system, reinforcement learning will greatly contribute to researchers in developing new chaos based metaheuristic algorithms.

Acknowledgment

The authors cordially thank the Sakarya University of Applied Sciences Robot Technologies and Intelligent Systems Application and Research Center (ROTASAM), Turkey for supporting this work.

References

- [1] Wolpert DH, Macready WG. No free lunch theorems for optimization. *IEEE Transactions on Evolutionary Computation* 1997; 1 (1): 67-82. doi: 10.1109/4235.585893
- [2] Civicioğlu P. Backtracking search optimization algorithm for numerical optimization problems. *Applied Mathematics and Computation* 2013; 219 (15): 8121-8144.
- [3] Mirjalili S, Mirjalili SM, Lewis A. Grey wolf optimizer. *Advances in Engineering Software* 2014; 69: 46-61.
- [4] Dogan B, Olmez TA. A new metaheuristic for numerical function optimization vortex search algorithm. *Information Sciences* 2015; 293: 125-145.
- [5] Zaharie D. Control of population diversity and adaptation in differential evolution algorithms. In: *Mendel 9th International Conference on Soft Computing*; Timișoara, Romania; 2003. pp. 41-46.
- [6] Ahmia I, Aïder M. A novel metaheuristic optimization algorithm: the monarchy metaheuristic. *Turkish Journal of Electrical Engineering and Computer Science* 2019; 27 (1): 362-376.
- [7] Yang XS, Deb S. Cuckoo search via Lévy flights. In: *World Congress on Nature & Biologically Inspired Computing (NaBIC)*; Coimbatore, India; 2009. pp. 210-214.
- [8] Taşpınar N, Şimşir Ş. An efficient technique based on firefly algorithm for pilot design process in OFDM-IDMA systems. *Turkish Journal of Electrical Engineering & Computer Sciences* 2018; 26 (2): 817-829.
- [9] Yang XS. Bat algorithm for multi-objective optimisation. *International Journal of Bio-Inspired Computation* 2011; 3 (5): 267-274.
- [10] Guney K, Basbug S. Interference suppression of linear antenna arrays by amplitude-only control using a bacterial foraging algorithm. *Progress In Electromagnetics Research* 2008; 79: 475-497. doi: 10.2528/PIER07110705
- [11] Chelly Z, Elouedi Z. A survey of the dendritic cell algorithm. *Knowledge and Information Systems* 2016; 48 (3): 505-535.
- [12] Kennedy J, Eberhart R. Particle swarm optimization. In: *Proceedings of ICNN'95-international conference on neural networks*; Perth, WA, Australia; 1995. pp. 1942-1948.
- [13] Erçin Ö, Çoban R. Identification of linear dynamic systems using the artificial bee colony algorithm. *Turkish Journal of Electrical Engineering Computer Sciences* 2012; 20 (1): 1175-1188.
- [14] Öztürk D, Cebeci M. Calculation of surface leakage currents on high voltage insulators by ant colony algorithm-supported FEM. *Turkish Journal of Electrical Engineering & Computer Sciences* 2015; 23 (4): 1009-1024.
- [15] Gündüz M, Kiran MS, Özceylan E. A hierarchic approach based on swarm intelligence to solve the traveling salesman problem. *Turkish Journal of Electrical Engineering & Computer Sciences* 2015; 23 (1): 103-117.
- [16] Storn R, Price K. Differential evolution: a simple and efficient adaptive scheme for global optimization over continuous spaces. *International Computer Science Institute* 1995; 1-12.
- [17] Goldberg DE, Holland JH. Genetic algorithms and machine learning. *Machine Learning* 1988; 3: 95-99.
- [18] Edmonds B. Meta-genetic programming: co-evolving the operators of variation. *Turkish Journal of Electrical Engineering & Computer Sciences* 2001; 9 (1): 13-29.
- [19] Jiao L, Li L, Shang R, Liu F, Stolkin R. A novel selection evolutionary strategy for constrained optimization. *Information Sciences* 2013; 239: 122-141.

- [20] Geem ZW, Kim JH, Loganathan GV. A new heuristic optimization algorithm: harmony search. *Simulation* 2001; 76 (2): 60-68.
- [21] Duman S, Yörükeren N, Altaş İH. Gravitational search algorithm for determining controller parameters in an automatic voltage regulator system. *Turkish Journal of Electrical Engineering & Computer Sciences* 2016; 24 (4): 2387-2400.
- [22] Kumar S, Datta D, Singh SK. Black hole algorithm and its applications. *Computational Intelligence Applications in Modeling and Control* 2015; 147-170.
- [23] Hatamlou A. Black hole: a new heuristic optimization approach for data clustering. *Information sciences* 2013; 222: 175-184.
- [24] Recioui A. Application of a galaxy-based search algorithm to MIMO system capacity optimization. *Arabian Journal for Science and Engineering* 2016; 41 (9): 3407-3414.
- [25] Precup RE, David RC, Petriu EM, Preitl S, Rădac MB. Novel adaptive charged system search algorithm for optimal tuning of fuzzy controllers. *Expert Systems with Applications* 2014; 41 (4): 1168-1175.
- [26] Duarte HMM, Dse Carvalho RL. Hybrid particle swarm optimization with spiral-shaped mechanism for solving high-dimension problems. *Academic Journal on Computing, Engineering and Applied Mathematics* 2020; 1 (1): 1-6.
- [27] Chen K, Zhou F, Liu A. Chaotic dynamic weight particle swarm optimization for numerical function optimization. *Knowledge-Based Systems* 2018; 139: 23-40.
- [28] Chen K, Xue B, Zhang M, Zhou F. Novel chaotic grouping particle swarm optimization with a dynamic regrouping strategy for solving numerical optimization tasks. *Knowledge-Based Systems* 2020; 10: 55-68.
- [29] Tian D, Zhao X, Shi Z. Chaotic particle swarm optimization with sigmoid-based acceleration coefficients for numerical function optimization. *Swarm and Evolutionary Computation* 2019; 51: 100573.
- [30] Koyuncu H. GM-CPSO: A new viewpoint to chaotic particle swarm optimization via Gauss map. *Neural Processing Letters* 2020; 52: 241-266.
- [31] Liu H, Zhang XW, Tu LP. A modified particle swarm optimization using adaptive strategy. *Expert Systems with Applications* 2020; 113353.
- [32] Nie SK, Wang YJ, Xiao S, Liu Z. An adaptive chaos particle swarm optimization for tuning parameters of PID controller. *Optimal Control Applications and Methods* 2017; 38 (6): 1091-1102.
- [33] Demir FB, Tuncer T, Kocamaz AF. A chaotic optimization method based on logistic-sine map for numerical function optimization. *Neural Computing and Applications* 2020; 1-13.
- [34] Alatas B. Chaotic bee colony algorithms for global numerical optimization. *Expert systems with applications* 2010; 37 (8): 5682-5687.
- [35] Li M, Zhao H, Weng X, Huang H. Artificial bee colony algorithm with comprehensive search mechanism for numerical optimization. *Journal of Systems Engineering and Electronics* 2015; 26 (3): 603-617.
- [36] Anand P, Arora S. A novel chaotic selfish herd optimizer for global optimization and feature selection. *Artificial Intelligence Review* 2020; 53 (2): 1441-1486.
- [37] Altay EV, Alatas B. Bird swarm algorithms with chaotic mapping. *Artificial Intelligence Review* 2020; 53 (2): 1373-1414.
- [38] Saxena A, Kumar R, Das S. β -chaotic map enabled grey wolf optimizer. *Applied Soft Computing* 2019; 75: 84-105.
- [39] Mitić M, Vuković N, Petrović M, Miljković Z. Chaotic metaheuristic algorithms for learning and reproduction of robot motion trajectories. *Neural Computing and Applications* 2018; 30 (4): 1065-1083.
- [40] Gaidhane PJ, Nigam MJ. A hybrid grey wolf optimizer and artificial bee colony algorithm for enhancing the performance of complex systems. *Journal of Computational Science* 2018; 27: 284-302.

- [41] Arora S, Singh S. An improved butterfly optimization algorithm with chaos. *Journal of Intelligent & Fuzzy Systems* 2017; 32 (1): 1079-1088.
- [42] Zhang L, Mistry K, Lim CP, Neoh SC. Feature selection using firefly optimization for classification and regression models. *Decision Support Systems* 2018; 106: 64-85.
- [43] Brajević I, Stanimirović P. An improved chaotic firefly algorithm for global numerical optimization. *International Journal of Computational Intelligence Systems* 2018; 12 (1): 131-148.
- [44] Brajević I, Ignjatović J. An upgraded firefly algorithm with feasibility-based rules for constrained engineering optimization problems. *Journal of Intelligent Manufacturing* 2019; 30 (6): 2545-2574.
- [45] Kaur G, Arora S. Chaotic whale optimization algorithm. *Journal of Computational Design and Engineering* 2018; 5 (3): 275-284.
- [46] Ding H, Wu Z, Zhao L. Whale optimization algorithm based on nonlinear convergence factor and chaotic inertial weight. *Concurrency and Computation: Practice and Experience* 2020; 32 (24): 5949.
- [47] Wei J, Yu Y. A novel cuckoo search algorithm under adaptive parameter control for global numerical optimization. *Soft Computing* 2019; 24: 4917-4940.
- [48] Sawant S, Manoharan P. A hybrid optimization approach for hyperspectral band selection based on wind driven optimization and modified cuckoo search optimization. *Multimedia Tools and Applications* 2020; 1-24.
- [49] Toz M. Chaos-based Vortex Search algorithm for solving inverse kinematics problem of serial robot manipulators with offset wrist. *Applied Soft Computing* 2020; 89: 106074.
- [50] Qu C, He W, Peng X, Peng X. Harris Hawks optimization with information exchange. *Applied Mathematical Modelling* 2020; 84: 52-75.
- [51] Hongwei LI, Jianyong LIU, Liang CHEN, Jingbo BAI, Yangyang SUN et al. Chaos-enhanced moth-flame optimization algorithm for global optimization. *Journal of Systems Engineering and Electronics* 2019; 30 (6): 1144-1159.
- [52] Huang M, Lei Y. Bearing damage detection of a reinforced concrete plate based on sensitivity analysis and chaotic moth-flame-invasive weed optimization. *Sensors* 2020; 20 (19): 54-88.
- [53] Yu H, Zhao N, Wang P, Chen H, Li C. Chaos-enhanced synchronized bat optimizer. *Applied Mathematical Modelling* 2020; 77: 1201-1215.
- [54] Wangkhamhan T. Adaptive chaotic satin bowerbird optimisation algorithm for numerical function optimisation. *Journal of Experimental & Theoretical Artificial Intelligence* 2020; 1-28.
- [55] Li X, Cai Z, Wang Y, Todo Y, Cheng J et al. TDSD: a new evolutionary algorithm based on triple distinct search dynamics. *IEEE Access* 2020; 8: 76752-76764.
- [56] Misaghi M, Yaghoobi M. Improved invasive weed optimization algorithm (IWO) based on chaos theory for optimal design of PID controller. *Journal of Computational Design and Engineering* 2019; 6 (3): 284-295.
- [57] Yousri D, Allam D, Babu TS, AbdelAty AM, Radwan AG et al. Fractional chaos maps with flower pollination algorithm for chaotic systems parameters identification. *Neural Computing & Applications* 2020; 32: 16291-16327.
- [58] Saha S, Mukherjee V. A novel multi-objective modified symbiotic organisms search algorithm for optimal allocation of distributed generation in radial distribution system. *Neural Computing and Applications* 2020; 1-21.
- [59] Zhang X, Xu Y, Yu C, Heidari AA, Li S et al. Gaussian mutational chaotic fruit fly-built optimization and feature selection. *Expert Systems with Applications* 2020; 141: 112976.
- [60] Sayed GI, Solyman M, Hassanien AE. A novel chaotic optimal foraging algorithm for unconstrained and constrained problems and its application in white blood cell segmentation. *Neural Computing and Applications* 2019; 31 (11): 7633-7664.
- [61] Ouertani MW, Manita G, Korbaa O. Chaotic lightning search algorithm. *Soft Computing* 2020; 1-17.
- [62] Xu Y, Mei Y. A modified water cycle algorithm for long-term multi-reservoir optimization. *Applied Soft Computing* 2018; 71: 317-332.