

# Multiobjective FET modeling using particle swarm optimization based on scattering parameters with Pareto optimal analysis

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## Abstract

*In this paper, design-oriented field effect transistor (FET) models are produced. For this purpose, FET modeling is put forward as a constrained, multiobjective optimization problem. Two novel methods for multiobjective optimization are employed: particle swarm optimization (PSO) uses the single-objective function, which gathers all of the objectives as aggregating functions; and the nondominated sorting genetic algorithm-II (NSGA-II) sorts all of the trade-off solutions on the Pareto frontiers. The PSO solution is compared with the Pareto optimum solutions in the biobjective plane and the success of the first method is verified. Furthermore, the resulting FET models are compared with similar FET models from the literature, and thus a comparative study is put forward with respect to the success of the optimization algorithms and the performances and utilizations of the models in the amplification circuits.*

**Key Words:** *FET modeling, scattering parameters, stability, particle swarm optimization, pareto optimality*

## 1. Introduction

In the microwave frequency range, characterization of a linear N-Port is based on the reflections and transmissions of the electromagnetic waves incident on the ports with respect to the finite reference terminations, commonly called scattering parameters (S-parameters) and defined on a rigorous mathematical basis by Kurokawa [1]. Hereafter, characterization of microwave transistors are given by the S-parameters from the manufacturer's data sheets depending on the bias conditions and operation frequency. Thus, analysis of the potential performance of a microwave transistor is generally performed using either S-parameters or other parameters, such as

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Z- or Y-, obtained by converting the S-parameters. Among the main performances, the stability, gain, noise, input voltage standing wave ratio (VSWR), and output VSWR of a microwave transistor can be considered, respectively presented in pioneering works [2-9]. In design of the linear microwave amplifier circuits, meanwhile, transistor models based on the S-parameters are employed. Paoloni and D'Agostino proposed a design-oriented field effect transistor (FET) model in conjunction with an appropriate design procedure for the distributed amplifiers, presented where the effects of the various elements of the complete FET model are identified and properly taken into account in the recalculated elements of the new simplified FET circuit proposed [10]. Günel applied a continuous hybrid approach (CHA) based on a continuous parameter genetic algorithm (CPGA) and the controlled random search-2 algorithm to determine the FET model elements [11]. Other typical nongradient algorithms such as the fuzzy hybrid approach (FHA), the CPGA, and the genetic algorithm (GA) have been utilized to obtain FET model elements ensuring the maximum transducer power gain within an operation frequency band for the 50- $\Omega$  input/output reference terminations, and a comparison based on the gain performances of FET models obtained by CHA and the nongradient algorithms was realized.

In this work, the FET model elements were determined so that all of the S-parameters could be optimized, and particle swarm optimization (PSO) was implemented as a high-performance evolutionary algorithm capable of solving general N-dimensional, linear, or nonlinear optimization problems. FET modeling for the transducer power gain is very essential, but reflection and reverse transmission losses should also be taken into account in the design stage. For the purpose of comparison, 2 kinds of FET modeling within the technological limits were carried out. The first FET model was constructed from the results of the optimization process with a single-objective function that included only the maximization of transmission parameter  $S_{21}$ , corresponding to the maximum transducer power gain ( $G_T$ ) with 50- $\Omega$  input/output terminations. The second FET model resulted from the multiobjective optimization process, in which the multiple objectives were normalized and processed into a single function as contributors commonly known as aggregating functions. The multiple objectives consist of all of the expected performance requirements from an active device, including the maximum transducer power gain ( $G_T \Leftrightarrow S_{21}$ ), minimum input reflection ( $S_{11}$ ), minimum reverse transmission ( $S_{12}$ ), and minimum output reflection ( $S_{22}$ ). The resulting FET models obtained by the PSO algorithm were compared with the models in [11], and thus a comparative study was also provided with respect to the success of the optimization algorithms and the performances and utilizations of the models in the amplification circuits. Moreover, the Pareto frontier was obtained using all of the trade-off solution sets for the biobjective case between the magnitudes of ( $S_{11} \times S_{12} \times S_{22}$ ) and  $S_{21}$ , employing the nondominated sorting genetic algorithm-II (NSGA-II). Thus, the location of the solution resulting from the PSO process was determined in the biobjective plane.

In the following section, determination of the FET model is described as a single- and multiobjective optimization problem using the relationships between the FET model elements and the S-parameters, and the PSO method is also explained briefly. In the third section, application of PSO to the determination of the FET model elements is given together with the Pareto optimal analysis. In the final section, the resulting FET models are compared with their counterparts from [11] with respect to their S-parameters and utilizations in the linear amplification circuits.

## 2. Determination of the FET model as a single- and multiobjective optimization problem

### 2.1. Problem formulation: the conventional complete FET model and its performance parameters

The conventional complete model of a FET model commonly available to designers is shown in Figure 1. The complete FET model consists of intrinsic parameters such as  $C_{gd}$ ,  $C_{gs}$ ,  $R_i$ ,  $g_m$ ,  $C_{ds}$ , and  $R_{ds}$  and extrinsic elements such as  $R_g$ ,  $L_g$ ,  $R_d$ ,  $L_d$ ,  $R_s$ , and  $L_s$ . The S-parameters of the complete FET model in Figure 1 can be derived using their definitions, as follows [12]:

$$S_{11}(f) = \frac{1 + j2\pi f C_{gse}(R_{ie} - R_o)10^{-3}}{1 + j2\pi f C_{gse}(R_{ie} + R_o)10^{-3}}, \quad (1.1)$$

$$S_{12}(f) = \frac{j2\pi f R_o C_{gd} 10^{-3}}{1 - (2\pi f R_o 10^{-3})^2 C_{gd} C_{gsw} + j2\pi f (2C_{gd} + C_{gsw}) R_o 10^{-3}}, \quad (1.2)$$

$$S_{21}(f) = \frac{2R_{dso} g_m}{(1 + j2\pi f R_{io} C_{gse} 10^{-3}) \times (1 + j2\pi f R_{dso} C_{dse} 10^{-3})}, \quad (1.3)$$

$$S_{22}(f) = \frac{(1 + j4\pi f C_{gd} R_o 10^{-3}) R_{dsh}}{1 + j2\pi f C_{gdh} (R_{dsh} + 2R_o) 10^{-3} - (2\pi f C_{gdh} 10^{-3})^2 R_{dsh} R_o}, \quad (1.4)$$

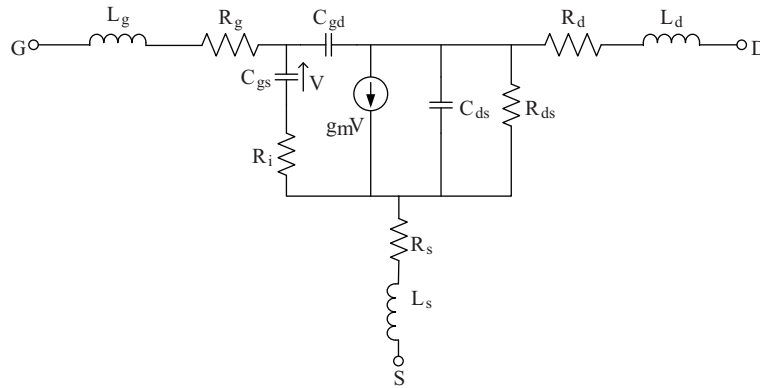
where  $R_o = 50 \Omega$  as the reference terminations, and the other parameters in terms of the model parameters are given below.

$$R_{dso} = R_o R_{dse} / (R_o + R_{dse}), R_{dso} = R_o R_{dse} / (R_o + R_{dse}), R_{dsh} = R_d + R_s + R_{ds}(1 + g_m R_s) \quad (1.5)$$

$$C_{gdh} = C_{gd} \left( 1 + R_o g_m + \frac{C_{ds}}{C_{gd}} \right), C_{gsw} = R_o C_{gd} \left( g_m + \frac{1}{R_{ds}} \right) + C_{gs} \left( 1 + \frac{R_o}{R_{ds}} \right) + C_{ds}, C_{gse} = C_{gs} + C_{gd}(1 - g_a) \quad (1.6)$$

$$C_{dse} = C_{ds} + C_{gd}(g_a - 1)/g_a, R_{ie} = R_i + R_s + R_g, \quad g_a = -g_m \frac{R_o R_{dse}}{R_o + R_{dse}} \quad (1.7)$$

$$R_{dse} = \frac{R_o + R_{ds}}{r_a + R_o}, r_a = R_d + R_s(1 + g_m R_{ds}) \quad (1.8)$$



**Figure 1.** The complete FET model.

In the above equations,  $f$  denotes discrete frequencies in the desired operation band. As clearly seen from Eqs. (1.1)-(1.8), the S-parameters are expressed in terms of FET model elements by neglecting parasitic inductances  $L_g$ ,  $L_d$ , and  $L_s$ . For 50-Ω input/output terminations, the transducer power gain of the FET device is equal to:

$$G_T(f) = |S_{21}(f)|^2. \tag{2}$$

Hereafter, the stability parameters  $\mu$ ,  $k$ , and  $\Delta$  can be given in terms of the S-parameters, together with the unconditional stability conditions, as follows [3,5,7]:

$$\mu \triangleq \frac{1 - |s_{11}|^2}{|s_{22} - s_{11}^* \Delta| + |s_{21} s_{12}|} > 1 \Leftrightarrow k \triangleq \frac{1 - |s_{11}|^2 - |s_{22}|^2 + |s_{11}|^2 |s_{22}|^2 + |s_{12}|^2 |s_{21}|^2}{2 |s_{12}| |s_{21}|} > 1, \tag{3}$$

$$\Delta \triangleq |s_{11} s_{22} - s_{12} s_{21}| < 1.$$

The maximum available gain,  $G_{max}$ , can be given respectively [7]:

$$G_{max} = \frac{|s_{21}|}{|s_{12}|} (k - \sqrt{k^2 - 1}), \tag{4}$$

which is obtained by the simultaneous conjugate matching of the input and output ports in the case of unconditional stability, given below.

$$\Gamma_S = \Gamma_{in}^*, \quad \Gamma_L = \Gamma_{out}^* \tag{5.1}$$

Here,  $\Gamma_S$  and  $\Gamma_L$  are the reflection coefficients of the source and load, respectively, and  $\Gamma_{in}$  and  $\Gamma_{out}$  are reflection coefficients of the input and output ports, given respectively as [7]:

$$\Gamma_{in} = f(\Gamma_L) = s_{11} + \frac{s_{12} s_{21} \Gamma_L}{1 - s_{22} \Gamma_L}, \quad \Gamma_{out} = g(\Gamma_S) = s_{22} + \frac{s_{12} s_{21} \Gamma_S}{1 - s_{11} \Gamma_S}. \tag{5.2}$$

In our case, the transducer power gain, input/output reflection, and feedback transmission losses are specified main performance parameters for the FET device for 50-Ω input/output terminations, since the source and load reflection coefficients are  $\Gamma_S = \Gamma_L = 0$ ,  $\Gamma_{in} = s_{11}$ , and  $\Gamma_{out} = s_{22}$ . In order to keep the reflection losses at the input and the output of the device at lower values, S-parameters  $S_{11}$  and  $S_{22}$  should be taken into account in device modeling for utilization in the amplification circuits. Feedback transmission coefficient

$S_{12}$  is another parameter for minimizing reverse transmission losses. For the 50- $\Omega$  system ( $\Gamma_S = \Gamma_L = 0$ ), the effects of the input and the output on the transducer power gain are disabled, and transducer power gain is only and directly dependent upon  $S_{21}$ . For the utilization of the FET device in the amplifier circuits, the device should be modeled with respect to these main performance parameters. The S-parameters given in Eqs. (1.1)-(1.8) and the performance parameters in Eqs. (2)-(5.2) will be utilized in determining and evaluating the FET model in the next sections.

## 2.2. Determination of FET models as a single- and multiobjective optimization problem

The FET model elements ( $g_m, C_{gs}, R_i, C_{ds}, R_{ds}, C_{gd}, R_g, R_d, R_s$ ) are considered unknown parameters whose search intervals are chosen as given in the Table. We defined the single- and multiobjective functions as “cost” functions in terms of the S-parameters given by Eqs. (1.1)-(1.8) to build the 2 different FET models below.

$$CF - I = - \sum_{i=1}^n |S_{21}(f_i)|^2 \quad (6)$$

$$CF - II = \sum_{i=1}^n \frac{|S_{11}(f_i) \times S_{12}(f_i) \times S_{22}(f_i)|}{|S_{21}(f_i)|^2} \quad (7)$$

Thus, while in Eq. (6) only gain maximization is pursued, in Eq. (7), in addition to this, simultaneous minimization of input  $|S_{11}|$  and output  $|S_{22}|$  reflections and feedback scattering  $|S_{12}|$  are taken into account as objectives, respectively. Minimizing the cost function given by Eq. (6) is equal to the maximization of the gain. Similarly, minimizing the other cost function defined by Eq. (7) is equal to the simultaneous minimization of losses and maximization of gain.

## 2.3. Particle swarm optimization

The PSO algorithm is an evolutionary algorithm capable of solving difficult multidimensional optimization problems in various fields. Since its introduction in 1995 by Kennedy and Eberhart [13], PSO has gained increasing popularity as an efficient alternative to the GA and simulated annealing in solving various optimization design problems. As an evolutionary algorithm, the PSO algorithm is similar to the GA, since it works with populations of individuals randomly initialized and calculates a fitness computation after each step, updates the population based on the fitness value, and stops the iterative algorithm when certain criteria are met [13]. However, there are neither crossover nor mutation operations in PSO to update the population; only the best particles are used.

For an N-dimensional problem, the position and velocity can be specified by  $M \times N$  matrices, where  $M$  is the number of particles in the swarm. Each row of the position matrix represents a possible solution to the optimization problem, and the  $i$ th particle of the swarm is represented by N-dimensional vector  $X_i = [x_{i1}, x_{i2}, \dots, x_{iN}]^T$ . Similarly, the velocity of the  $i$ th particle is represented by N-dimensional vector  $V_i = [v_{i1}, v_{i2}, \dots, v_{iN}]^T$ . Each particle has a memory of the best position of the search space that has ever been obtained at each iteration, and the personal best performance of the  $i$ th particle is defined as  $Pbest_i = [pbest_{i1}, pbest_{i2}, \dots, pbest_{iN}]^T$ . The global best position vector defines the position in the solution space at

which the lowest cost value was achieved by all particles; it is defined as  $Gbest = [gbest_1, gbest_2, \dots, gbest_N]^T$ . The velocity of each particle depends on the distance of its current position from that resulting in the lower cost values. To update the velocity matrix at each iteration, every particle should know its Pbest and Gbest position vectors. Thus, all of the information needed by the PSO algorithm is contained in X, V, Pbest, and Gbest. The core of the PSO algorithm is the method by which these matrices are updated in every iteration of the algorithm. In our work, the velocity matrix and the position matrix are updated according to the following equations:

$$v_i^t(n) = w^t * v_i^{t-1}(n) + c_1 * U_{i1}^t * (Pbest_i^t(n) - x_i^{t-1}(n)) + c_2 * U_{i2}^t * (Gbest^t(n) - x_i^{t-1}(n)), \quad (8)$$

$$x_i^t(n) = x_i^{t-1}(n) + v_i^t(n) \quad i = 1, \dots, M; n = 1, \dots, N, \quad (9)$$

where the superscripts t and t-1 refer to the time index of the current and the previous iterations, w is the inertia weight,  $c_1$  and  $c_2$  are the learning factors of the swarm, and  $U_{i1}^t$  and  $U_{i2}^t$  are 2 uniformly distributed random numbers in the interval [0,1]. These random numbers are different for each of the n components of the particle's velocity vector. As given in Eq. (8), particles update their velocities by keeping their previous velocity with  $w^t * v_i^{t-1}(n)$ , by remembering their own past personal best performances with  $c_1 * U_{i1}^t * (Pbest_i^t(n) - x_i^{t-1}(n))$ , and by utilizing the best performance of the swarm with  $c_2 * U_{i2}^t * (Gbest^t(n) - x_i^{t-1}(n))$ . Each particle then updates its position by using the previous position and the new velocity information as given by Eq. (9).

In our case,  $N = 9$ , each of which corresponds to an unknown element of the FET model's equivalent circuit, and for both single- and multiobjective optimization problems,  $M = 25$  particles were chosen, each of which corresponded to a possible solution set for the FET equivalent circuit. Learning factors  $c_1$  and  $c_2$  from Eq. (8), which were chosen by trial and error, were set to 2 and inertia weight w was set to 0.9 at the start in order to perform a global search, and they decreased linearly during the execution of the algorithm to make a local search around the global optimum in the late iterations of the algorithm:

$$w^{t+1} = w^t - w^t * \frac{t}{t_{Max}}, \quad t = 1, \dots, t_{Max}. \quad (10)$$

In Figure 2, the flowchart of the PSO algorithm is presented. In the first stage, the solution space, fitness function, and number of particles (M) for the algorithm are assigned. In our case, the solution space is defined within the technological limitations given in section 2.2, and the cost functions (CF-I and CF-II) are defined in Eqs.(6) and (7). Position, velocity, Pbest, and Gbest matrices are initialized randomly in the second stage. At each iteration, the cost function value is computed for each particle that represents a candidate solution; these values define each particle's Pbest and Gbest values from the swarm. This convergence ends when the target value is met or the algorithm reaches its maximum iteration number.

## 2.4. Pareto optimal analysis

Real-world problems require a simultaneous optimization process with respect to multiple objectives that are mostly in conflict with each other; this case is referred to as the multiobjective optimization problem. The existence of conflicting objectives necessitates obtaining a set of optimal solutions in multiobjective optimization problems, since any single optimum point cannot be guaranteed to be better than the others. However, a set of optimal solutions, which are also called nondominated solutions, include the ones whose performances for

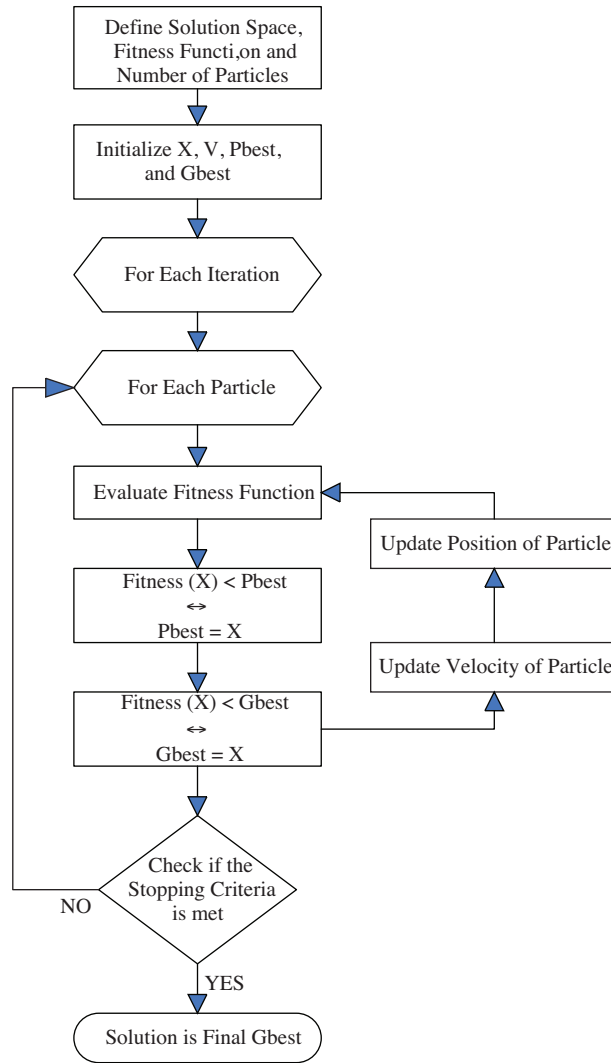


Figure 2. Flowchart of PSO algorithm.

one objective cannot be developed without sacrificing the performances for at least one other. In this work, we consider the Pareto domination relation. Let  $\vec{x}_1, \vec{x}_2 \in X$  be the 2 solution vectors in a multiobjective optimization problem having  $m$  objectives. The Pareto domination relation between these solution vectors can be defined as follows:

- $\vec{x}_1 \prec \vec{x}_2$  ( $\vec{x}_1$  weakly dominates  $\vec{x}_2$ ) if and only if  $f_i(\vec{x}_1) \leq f_i(\vec{x}_2)$  for all  $i \in \{1, 2, \dots, m\}$ ,
- $\vec{x}_1 \prec \vec{x}_2$  ( $\vec{x}_1$  dominates  $\vec{x}_2$ ) if and only if  $\vec{x}_1 \prec \vec{x}_2$  and  $f_j(\vec{x}_1) < f_j(\vec{x}_2)$  for at least one  $j \in \{1, 2, \dots, m\}$ ,
- $(\vec{x}_1 \sim \vec{x}_2)$  ( $\vec{x}_1$  is indifferent to  $\vec{x}_2$ ) if and only if  $\vec{x}_1$  does not dominate  $\vec{x}_2$  and  $\vec{x}_2$  does not dominate  $\vec{x}_1$ .

In the case where  $\vec{x}_1$  and  $\vec{x}_2$  dominate other solution vectors but not each other, they are deemed mutually optimal solutions and are referred to as the Pareto optimal. The set of Pareto optimal solutions reflects the trade-off surfaces between the different objectives. This set of Pareto optimal solutions is referred to as the Pareto front.

In our work, the multiobjective PSO solution is compared with the Pareto optimal solution sets resulting from the Pareto optimal analysis for the biobjective case between the magnitudes of  $(S_{11} \times S_{12} \times S_{22})$  and  $S_{21}$ , employing the NSGA-II. Thus, the location of the solution resulting from PSO is determined in the biobjective plane. The application for the worked example will be explained in the next section.

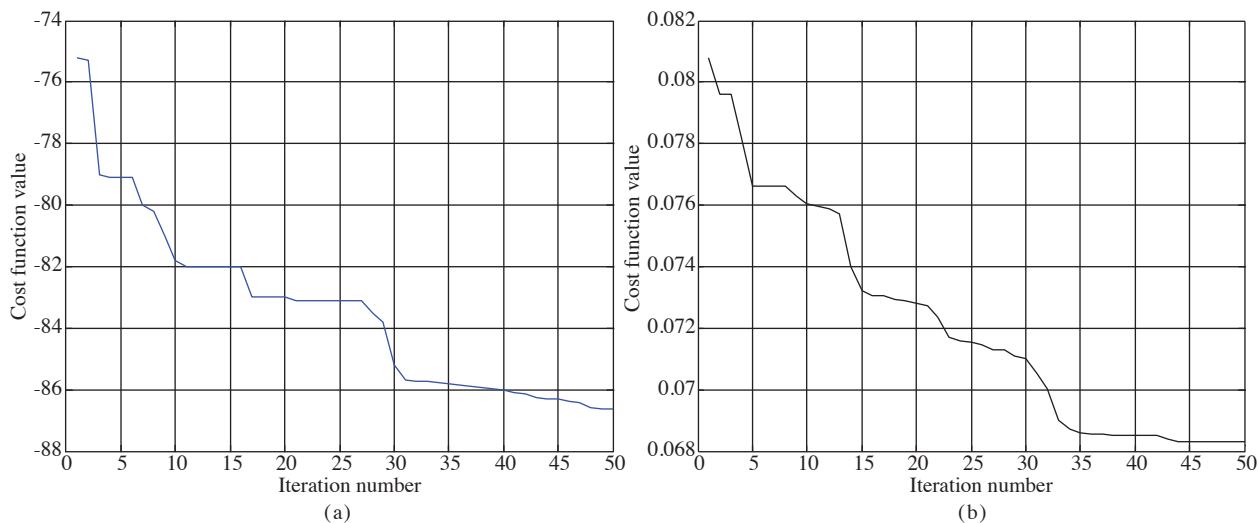
### 3. Worked example

#### 3.1. The PSO FET models

In this work, 2 FET models are built by minimizing the cost functions given by Eqs. (6) and (7), subject to technological limits within the same bandwidth as stated in [11], by utilizing the PSO algorithm. Thus, the 2 sets of FET model elements are determined by the resulting single- and multiobjective PSO FET models (CF-I and CF-II, respectively), whose equivalent circuit element values are given in the Table.

The iteration number for convergence in our PSO processes changes between 50 and 35 depending on the initialization, which takes 2.25 s and 1.55 s, respectively, with a Pentium 4 CPU, a 3 GHz processor, and 512 MB of RAM. Typical convergence curves for both cost functions are given in Figure 3.

In the next stage, performances of the PSO FET models were examined by comparing their scattering performances with the FET models obtained by the different algorithms used in [11]. Figures 4a-4d give magnitudes of the S-parameters of the CF-I and CF-II FET models as obtained by the GA, CPGA, FHA, CHA, and PSO.



**Figure 3.** Convergence curves for PSO application for a) CF-I and b) CF-II.

It can be seen from Figures 4a-d that the multiobjective PSO optimization (CF-II) gives the best scattering performance when compared with the single-objective (CF-I) FET models. However, the single-objective PSO optimization results in the FET model with maximum gain at the expense of worsening the input  $|S_{11}|$  and output  $|S_{22}|$  reflections and feedback scattering  $|S_{12}|$  in comparison with the other FET models. Figure 5 gives the stability performances of the CF-I and CF-II PSO FET models over the working operation bandwidth using Eq. (3); the multiobjective FET model can be seen to be unconditionally stable over the operation band in spite of the conditionally stable performance of the single-objective PSO FET model.

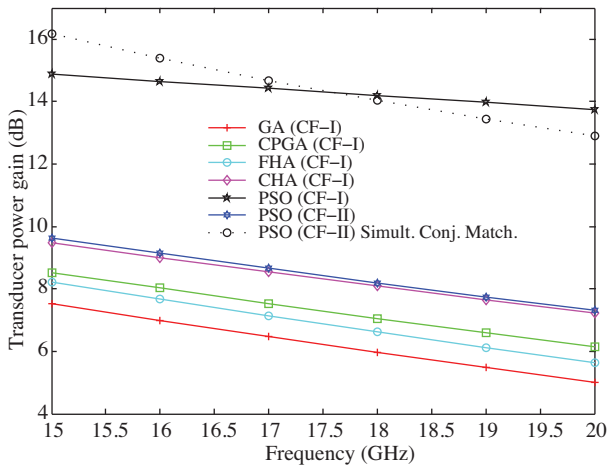


**Table.** FET model element values for CF-I and CF-II.

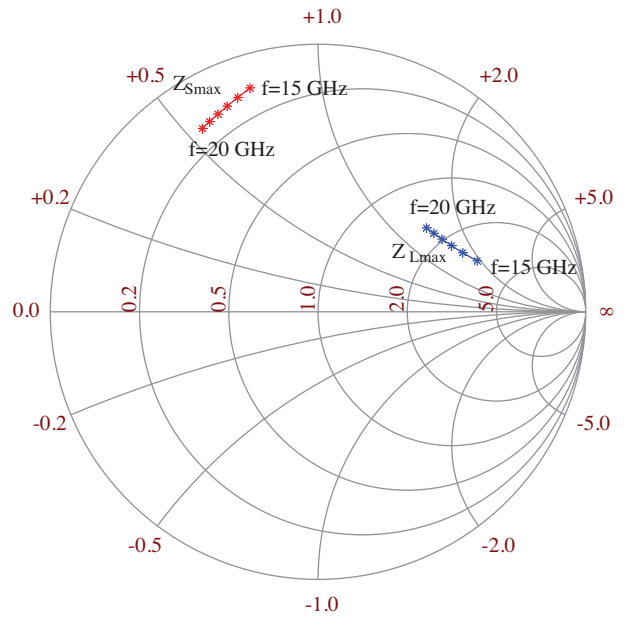
FET model elements	PSO results, CF-I	PSO results, CF-II	Solution space	
$g_m$ (S)	0.08	0.08	0.04	0.08
$C_{gs}$ (pF)	0.1	0.279	0.1	0.6
$R_i$ ( $\Omega$ )	3.1056	5.997	2	6
$C_{ds}$ (pF)	0.0237	0.076	0.02	0.08
$R_{ds}$ ( $\Omega$ )	594.4867	200.424	200	600
$C_{gd}$ (pF)	0.01	0.01	0.01	0.02
$R_g$ ( $\Omega$ )	0.7858	0.932	0.1	1
$R_d$ ( $\Omega$ )	0.2775	0.142	0.1	1
$R_s$ ( $\Omega$ )	0.7206	0.102	0.1	1

The transducer gain variations of the FET models for the reference terminations, 50  $\Omega$  in this case, are shown in Figure 6, where the maximum available gain variation of the multiobjective FET model in Eq. (4) is also given. The input and output terminations ensuring simultaneous conjugate matching for maximum available gain are presented in a Smith chart in Figure 7.

From Figure 6, it can be observed that the FET models resulting from PSO algorithms offer the 3 alternatives for maximum gain. The CF-I PSO achieves the maximum gain with 50- $\Omega$  terminations at the expense of worsening the other scattering performances, while the CF-II PSO results in minimum mismatching and greater gain than the other models with the same terminations. If the matching network can be used, however, the latter model is capable of maximum available gain with the best scattering performance, since it is unconditionally stable throughout the entire operation bandwidth, as shown in Figure 5.



**Figure 6.** Transducer gain variations of the FET models obtained by the GA, CPGA, FHA, CHA, and single- and multiobjective PSO with reference terminations (50  $\Omega$ ) and simultaneous conjugate matching.



**Figure 7.** Input and output terminations for simultaneous conjugate matching conditions.

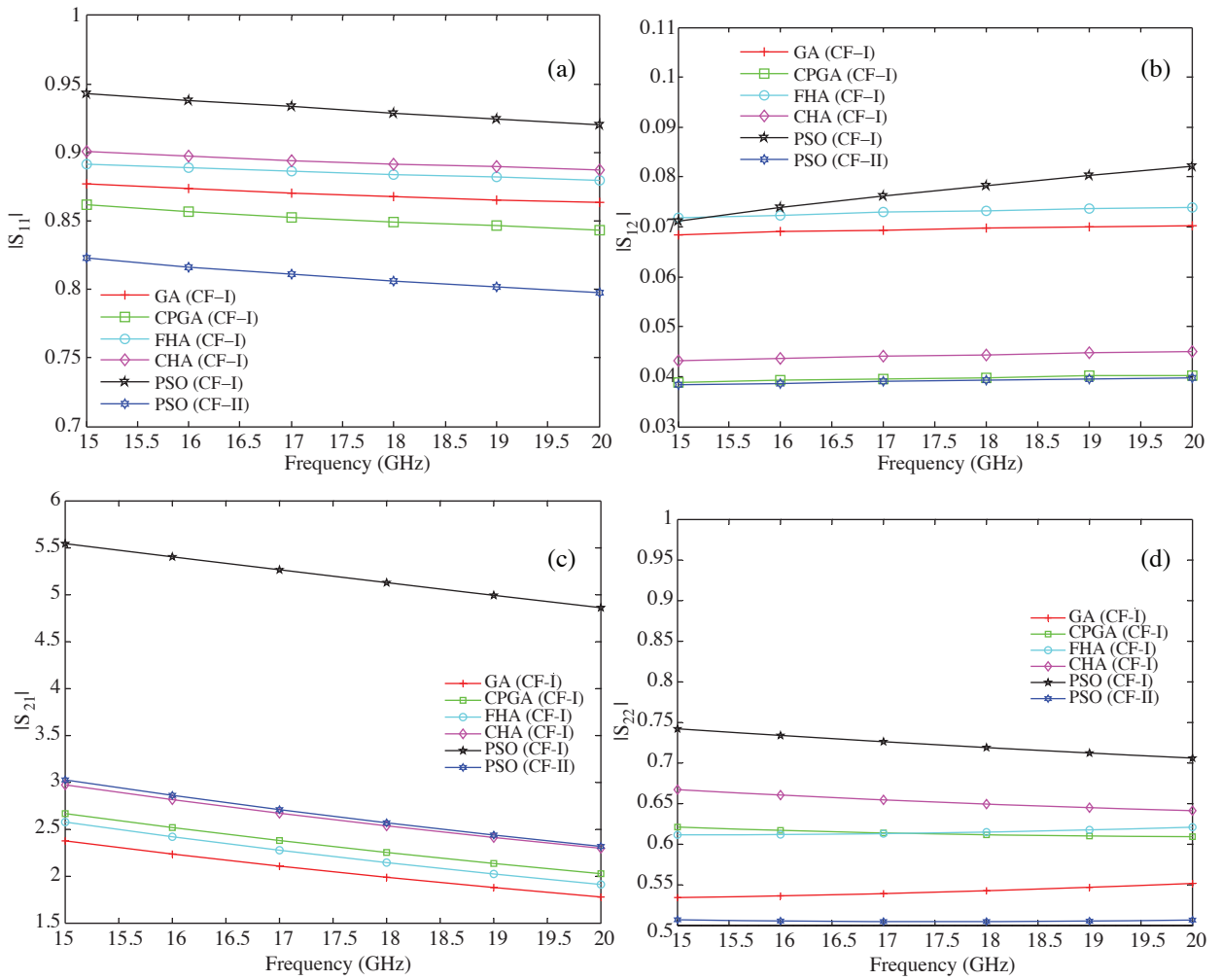


Figure 4. S-parameters of FET models obtained by the GA, CPGA, FHA, CHA, and PSO: a)  $|S_{11}|$ , b)  $|S_{12}|$ , c)  $|S_{21}|$ , and d)  $|S_{22}|$ .

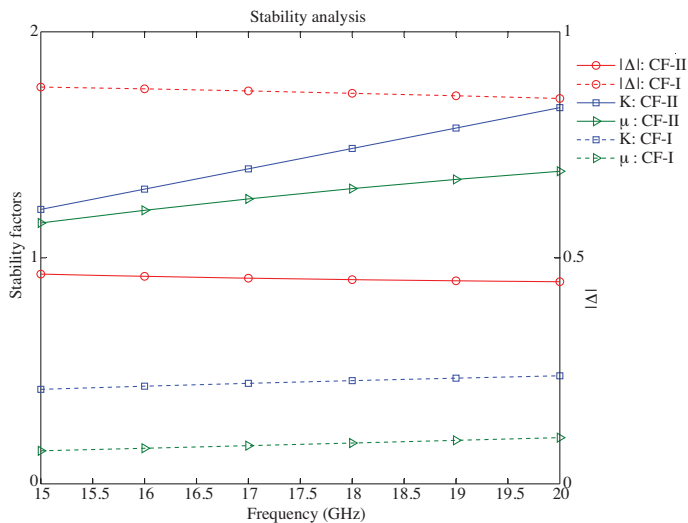


Figure 5. Stability analysis for the CF-I and CF-II PSO FET models.

### 4. Biobjective Pareto optimal analysis

In this work, multiple objectives are grouped under the 2 objective functions  $\vec{f}(\vec{x}) = (f_1(\vec{x}), f_2(\vec{x}))$  to be simultaneously minimized with respect to n decision variables  $\vec{x} = (x_1, x_2, x_3, \dots, x_n)$ , subject to the given constraints in decision space X. Thus, the multiobjective function  $\vec{f} : X \Rightarrow Y$  evaluates the quality of the specific solution by assigning objective vector  $\vec{y} = (y_1, y_2)$  in the Y-objective space:

$$\text{Minimize } f_1(\vec{x}) = \sum_{i=1}^m |S_{11}(\vec{x}, \omega_i)| \times |S_{12}(\vec{x}, \omega_i)| \times |S_{22}(\vec{x}, \omega_i)|, \quad (12.a)$$

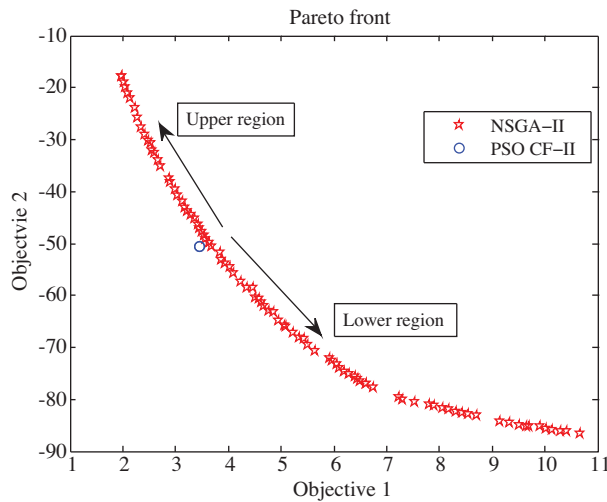
$$\text{Minimize } f_2(\vec{x}) = - \sum_{i=1}^m |S_{21}(\vec{x}, \omega_i)|^2, \quad (12.b)$$

where the  $\vec{x}$  decision variable vector consists of n = 9 unknowns of the FET model elements and each element has its lower and upper technological limitations:

$$x_{i_l} \leq x_i \leq x_{i_u} \quad i = 1, 2, \dots, 9. \quad (12.c)$$

The constraints given by Eq. (12.c) determine a feasible D region in the decision space  $x \in R^n$ , and the multiobjective function  $\vec{f}$  maps this feasible D region onto an objective function space,  $y \in R^n$ .

The evolutionary multiobjective optimization procedure that we use is the denominated NSGA-II proposed by Deb et al. [14] and Deb [15], in which a fast, nondominated sorting genetic algorithm is presented to alleviate all of the main difficulties of the previous nondominated sorting multiobjective evolutionary algorithms. Specifically,  $O(MN^3)$  computational complexity, where M is the number of objectives and N is the population size, is overcome by a fast, nondominated sorting approach. A selection operator is also presented, which creates a mating pool by combining the parent and offspring populations and selecting the best with respect to fitness and spread solutions. Simulation results from difficult test problems show that the NSGA-II, in most problems, is able to find a much better spread of solutions and better convergence near the true Pareto optimal frontier compared to Pareto-archived evolution strategies and strength Pareto evolutionary algorithms. Figure 8 gives the Pareto front and the PSO solution point in the plane defined by the objectives given in Eqs. (12.a) and



**Figure 8.** Pareto front defined by  $f_1(\vec{x}) = \sum_{i=1}^m |S_{11}(\vec{x}, \omega_i)| \times |S_{12}(\vec{x}, \omega_i)| \times |S_{22}(\vec{x}, \omega_i)|$  and  $f_2(\vec{x}) = - \sum_{i=1}^m |S_{21}(\vec{x}, \omega_i)|^2$ , and the PSO solution.

(12.b). From Figure 8, one can observe that the solution resulting from the PSO process takes place within a reasonably good region of the Pareto front in view of the objective values.

Thus, in the upper region of the Pareto front with respect to the PSO solution point (circle), the gain is decreasing, while the other scattering performances, including losses, get increasingly better. In the lower region, from the counter to upper region, nondominated solutions have increasing gain performances but worsening loss performances.

## 5. Conclusions

FET modeling, subject to optimum scattering performances, was put forward as a constrained multiobjective optimization problem. In this problem, the elements of the complete FET model were examined to satisfy all expected performance requirements of an active device, including the maximum transducer power gain ( $G_T \Leftrightarrow S_{21}$ ), minimum input reflection ( $S_{11}$ ), minimum reverse transmission ( $S_{12}$ ), and minimum output reflection ( $S_{22}$ ), within the limitations of the solid-state technology. For this purpose, 2 novel methods were employed. In the first method, the multiple objectives were normalized and processed into a single function as contributors commonly known as aggregating functions. Among the FET models obtained from different algorithms, the multiobjective PSO FET model provided the best scattering performance by ensuring minimum losses and by having relatively good gain performance. Stability analysis also demonstrated that the FET model worked unconditionally stably over the entire operation band. The second method was the Pareto optimal analysis, in which the Pareto frontier was obtained using all of the trade-off solution sets for the biobjective case between the magnitudes of ( $S_{11} \times S_{12} \times S_{22}$ ) and  $S_{21}$ . In this method, nondominated sorting was done by the NSGA-II. Thus, all optimum FET models satisfying the performance requirements subject to the limitations of the solid-state device technology were obtained together, the location of the PSO solution was determined in the biobjective plane, and the success of the first method was verified. Furthermore, the resulting FET models were compared with similar FET models from the literature, thus providing a comparative study in respects to the success of the optimization algorithms and performances and utilizations of the models in the amplification circuits.

## References

- [1] K. Kurokawa, "Power waves and the scattering matrix", IEEE Transactions on Microwave Theory and Techniques, Vol. 3, pp. 194-202, 1965.
- [2] H. Fukui, "Available power gain, noise figure and noise measure of two-ports and their graphical representations", IEEE Transactions on Circuit Theory, Vol. 13, pp. 137-142, 1966.
- [3] D. Woods, "Reappraisal of the unconditional stability criteria for active 2-port networks in terms of S parameters", IEEE Transactions on Circuits and Systems, Vol. 23, pp. 73-81, 1976.
- [4] A.M. Björn, "A graphic design method for matched low-noise amplifiers", IEEE Transactions on Microwave Theory and Techniques, Vol. 38, pp. 118-122, 1990.
- [5] M.L. Edwards, J.H. Sinsky, "A new criterion for linear 2-port stability using a single geometrically derived parameter", IEEE Transactions on Microwave Theory and Techniques, Vol. 40, pp. 2303-2311, 1992.
- [6] F. Güneş, M. Güneş, M. Fidan, "Performance characterisation of a microwave transistor", IEE Proceedings Circuits, Devices & Systems, Vol. 141, pp. 337-344, 1994.

- [7] F. Güneş, B.A. Çetiner, "A novel Smith chart formulation of performance characterisation for a microwave transistor", *IEE Proceedings Circuits, Devices & Systems*, Vol. 145, pp. 419-428, 1998.
- [8] F. Güneş, C. Tepe, "Gain-bandwidth limitations of microwave transistor", *International Journal of RF and Microwave CAE*, Vol. 12, pp. 483-495, 2002.
- [9] F. Güneş, S. Demirel, "Gain gradients applied to optimization of distributed-parameter matching circuits for a microwave transistor subject to its potential performance," *International Journal of RF and Microwave CAE*, Vol. 18, pp. 99-111, 2008.
- [10] C. Paoloni, S. D'Agostino, "An approach to distributed amplifier based on a design-oriented FET model", *IEEE Transactions on Microwave Theory and Techniques*, Vol. 43, pp. 272-277, 1995.
- [11] T. Günel, "A continuous hybrid approach to the FET modelling for the maximum transducer power gain", *Microwave and Optical Technology Letters*, Vol. 35, pp. 348-352, 2002.
- [12] C. Paoloni, "A simplified procedure to calculate the power gain definitions of FET's," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 48, pp. 470-474, 2002.
- [13] J. Kennedy, R.C. Eberhart, "Particle swarm optimization", in *Proceedings IEEE Conference on Neural Networks*, Vol. 4, pp. 1942-1948, 1995.
- [14] K. Deb, A. Pratap, S. Agrawal, T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II", *IEEE Transactions on Evolutionary Computation*, Vol. 6, pp. 182-197, 2002.
- [15] K. Deb, *Multi-Objective Optimization Using Evolutionary Algorithms*, Chichester, John Wiley & Sons, 2004.