Turk J Elec Eng & Comp Sci, Vol.18, No.4, 2010, © TÜBİTAK
doi:10.3906/elk-0908-137

A novel method for lung segmentation on chest CT images: complex-valued artificial neural network with complex wavelet transform

Murat CEYLAN[1], Yüksel ÖZBAY[1], O. Nuri UÇAN[2], Erkan YILDIRIM[3]
[1]*Selçuk University, Electrical & Electronics Department, 42075-Konya-TURKEY*
e-mail: {mceylan,yozbay}@selcuk.edu.tr
[2]*İstanbul University, Department of Electrical and Electronics Engineering, 34850-İstanbul-TURKEY*
e-mail: uosman@istanbul.edu.tr
[3]*Başkent University, Department of Radiology, Ankara-TURKEY*
e-mail: drerkany@yahoo.com

Abstract

Image segmentation is an important step in many computer vision algorithms. The objective of segmentation is to obtained an optimal region of convergences (ROC). Error in this stage will impact all higher level activities. This paper focuses on a new efficient method denoted as Complex-Valued Artificial Neural Network with Complex Wavelet Transform (CWT-CVANN) for the segmentation of lung region on chest CT images. In this combined architecture is composed of two cascade stages: feature extraction with various levels of complex wavelet transform and segmentation with complex-valued artificial neural network. Here, 32 CT images of 6 female and 26 male patients were recorded from Baskent University Radiology Department. (This collection includes 10 images with benign nodules and 22 images with malign nodules. Averaged age of patients is 64. Each CT slice used in this study has dimensions of 752× 752 pixels with grey level) In only two seconds of processing time per each CT image, 99.79% averaged accuracy rate is obtained using 3^{rd} level CWT-CVANN for segmentation of the lung region. Thus, it is concluded that CWT-CVANN is a comprising method in lung region segmentation problem.

Key Words: *Lung segmentation, complex wavelet transform, complex-valued artificial neural network*

1. Introduction

The principal goal of the segmentation process is to partition an image into classes or subsets that are homogeneous with respect to one or more characteristics or features [1]. In medical imaging, segmentation is important for feature extraction, image measurements, and image display. In some applications it may be useful to classify image pixels into anatomical regions, such as bones, muscles, and blood vessels, while in others into pathological regions, such as cancer, tissue deformities, and multiple sclerosis lesions [1].

A wide variety of segmentation techniques has been proposed in the literature [2–5]. However, there is no one standard segmentation technique that can produce satisfactory results for all imaging applications. Segmentation tecniques can be divided in to classes as follows:

1. Manual, semiautomatic, and automatic [6];

2. Pixel-based and region-based [7];

3. Low level segmentation (thresholding, region growing) and model-based segmentation (feature map tecniques, dynamic programming) [8]; and

4. Classical (thresholding, edge-based, and region-based techniques), statistical, fuzzy, and artificial neural network techniques (ANN) [9].

Compared with these classical methods, the ANN approach has the advantage of paralel processing (with appropriate hardware), robustness, noise tolerance, and adaptability. Neural networks provide pixel classification paradigm that can be used for image segmentation [10–13]. Neural network based segmentation approaches may provide good results for medical images with considerable variance in structures of interest [10]. Sha and Sutton [14] proposed a neural network system for segmentation and classificiation of digital brain images. Another application of neural network for automatic segmentation was done by Nattkemper et al. [15]. Papadopoulos et al. [16], has used a hybrid neural network, which consist of two components: a rule-based and a neural network. In [17], ultrasound images were segmented using hybrid neural network. Vilarino et al. [18], were applied cellular neural network to image segmentation based on active contour techniques. Middleton et al. [19], used combination of neural networks and active contour models for segmentation of MR images. In [20], region-based segmentation and neural network edge detection were used. Fuzzy clustering approach for image segmentation was proposed by Cinque et al. [21]. Fu et al. [22] used a cascade system for mamographic microcalcifications. This system includes feature extraction, general regression neural network and support vector machine blocks. For MR, CT and ultrasound images segmentation, a new method which called as incremental neural network was developed by Dokur [23] and Kurnaz et al. [24, 25]. Wismuller et al. [26] and Ong et al. [27] were used self-organized model for the segmentation.

A complex-valued artificial neural network (CVANN) is a neural network which consist of complex-valued inputs, weights, thresholds, activation functions and outputs. CVANN have been widening the scope of applications not only in signal processing [28–30] but also in image processing [30, 31]. In this paper, segmentation of biomedical images (lung images for this sudy) using proposed networks based CVANN is a new approach.

2. Materials and methods

In this paper, segmentation of lung region is considered in order to prevent time wasting while searching nodules out of the lung region but inside the CT image. For this process to be able to make searching in the interested region only, we propose a novel cascade structures, called CWT-CVANN and CVWANN based on CVANN.

2.1. Image data

For the development and evaluation of the proposed system we used our image collection (32 chest CT images of 6 female and 26 male patients). These images were recorded at Baskent University Radiology Department. This collection includes 10 images with benign nodules and 22 images with malign nodules. Averaged age of patients is 64. Each CT slice used in this study has dimensions of 752 $\times$ 752 pixels with grey level.

2.2. Complex discrete wavelet transform (CWT)

Wavelet techniques are successfully applied to various problems in signal and image processing. Data compression [32], motion estimation [33], segmentation and classification [34, 35] and denoising [36] are only some examples. It is perceived that the wavelet transform is an important tool for analysis and processing of signals and images. In spite of its efficient computational algorithm, the wavelet transform suffers from three main disadvantages.

2.2.1. Limitations of wavelet transform

Although the standard discrete wavelet transform (DWT) is a powerful tool, it has three major disadvantages that undermines its application for certain signal and image processing tasks [37, 38].

2.2.2. Shift sensitivity

A transform is shift sensitive, if the shifting in time, for input-signal causes an unpredictable change in transform coefficients. It has been observed that the Standard DWT is seriously disadvantaged by the shift sensitivity that arises from down samplers in the DWT implementation [37, 39]. Shift sensitivity is an undesirable property because it implies that DWT coefficients fail to distinguish between input-signal shifts.

2.2.3. Poor directionality

An m-Dimensional transform ($m > 1$) suffers poor directionality when the transform coefficients reveal only a few feature orientations in the spatial domain. Wavelet transform has been poor directional selectivity for diagonal features, because the wavelet filters are separable and real.

2.2.4. Absence of phase information

For a complex-valued signal or vector, its phase can be computed by its real and imaginary projections. Phase information is valuable in many signal and image processing applications [40] such as e.g. in image compression and power measurement [41, 42].

Most DWT implementations use separable filtering with real coefficient filters associated with real wavelets resulting in real-valued approximations and details. Such DWT implementations cannot provide the local phase information. All natural signals are basically real-valued, hence to avoid the local phase information, complex-valued filtering is required [43, 44].

Recent research in the development of CWTs can be broadly classified in two groups: RCWT (Redundant CWTs) and NRCWT (Non-redundant CWTs). Standard DWT is critically decimated and gives N samples in transform domain for the same N samples of a given signal. While the redundant transform gives M samples in transform domain for N samples of given input signal (where $M > N$) and hence it is expensive by the factor M/N. The NRCWT follows the design aim to approach towards N samples in transform domain for a given N input samples [37, 38].

The RCWT include two almost similar CWT. They are denoted as DT-DWT (Dual-Tree DWT based CWT, with two almost similar versions namely Kingsbury's and Selesnick's [45]. In this paper, we used Kingsbury's CWT [30, 40] for feature extraction of image to be segmented.

2.3. Complex-valued artificial neural network (CVANN)

Recently, there has been an increased interest in applications of the CVANN to process complex signals [46–48]. In this study, a complex back-propagation (CBP) algorithm has been used for image segmentation. We will first give the theory of the CBP algorithm as applied to a multi layer CVANN. The input signals, weights, thresholds, and output signals are all complex numbers. The activity Y_n of neuron n is defined as

$$Y_n = \sum_m W_{nm} X_m + V_n, \tag{1}$$

where W_{nm} is the complex-valued (CV) weight connecting neuron n and m, X_m is the CV input signal from neuron m, and V_n is the CV threshold value of neuron n. To obtain the CV output signal, the activity value Y_n is converted into its real and imaginary parts as

$$Y_n = x + iy = z, \tag{2}$$

where i denotes $\sqrt{-1}$. Although various output functions of each neuron can be considered, the output function used in this study is defined by the equation

$$f_C(z) = f_R(x) + i.f_R(y), \tag{3}$$

where $f_R(u)$ is called the activation function of neural network. One of the difficulties encountered in applying the CBP algorithm to the complex domain involves the appropriate choice of activation function. For a practical implementation of the complex multilayer perceptron, it is necessary that the activation function be bounded. Several researchers developed a set of properties that a complex activation function must satisfy in order to be useful in a multilayer perceptron trained with the back-propagation algorithm [49]. Complex activation function that used in this study is a superposition of real and imaginary logarithmic sigmoids:

$$f_R(u) = \frac{1}{1+\exp(-u_R)} + j\frac{1}{1+\exp(-u_R)} \tag{4}$$

The CBP algorithm can be summarized as follows.

1. Initialization

Set all the weights and thresholds to small complex random values.

2. Presentation of input and desired (target) outputs

Present the input vector $X(1)$, $X(2)$,..., $X(N)$ and corresponding desired (target) response $T(1)$, $T(2)$,...$T(N)$, one pair at a time, where N is the total number of training patterns.

3. Calculation of actual outputs

To obtain the complex-valued output signal, the activity value Y_n is converted into its real and imaginary parts as equation 2.

4. Calculation of the stopping criteria with respect to (Equation 5) [47].

The condition is required that

$$\sqrt{\sum_{p}\sum_{n=1}^{N}\left|T_n^{(p)} - O_n^{(p)}\right|^2} = 10^{-1} \tag{5}$$

And if satisfied, the algorithm is stopped and weights and biases are frozen. Here, $T_n^{(p)}$ and $O_n^{(p)}$ are complex numbers and denote the desired and output value, respectively. The actual output value of the neuron n for the pattern p, i.e. the left side of (6) denotes the error between the desired output pattern and the actual output pattern. N denotes the number of neurons in the output layer.

3. Measurement for performance evaluation

To assess results, a method of measuring the accuracy of the segmentation techniques is required. This is a common problem in medical image segmentation [23]. Visual inspection is sometimes used to evaluate performance since "perfect" segmentations cannot be defined [23]. This evaluation is extremely subjective. In this paper, we used an algorithm to evaluate the classification results of CVANN's outputs for training and test images that include lung –region pixels and surrounding pixels of lung-region pixels. Number of correct classified pixels in complete image (752×752) were calculated according to the following algorithm:

IF

$I_T - I_O = 0$

THEN

Number of correct classified pixel (CCP) increase by 1

ELSE

Number of incorrect classified pixel (iCCP) increase by 1

Here, I_T and I_O are target of network and actual output of network, respectively. Finally, we measured the accuracy of proposed methods using the equation

$$\text{Accuracy (\%)} = (\text{CCP} / (\text{CCP} + \text{iCCP})) \times 100 \tag{6}$$

4. Results and discussion

In this paper, segmentation of biomedical images is considered in order to prevent time wasting while searching nodules out of the lung region but inside the CT image. For this process to be able to make searching in the interested region only, we propose a novel cascade structure including complex-valued feature extractor (CWT) and complex-valued classifier (CVANN) called as CWT-CVANN for segmentation.

In proposed structure, segmentation of lung region was performed using CVANN based CWT. Complex discrete wavelet transform was used to reduce the size of input matrix of training and test images in CWT-CVANN. The basic idea in using complex wavelet transform was to eliminate unnecessary features by compressing image to be segmented, so it was to make a complex-valued neural network more efficient for segmentation of images. In proposed cascade structure, feature vector of original image (real-valued lung image) was extracted using CWT with four different level. Size of feature vectors were 376×376, 188×188, 94×94, 47×47 for 1st level, 2nd level, 3rd level and 4^{th} level, respectively. Obtained feature vectors were presented as inputs to CVANN. Figure 1 shows block representation of proposed method.

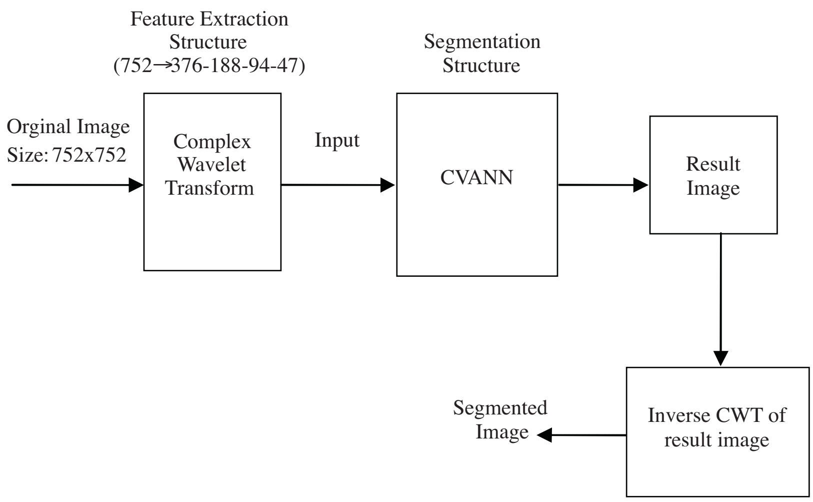


Figure 1. Block representation of CWT-CVANN for image segmentation.

The complex-valued backpropagation algorithm was used for training of the proposed networks. In training phase, the weights and biases of CVANNs were initialised with small random complex numbers. An error goal (stopping criteria of training of 10^{-1}) was specified as in (5). The training of CWT-CVANN was stopped when the error goal was achieved. After that, the performance of this network was tested by presenting test image from image database. The optimum learning rate and number of hidden nodes were determined as 0.1 and 10, respectively, via experimentation. Number of iteration was determined as 10 for network.

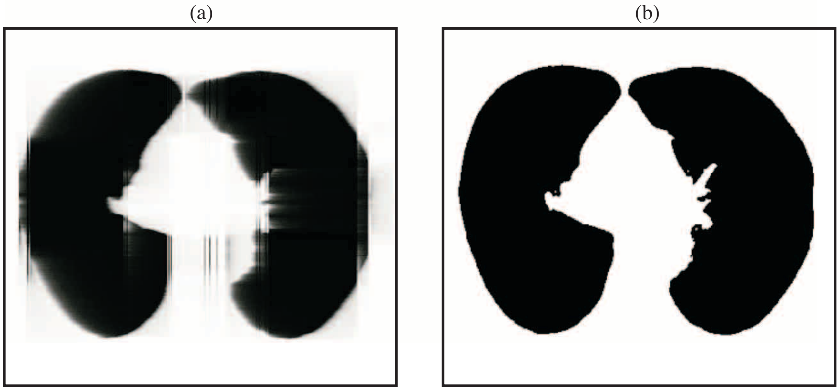
Test result is given in Table 1 for four level CWT-CVANN using obtained best and worst accuracy. In Table 1, B denotes an image with benign nodule and M denotes an image with malign nodule. According to Table 1, best accuracies were obtained using 3^{rd} level CWT - CVANN as 99.791% and 99.774% for images with benign nodule and malign nodule, respectively. Moreover, this structure was required only 2 second per image for segmentation task. If time consumptions for all structures were observed, it was seen that 4th level CWT-CVANN required less time (only 1.2 second) than others for segmentation task. Other structures have wasted time that is very important for the online biomedical applications.

Figure 2 shows the result images of 1st level CWT-CVANN with worst accuracy rate and time consumption and 3rd level CWT-CVANN with best accuracy rate for one of used image from image database. As seen in Figure 2(c), CVANN was obtained small artifacts in the neighbourhood of lung region. These artifacts were caused the errors and decreasing of accuracy rate.

Table 1. Segmentation results of CWT-CVANN for obtained best and worst accuracy.

Level of CWT	Image No.	CCP	iCCP	Accuracy (%)	Time (second)
1	B10 (best)	494053	71451	87.3651	79.5807
	B8 (worst)	450892	114612	79.7328	77.1671
	M16 (best)	483767	81737	85.5462	77.9334
	M10 (worst)	433953	131551	76.7374	77.3682
2	B10 (best)	564279	1225	99.7834	7.1249
	B1 (worst)	563483	2021	99.6426	6.7355
	M16 (best)	564150	1354	99.7606	7.0482
	M19 (worst)	563483	2021	99.6426	7.2498
3	B10 (best)	564322	1182	99.7910	2.0225
	B1 (worst)	562808	2696	99.5233	2.0437
	M6 (best)	564226	1278	99.7740	2.0947
	M10 (worst)	563548	1956	99.6541	2.0133
4	B10 (best)	564169	1335	99.7639	1.1896
	B1 (worst)	561201	4303	99.2391	1.1883
	M16 (best)	564079	1425	99.7480	1.1921
	M4 (worst)	563349	2155	99.6189	1.1918

(a) (b)



(c) (d)

Figure 2. (a) Original image. (b) Segmented image (target for networks). (c) Segmented output of 1st level CWT-CVANN. (d) Segmented output of 3rd level CWT-CVANN.

For comparison, lung segmentation was done using cascade structure of WT-ANN. Features of lung images were extracted using WT and extracted features were presented to ANN to segmentation task. In WT-ANN structure, size of input image (752×752) was reduced to 377×377, 190×190, 96×96 and 49×49 using 1st level, 2^{nd} level, 3^{rd} level and 4^{th} level WT, respectively. Obtained feature vectors were utilized to train ANN with gradient descent back-propagation algorithm [49]. End of training, test phase was realized using lung images which segmented with best and worst accuracy by CWT-CVANN in Table 1. The optimum network parameters of CWT-CVANN was used in both training and test phases of WT-ANN. Table 2 demonstrates test results of lung segmentation with WT-ANN.

Table 2. Segmentation results of WT-ANN for obtained best and worst accuracy using CWT-CVANN.

Level of WT	Image No.	CCP	iCCP	Accuracy (%)	Time (second)
1	B10 (best)	505253	60251	89.35	53.52
	B8 (worst)	466106	99398	82.42	55.62
	M16 (best)	500576	64928	88.52	51.70
	M10 (worst)	452408	113096	80	52.24
2	B10 (best)	504295	61209	89.17	23.78
	B1 (worst)	495965	69539	87.70	20.93
	M16 (best)	501818	63686	88.74	21.04
	M19 (worst)	455038	110466	80.47	20.60
3	B10 (best)	503645	61859	89.01	14.44
	B1 (worst)	495387	70117	87.6	14.56
	M6 (best)	511988	53516	90.54	15.14
	M10 (worst)	449694	115810	79.52	14.26
4	B10 (best)	503398	62106	89.02	11.57
	B1 (worst)	495767	69737	87.67	11.80
	M16 (best)	501351	64153	88.66	11.53
	M4 (worst)	462520	102984	81.79	11.69

Results of CWT-CVANN (see Table1) and WT-ANN (see Table 2) were compared in Figure 3. According

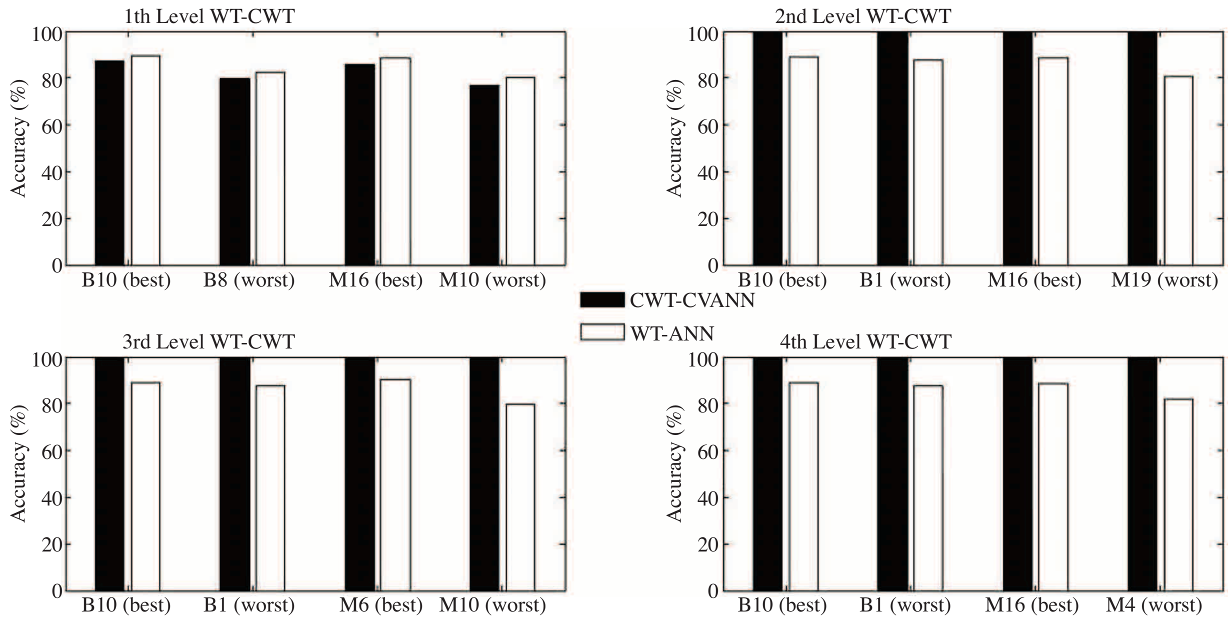


Figure 3. Comparison of CWT-CVANN and WT-ANN methods in lung segmentation.

to Figure 3, 3^{rd} level CWT-CVANN structure can segment lung region better than all of the other structures and is faster than all WT-ANNs.

These results shown that CWT-CVANN structures were segmented lung region and surrounding image, succesfully. Not only the number of ROIs and computation time will decrease but also the sensitivity of the system will increase using CWT-CVANN.

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