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Programmable design and implementation of a chaotic system utilizing multiple nonlinear functions

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Abstract

In addition to exhibiting a rich variety of bifurcation and chaos via tuning parameters, a chaotic system introduced by Sprott can be modeled and realized with a fixed main system block and many different changeable nonlinear function blocks such as piecewise-linear function, cubic function and other trigonometric functions. This system is very suitable for implementing a programmable chaos generator according to its changeable nonlinearity. This paper presents a FPAA (Field Programmable Analog Array)-based programmable implementation of this system. Nonlinear function blocks used in this chaotic system are modeled with FPAA programming and a model is rapidly changed for realizing other nonlinear functions.

Key Words: *Field Programmable Analog Array, Chaos, Chaotic System, Mathematical Modeling*

1. Introduction

FPAA (Field Programmable Analog Array) is a programmable device for implementing a rich variety of systems including analog functions. Because this electronic device has a feature that can be used to programmatically change component values and interconnections, it can be dynamically reconfigured. This meant that a design modification or a completely new design can be implemented with reconfigurable FPAA device. For this process, there is no need to power down or to reset the system. In addition to these features, FPAA provides more efficient and economical solutions for analog dynamical system designs. Using FPAAs, various dynamical systems including different chaotic systems can be implemented at less cost, in a much smaller size and with increased reliability and component stability [1–10].

On the other hand, among the nonlinear dynamical systems, chaotic systems based on jerk equations [11–15] have received a considerable attention. A chaotic system introduced by Sprott [11, 12] is also based on jerk equations and has very high potential in terms of programmability and reconfigurability. In addition to exhibiting a rich variety of bifurcation and chaos via tuning its parameters, this system can be modeled and realized with a fixed main system block and many different changeable nonlinear function blocks such as piecewise-linear function, cubic function and other trigonometric functions. The implementation and experimentally investigation of this chaotic system, which can be modeled with many different nonlinear functions require a significant variety of circuit hardware. FPAA can be effectively used instead of these discrete implementations. This programmable device is more efficient, simpler and economical than using individual op-amps, comparators, analog multipliers and other discrete components used for implementing nonlinear functions. Nonlinear function blocks used in this chaotic system can be modeled with FPAA programming and a model can be rapidly changed for realizing another nonlinear function. Using this design approach, it can easily be investigated the roles and effects of different nonlinear functions in chaotic system structure under a programmable realistic model.

The organization of this paper is as follows. In Section 2 the system definitions with different nonlinear functions are summarized. FPAA-based circuit implementations using different nonlinear functions for this model will be introduced in Section 3. Finally, some concluding remarks will be discussed in the last Section.

2. System definitions

Chaotic system studied here is based on "Jerk" equations [11] and it is defined by the following ordinary differential equations:

$$\begin{aligned} \frac{du}{d\tau} &= v \\ \frac{dv}{d\tau} &= w \\ \frac{dw}{d\tau} &= f(u) - pw - v. \end{aligned} \tag{1}$$

Here, u, v and w denote the state variables of the system; p is the system parameter; $f(u)$ represents a nonlinear function which plays important role in the system's chaos mechanism. It has been verified that this chaotic system can be realized with many different nonlinear functions including different types of piecewise-linear functions [11, 12]. The most preferred functions in modeling this system are summarized in Table 1 with the typical parametric definitions. The implementation and experimentally investigation of the generalized chaotic system defined by equation (1) with different nonlinear functions require a significant variety of circuit hardware. In these implementations, several circuit topologies using op-amps, diodes and other passive components have been used for realizing the referred piecewise-linear nonlinear functions. To electronically implement other trigonometric nonlinear functions such as $f(u) = -B\left[u - 2\tanh\left(Cu\right)/C\right]$, it is required to use an IC device that operates as universal trigonometric function generator. FPAA can be effectively used instead of nearly all of discrete analog implementations. This programmable device is more efficient, simpler and economical than using individual op-amps, comparators, analog multipliers, trigonometric function generator and other discrete components. Nonlinear function blocks used in this generalized chaotic system can be modeled with FPAA programming and a model can be rapidly changed for realizing other nonlinear functions. In the next section, we will introduce some FPAA-based system implementations consisting different nonlinear functions described in Table 1.

Table 1. The most used nonlinear function definitions for modeling chaotic system defined by equation (1).

Nonlinear Functions		Function Parameters
$f(u) = \|u\| - r$	(2)	r=2
$f(u) = -B\max(u,0) + C$	(3)	B=6, C=-0.5
$f(u) = -Bu + Csgn(u)$	(4)	B=1.2, C=2
$f(u) = B(u^2/C - C)$	(5)	B=0.58, C=1
$f(u) = Bu(u^2/C - 1)$	(6)	B=1.6, C=5
$f(u) = -Bu(u^2/C - 1)$	(7)	B=0.9, $1/C$ =0.47
$f(u) = -B[u - 2\tanh(Cu)/C]$	(8)	B=2.15, C =1

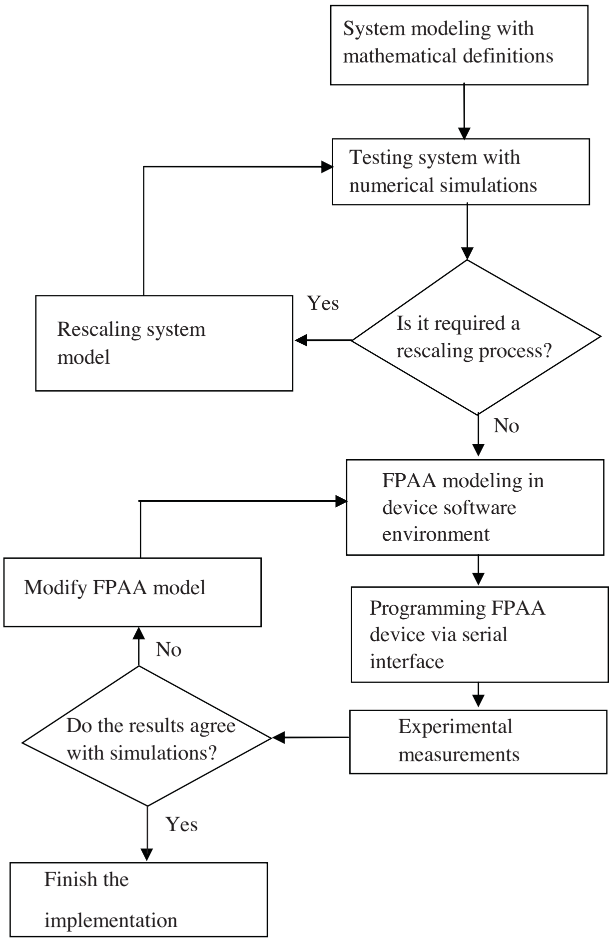


Figure 1. FPAA-based design and implementation procedure.

3. FPAA-based system implementations

3.1. Design procedure and experimental setup

Figure 1 shows typical FPAA implementation flow diagram. As shown in the diagram, the system is tested with numerical simulations before FPAA modeling. Because FPAA device has ±2 V saturation level, rescaling process may be required according to simulation results. After modeling system implementation in FPAA software tool, this model is downloaded to the FPAA development board via serial interface. Experimental results are compared to simulation results and/or experimental results from discrete electronic implementation of the same system. If the results are satisfactory, the implementation is finished or not FPAA modeling is modified. Last, let's mention about the limitations of FPAA. FPAA has limited capacity and for large-scale designs, additional FPAA boards can be used. In the experimental studies with FPAAs, the speed of the device should take into consideration. The operation frequency of FPAA is 2 MHz and it can only be extended to 8 MHz.

Figure 2. Experimental setup for FPAA-based implementation. AN221E04 type FPAA produced by Anadigm [1] has been used in this setup.

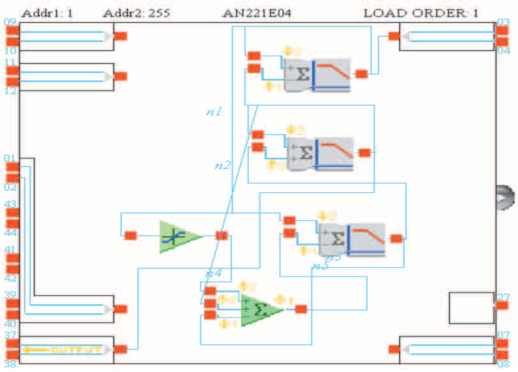
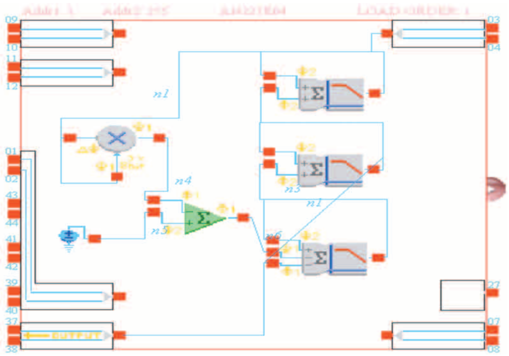
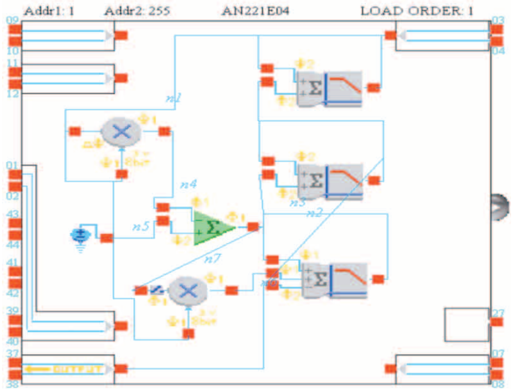
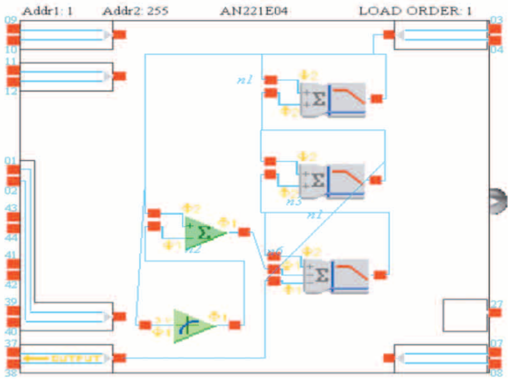
Experimental measurements are obtained from I/O connections of the FPAA board [1]. For monitoring chaotic dynamics in time and frequency domain, and X-Y mode, we used a virtual measurement system shown in Figure 2. This system is a PC-compatible virtual measurement system using a PC oscilloscope module [16]. PC oscilloscope module incorporates software that allows a PC to be used as an oscilloscope or spectrum analyzer. This system is flexible, easy to use and has many advantages over conventional instruments, including multiple views of the same signal, and on-screen display of voltage and time.

3.2. FPAA-based Chaotic System Realizations

By using FPAA programmable device, a common part of the chaotic system and all of the nonlinear functions listed in Table 1 can be easily realized in the reconfigurable form. Here, we will present four FPAA-based system implementations of generalized chaotic system defined by Equation (1) and using four different nonlinear functions listed in Table 1. After testing the system associated with four nonlinear functions via numerical simulations, we constructed four FPAA models. The system definitions and FPAA implementation schemes of these four models are illustrated in Table 2.

Among of these models, rescaling process has been applied to only the first model using piecewise-linear nonlinearity. Because the dynamic ranges of the state variables u, v, w of the first model are (-4.5, 4.5), (-4.4, 4.4) and (-4.4, 4.4), respectively, these values are over the saturation limits (± 2 V) of the FPAA. The first FPAA-based implementation uses a piecewise-linear nonlinear function. The blocks used in this implementation have been described in Table 3 with parameter settings. As shown in the FPAA implementation scheme in Table 1, system state-variables u, v and w are represented at the output of SUMFILTER blocks. Circuit gains are implemented by SUMFILTER block gains. GAIN LIMITER and SUM/DIFFERENCE blocks were used for

Table 2. FPAA implementation schemes of the chaotic system with four nonlinear functions.

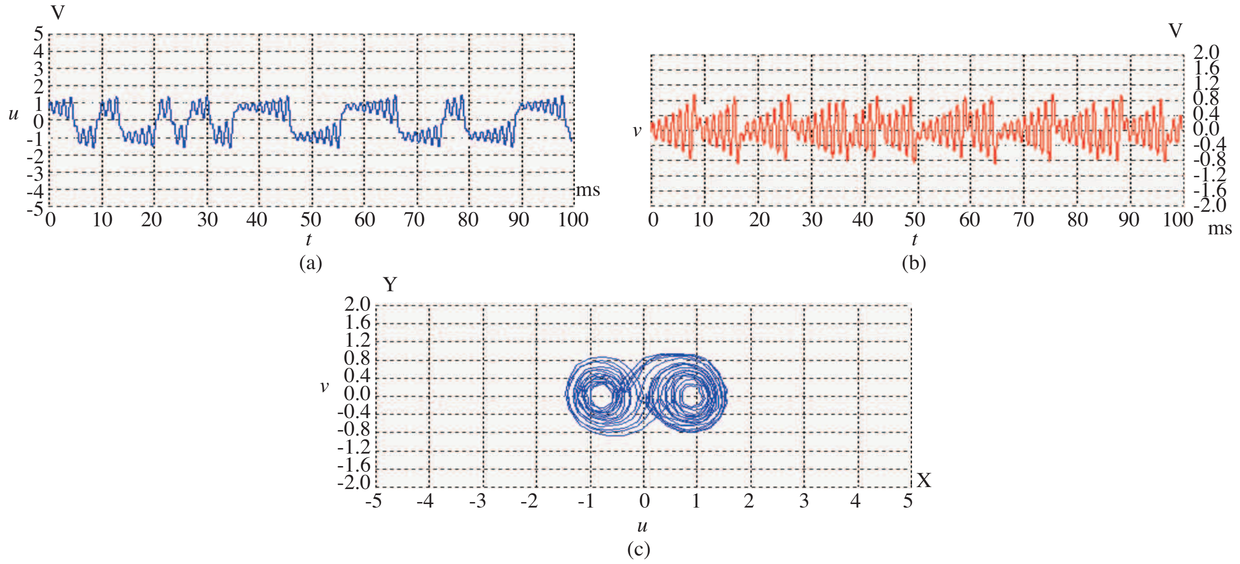
CHAOTIC SYSTEM_1 $\dot{u} = 0.4v$ $\dot{v} = 0.4w \qquad (9)$ $\dot{w} = -0.4v - 0.24w + 0.4[-2\,\mathrm{sgn}\,u) + 1.2u]$	
CHAOTIC SYSTEM_2 $\dot{u} = v$ $\dot{v} = w \qquad (10)$ $\dot{w} = -v - 0.6w - 0.58(1 - u^2)$	
CHAOTIC SYSTEM_3 $\dot{u} = v$ $\dot{v} = w \qquad (11)$ $\dot{w} = -v - 0.6u(0.47u2 - 1)$	
CHAOTIC SYSTEM_4 $\dot{u} = v$ $\dot{v} = w \qquad (12)$ $\dot{w} = -v - 0.6w - 2.15u(u - 2\tanh(u))$	

implementing piecewise-linear nonlinear function including an sgn(.) term. The chaotic dynamics and double scroll chaotic attractor obtained from the first FPAA-based chaotic system using piecewise-linear nonlinearity are shown in Figure3.

Table 3. FPAA modules and parameter settings for the first chaotic system defined by Equation (9).

Name	Options		Parameters	
SumFilter1 (SumFilter v.1.1.2) Anadigin (Approved)	Output Changes on	*Phase 1*	Corner Frequency [kHz]	0.339
	Input 1	*Non-inverting*	Gain 1 (UpperInput)	1.00
	Input 2	*Inverting*	Gain 2 (LowerInput)	0.889
	Input 3	*Off*		
SumFilter2 (SumFilter v.1.1.2) Anadigin (Approved)	Output Changes on	*Phase 1*	Corner Frequency [kHz]	0.800
	Input 1	*Non-inverting*	Gain 1 (UpperInput)	1.00
	Input 2	*Inverting*	Gain 2 (LowerInput)	1.00
	Input 3	*Off*		
SumFilter3 (SumFilter v.1.1.2) Anadigin (Approved*)	Output Changes on	*Phase 1*	Corner Frequency [kHz]	0.799
	Input 1	*Non-inverting*	Gain 1 (UpperInput)	1.35
	Input 2	*Inverting*	Gain 2 (LowerInput)	1.00
	Input 3	*Off*		
Gain Limiter1 (GainLimiter v.1.0.3) Anadigin (Approved)			Gain	20.0
			Output Voltage Limited	1.00
SumDiffl (SumDiffl v.1.1.4) Anadigin (Approved)	Output Phase	*Phase 1*	Corner Frequency	2.25
	Input 1	*Non-inverting*	Gain 2 (MiddleInput)	1.00
	Input 2	*Non-inverting*	Gain 3 (LowerInput)	0.400
	Input 4	*Inverting*		
	Input 4	*Off*		

After completing an experiment on a FPAA-based model and getting experimental results, the same FPAA device can be reconfigured for another modeling on the fly. So, in our modeling and experimental study for the generalized chaotic system, by keeping the common part of the circuit, we changed the nonlinear function used in the first implementation.

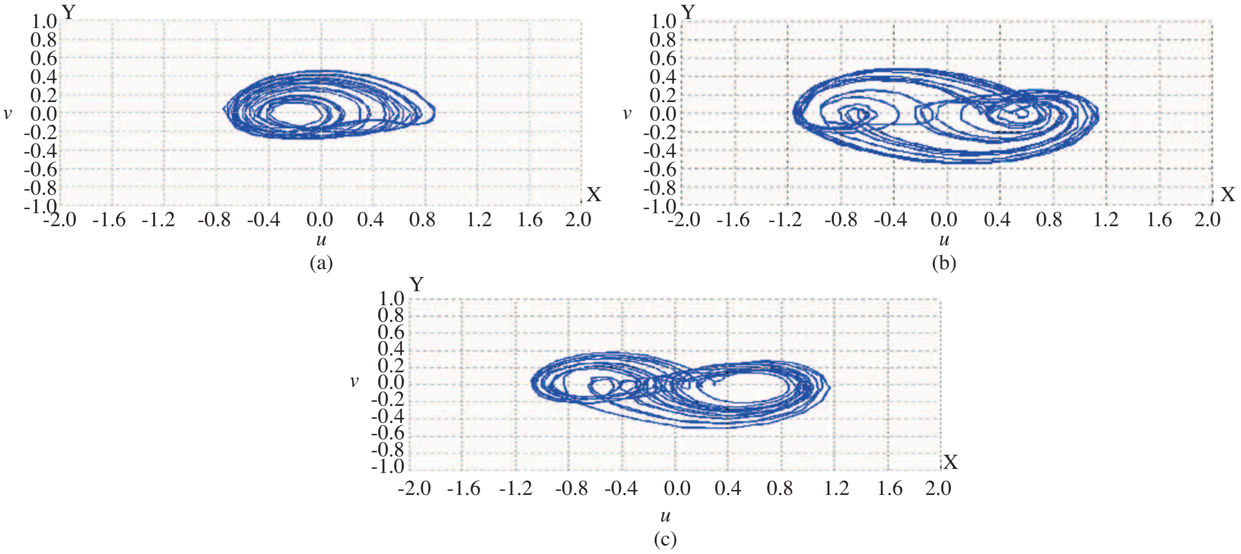


(a)

(b)

(c)

Figure3. Experimental results for the first FPAA implementation, (a)–(b) Time responses of $u(t)$ and $v(t)$ chaotic dynamics, (c) Chaotic attractor projection in $u - v$ plane.



(a)

(b)

(c)

Figure 4. Chaotic attractor projections in $u - v$ plane, (a) for the second FPAA-based implementation, (b) for the third FPAA-based implementation, (c) for the fourth FPAA-based implementation.

The second FPAA-based implementation has been constructed in FPAA environment according to mathematical model defined by Equation (10) in Table 2. In addition to common SUMFILTER blocks, MULTIPLIER, SUM/DIFFERENCE and VOLTAGE REFERENCE blocks were used to implement the second nonlinear function. After reconfiguration and downloading process, the experimental measurements can be done, as in the first one. The chaotic attractor illustration belonging to this modeling has been shown in Figure 4(a).

The third FPAA-based implementation defined by Equation (11) uses a cubic-like nonlinear function. Similar to the former implementation, MULTIPLIER, SUM/DIFFERENCE and VOLTAGE REFERENCE blocks were used to implement cubic-like nonlinear function. After reconfiguration and downloading process, the experimental measurements are obtained as in the others. The chaotic attractor illustration belonging to this modeling has been shown in Figure 4(b).

The last FPAA-based implementation has been constructed in FPAA environment according to mathematical model defined by Equation (12) in Table 2. This implementation uses a trigonometric nonlinear function. In addition to common SUMFILTER blocks used in other models, a user-defined TRANSFER FUNCTION block was used for implementing trigonometric function including a tanh(.) term. This transfer function module produces an output voltage with 256 quantization steps according to a Lookup Table constituted by user. After reconfiguration and downloading process, the experimental measurements are obtained as in the others. The chaotic attractor illustration is shown in Figure 4(c).

4. Conclusions

FPAA-based programmable implementation of a chaotic system which is suitable for reconfigurable design according to its nonlinear structure has been presented. Experimental results agree with the results obtained from experiments established by discrete components. This system can be effectively used as a programmable chaos generator in many chaos-based applications. In its programmable structure, different nonlinear function blocks can be modeled with FPAA programming and a model can be rapidly changed for realizing another nonlinear function. Since a FPAA device can be effectively used instead of nearly all electronic hardware for discrete implementations of related chaotic system, this programmable and reconfigurable implementation offers more efficient, simpler and economical solutions. Because such implementations provide the possibility of flexible and reconfigurable design and implementation of many analog chaotic systems based on mathematical modeling without need to complex electronic hardware, FPAA-based chaos studies will be very useful for the researchers from different science and engineering disciplines.

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