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Parametrically Tunable Audio Shelving And Equalizing Ladder Wave Digital Filters

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Abstract

Parametrically tunable audio equalizers are conventionally realized using allpass digital filter networks. They consist of first-order shelving filters and second-order equalizing filters. In this paper, ladder wave digital filters (WDFs) with parametrically tunable coefficients are proposed as shelving and equalizing filters. Similar to the allpass realization, the transfer function power complementary property of WDFs is used to obtain efficient shelving and equalizing filters. However, unlike the allpass structures, the transfer function and the tunable parameters of the ladder WDF are derived from the analog filter equivalent of the digital shelving and equalizing filters. For the shelving WDF, the cut-off frequency and gain are tunable, while for the equalizing WDF, the tunable parameters are the center frequency, 3-dB bandwidth and gain. The transfer functions are derived and shown in terms of the tunable coefficients for both the shelving and equalizing WDFs.

Key Words: *Tunable parameters, audio shelving filters, equalizing filters, wave digital filters.*

1. Introduction

Wave digital filters (WDFs) represent a class of digital filters that are closely related to lossless resistively terminated classical filter networks. WDFs are a viable type of digital filter suitable for implementation due to their excellent sensitivity properties with respect to coefficient value variation and due to the modularity of their structures [1-3].

Every WDF has a reference analog filter from which it is derived. A WDF obtains its transfer function from the reference filter, as do other infinite impulse response (IIR) filters, using the bilinear transformation method [4-6]. However, unlike conventional IIR filters, a WDF derives its structure from the reference filter. There are many types of WDFs based on reference analog filters, the most popular being lattice and ladder WDFs [7-8].

Ladder WDFs are more flexible than lattice structures, and recently there has been an increased interest in applying ladder WDFs in audio systems applications. Developments in multimedia audio and in auditory display have created a special interest in physical models of audio sources with optimal tunable parameters [9]. In complex sound synthesis using physical modeling, ladder WDFs are used to synthesize the sound source using multi-port adaptors [10]. As such, new ladder WDF structures are needed to create the many digital audio effects currently realized with conventional digital filter structures.

Equalization is a processing technique used to change the properties of audio signals within a specific frequency range. Tunable equalization filters, classified as first-order shelving and second-order parametric equalization filters, are the most common building blocks that are used in the design of digital graphic equalizers. These filters are conventionally realized using allpass structures [11-13]. In all these cases the allpass sections are obtained from the digital domain transfer function. The power complementary property of the allpass structures is exploited to obtain efficient shelving and equalizing filters.

In this paper, ladder WDFs with parametrically tunable parameters are proposed as shelving and equalizing filters. The transfer function and the WDF structure are derived from the analog equivalent of the digital shelving and equalizing filters. As WDFs are inherently power complementary, an efficient realization is obtained by using the simultaneously available lowpass and highpass responses of a first-order filter, and the bandpass and bandstop responses of a second-order filter. The resulting WDFs have tunable parameters that control the gain and the 3-dB bandwidth of the shelving filter, and for the equalizing filter there is an additional parameter: the center frequency. The digital domain transfer functions of the WDFs are derived in terms of these tunable parameters.

2. Equalization Filters

2.1. Shelving filters

A digital low-frequency shelving filter consists of a first-order lowpass filter with adjustable positive gain G in parallel with a highpass filter. Its transfer function may be written as

$$H_{SL}(z) = GH_{LP}(z) + H_{HP}(z) \tag{1}$$

where H_{LP} and H_{HP} are the transfer functions of the first-order lowpass and highpass filters, respectively.

A digital high-frequency shelving filter consists of a first-order highpass filter with adjustable positive gain G in parallel with a lowpass filter. Its transfer function may be written as

$$H_{SH}(z) = GH_{HP}(z) + H_{LP}(z) \tag{2}$$

Practical realizations of shelving filters typically employ allpass structures obtained from a pair of power complementary lowpass and highpass IIR filter transfer functions. The low-frequency shelving filter can be written in terms of the allpass structure as

$$H_{SL}(z) = \frac{G}{2}\{1 - A_1(z)\} + \frac{1}{2}\{1 + A_1(z)\} \tag{3}$$

where $A_1(z)$ is a first order allpass transfer function given by

$$A_1(z) = \frac{\alpha - z^{-1}}{1 - \alpha z^{-1}} \tag{4}$$

The high-frequency shelving filter can be written in terms of the allpass function as

$$H_{SH}(z) = \frac{G}{2}\{1 + A_1(z)\} + \frac{1}{2}\{1 - A_1(z)\} \tag{5}$$

The 3-dB cutoff frequency $\omega ?_c$ for both filters is given by

$$\omega_c = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right) \tag{6}$$

2.2. Equalizing filters

Most digital parametric equalizing filters are based on second-order IIR bandpass and bandstop filters. A second-order parametric filter consists of a bandpass filter H_{BP} with a positive gain G, in parallel with a bandstop filter H_{BS}. Its transfer function can be written as

$$H_{EQ}(z) = GH_{BP}(z) + H_{BS}(z) \tag{7}$$

The transfer function can be expressed in terms of a second-order allpass section as

$$H_{EQ}(z) = \frac{G}{2}\{1 - A_2(z)\} + \frac{1}{2}\{1 + A_2(z)\} \tag{8}$$

where $A_2(z)$ is a second-order allpass transfer function given by

$$A_2(z) = \frac{\alpha - \beta(1+\alpha)z^{-1} + z^{-2}}{1 - \beta(1+\alpha)z^{-1} + \alpha z^{-2}} \tag{9}$$

The center frequency of the bandpass filter and the notch frequency of the bandstop filter are given by

$$\omega_0 = \cos^{-1}(\beta) \tag{10}$$

while the 3-dB bandwidth for both filters is

$$\Delta\omega_{3dB} = \cos^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right) \tag{11}$$

3. Ladder Wave Digital Filters

A ladder WDF is designed using an analog filter as its reference. The conversion from the analog s domain to the digital z domain is accomplished using the bilinear transform

$$s = \frac{1-z^{-1}}{1+z^{-1}} \tag{12}$$

The relationship between the analog frequency, Ω, of the s-plane and the normalized digital frequency, ω, on the unit circle of the z-plane is

$$\Omega = \tan(\frac{\omega}{2}) \tag{13}$$

In addition to the transfer function, the structure of the WDF is derived from the reference analog filter. The recourse of a classical port network is used to obtain the WDF equivalents of analog filter components. A classical port designated by a port resistance, R, instantaneous voltage, v, and instantaneous current, i, has the incident wave, a, and reflected wave, b, defined by

$$a = v + iR; \quad b = v - iR \tag{14}$$

By representing each analog element as a 2-port network having the appropriate port resistance, and using the incident and reflected waves as the signal variables, a WDF element is constructed. The WDF equivalents are shown in Table 1 for the capacitor, inductor, resistor and resistive source.

Adaptors are required to connect the basic elements that have different port resistance values in a manner satisfying Kirchhoff's laws. Adaptors connect the signals either in series or parallel. For an n-port adaptor, each port, k, has its incident and reflected wave, a_k, and, b_k, respectively. Using the column vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ to represent these signals as

$$\mathbf{A} = \begin{vmatrix} a_1 \\ a_2 \\ : \\ : \\ a_n \end{vmatrix} \text{ and } \mathbf{B} = \begin{vmatrix} b_1 \\ b_2 \\ : \\ : \\ b_n \end{vmatrix} \tag{15}$$

the equation for the n-port series adaptor can be written as

$$\mathbf{B} = (\mathbf{I} - \mathbf{M}_s)\mathbf{A} \tag{16}$$

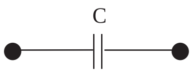
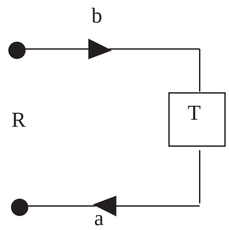
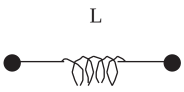
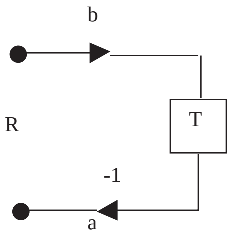
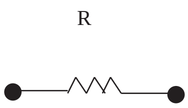
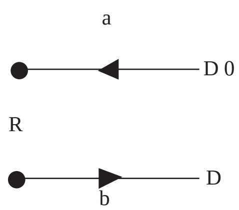
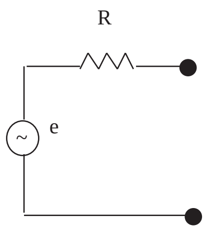
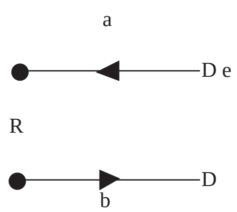
where $\boldsymbol{I}$ is an n by n unity matrix and the coefficient matrix is

$$\mathbf{M}_s = \begin{bmatrix} m_{s1} & m_{s1} & .. & .. & m_{s1} \\ m_{s2} & m_{s2} & .. & .. & m_{s2} \\ : & : & .. & .. & : \\ : & : & .. & .. & : \\ m_{sn} & m_{sn} & .. & .. & m_{sn} \end{bmatrix} \tag{17}$$

For port k the coefficient is

$$m_{sk} = \frac{2R_k}{\sum\limits_{k=1}^{n} R_k} \tag{18}$$

Table 1. WDF Equivalents of Analog Elements.

ELEMENT	PORT RESISTANCE (R)	WDF EQUIVALENT
 Capacitor	1/C	
 Inductor	L	
 Resistor	R	
 Resistive source	R	

In addition, the coefficients are related as

$$\sum_{k=1}^{n} m_{sk} = 2 \tag{19}$$

with

$$0 \leq m_{sk} \leq 2 \tag{20}$$

For an n-port parallel adaptor, the equation that relates the input and output signals is

$$\mathbf{B} = (\mathbf{M}_p - \mathbf{I})\mathbf{A} \tag{21}$$

where

$$\mathbf{M}_p = \begin{bmatrix} m_{p1} & m_{p2} & .. & .. & m_{pn} \\ m_{p1} & m_{p2} & .. & .. & m_{pn} \\ : & : & .. & .. & : \\ : & : & .. & .. & : \\ m_{p1} & m_{p2} & .. & .. & m_{pn} \end{bmatrix} \tag{22}$$

and

$$m_{pk} = \frac{2G_k}{\sum\limits_{k=1}^{n} G_k} \tag{23}$$

where G_k is the port conductance. The coefficients are related as

$$\sum_{k=1}^{n} m_{pk} = 2 \tag{24}$$

and

$$0 \leq m_{pk} \leq 2 \tag{25}$$

WDF structures use a combination of different adaptors to emulate the analog filter structures. For the ladder WDFs considered here, 2-port and 3-port adaptors are required. Realizing an n-port adaptor with a minimum number of multipliers is desirable to reduce cost. For a 2-port adaptor, a parallel realization is essentially equivalent to that of a series. A symmetrical realization of a 2-port adaptor is obtained by the manipulation of the adaptor equations and expressing the single adaptor coefficient as

$$m = \frac{R_1 - R_2}{R_1 + R_2} \tag{26}$$

where

$$-1 \leq m \leq 1 \tag{27}$$

The equation that relates the input and output signals of the 2-port adaptor is

$$\mathbf{B} = \mathbf{MA} \tag{28}$$

with

$$\mathbf{M} = \begin{bmatrix} -m & 1+m \\ 1-m & m \end{bmatrix} \tag{29}$$

For a 3-port adaptor, a single multiplier realization is obtained when 2 of the ports are equivalent in value, hence having equal coefficient value. The other remaining coefficient is derived from this coefficient. Due to the 2 ports having an equal resistance value, the resulting single coefficient, m, for both the parallel and series adaptor has a range of

$$0 \le m \le 1 \tag{30}$$

The symbols and signal flow diagrams of single multiplier adaptors are shown in Table 2.

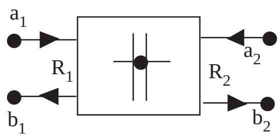
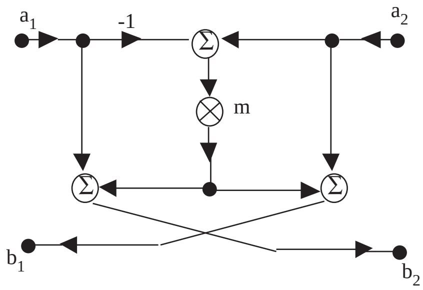
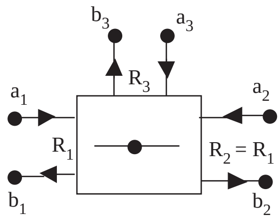
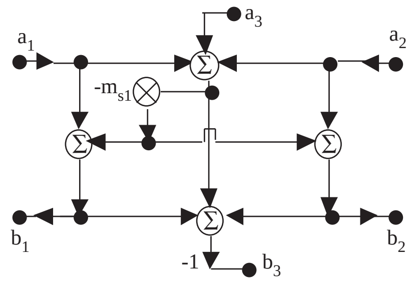
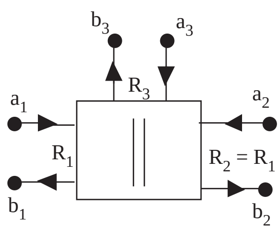
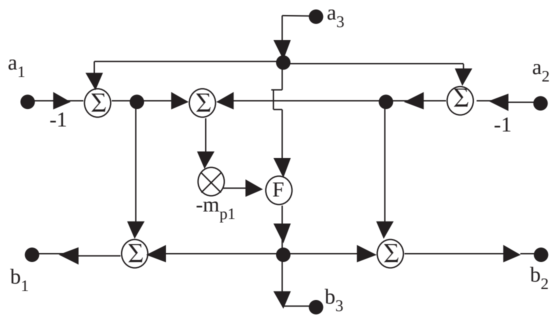
Adaptor	Signal-flow Diagram
 2-port parallel/series	
 3-port series	
 3-port parallel	

Table 2. WDF Adaptors with Single Multiplier.

4. Ladder WDF Shelving Filters

For analog audio systems, the shelving and equalizing filters consist of first- and second-order Butterworth filters, respectively [14]. The first-order analog filter is shown in Figure 1. For a Butterworth response with a normalized cut-off frequency Ω_c rad/s the values of the resistors are $R_1 = R_2 = 1\Omega$ and $L = (2/\Omega_c)H$.

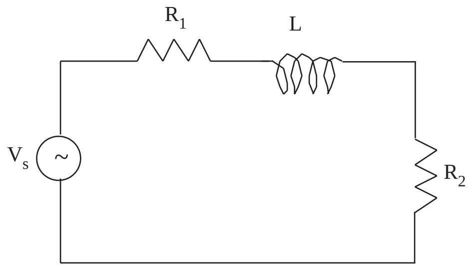


Figure 1. First-order Butterworth filter.

A ladder WDF is obtained by converting the analog elements to the WDF equivalent and using a 3-port series adaptor to connect these elements. Consider the 3-port series adaptor shown in Figure 2 where ports 1 and 2 are the resistive ports and port 3 is the inductive port. The coefficient for the 3-port series adaptor according to (18) is

$$m = m_{s1} = \frac{2R_1}{R_1 + R_2 + R_3} \tag{31}$$

where

$$\begin{array}{l} R_1 = R_2 = 1 \\ R_3 = \frac{2}{\Omega_c} \end{array} \tag{32}$$

The coefficient value is thus

$$m = \frac{\Omega_c}{1 + \Omega_c} \tag{33}$$

with its range given by (30). Alternatively, from (13), m can be written in terms of the digital frequency as

$$m = \frac{\tan(\omega/2)}{1 + \tan(\omega/2)} \tag{34}$$

The WDF has a doubly complementary response that is not available with a conventional IIR filter realization. The lowpass and highpass responses are obtained simultaneously as shown in Figure 2. The equation that relates the lowpass, H_{LP}, with highpass response, H_{HP}, is [1]

$$|H_{LP}|^2 + |H_{HP}|^2 = 1 \tag{35}$$

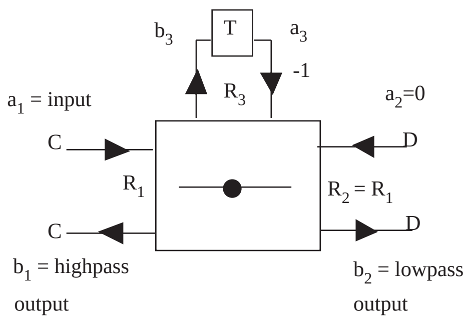


Figure 2. First-order WDF with series adaptor.

The WDF lowpass transfer function is obtained from the analog domain transfer function. The first order Butterworth filter transfer function is

$$H_{LP}(s) = \frac{\Omega c}{s + \Omega c} \tag{36}$$

The WDF transfer function is derived using (12) and (36) and can be written in terms of the coefficient m as

$$H_{LP}(z) = \frac{m(1 + z^{-1})}{1 + (2m - 1)z^{-1}} \tag{37}$$

The WDF highpass transfer function is obtained from the lowpass transfer function using the power complementary property and can be written as

$$H_{HP}(z) = \frac{(1 - m)(1 - z^{-1})}{1 + (2m - 1)z^{-1}} \tag{38}$$

The low-frequency shelving WDF transfer function is obtained from (37, 38) and (1) as

$$H_{SL}(z) = \frac{[m(G - 1) + 1] + [m(G + 1) - 1]z^{-1}}{1 + (2m - 1)z^{-1}} \tag{39}$$

The high-frequency shelving WDF transfer function is obtained from (37, 38) and (2) as

$$H_{SH}(z) = \frac{[m(1 - G) + G] + [m(1 + G) - G]z^{-1}}{1 + (2m - 1)z^{-1}} \tag{40}$$

The realization of the shelving ladder WDF filter is shown in Figure 3. Both the low-frequency and the high-frequency shelving filters use the same structure. With the values of coefficients set to $K_1 = 1$ and $K_2 = G$, a low-frequency shelving filter is obtained. A high-frequency shelving filter is obtained with $K_1 = G$ and $K_2 = 1$.

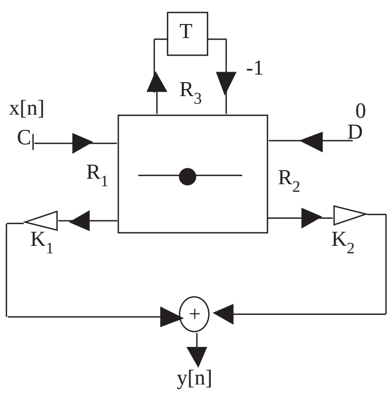


Figure 3. Ladder shelving WDF. for lowpass shelving $K_1 = 1$ and $K_2 = G$, for highpass shelving $K_1 = G$ and $K_2 = 1$.

For the low-frequency shelving filter, the parameter G controls the amount of boost or cut at low frequencies while the adaptor coefficient m controls the bandwidth of the boost or cut. Figures 4-6 show the gain responses for the low-frequency shelving filter for various values of G and m.

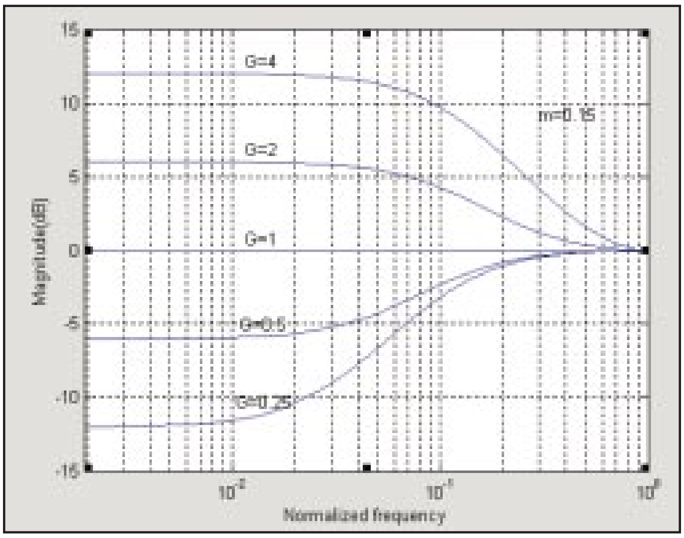


Figure 4. The effects of varying G with m = 0.15 on the gain responses of a low-frequency shelving WDF.

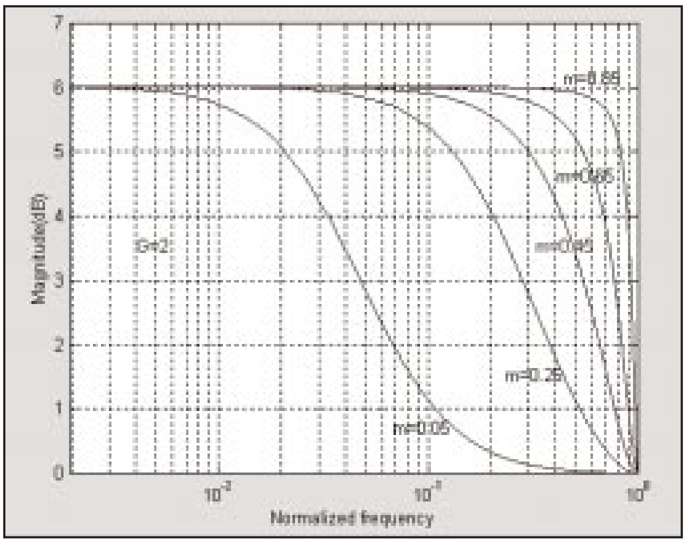


Figure 5. The effects (boost) of varying m with G = 2 on the gain response of a low-frequency shelving WDF.

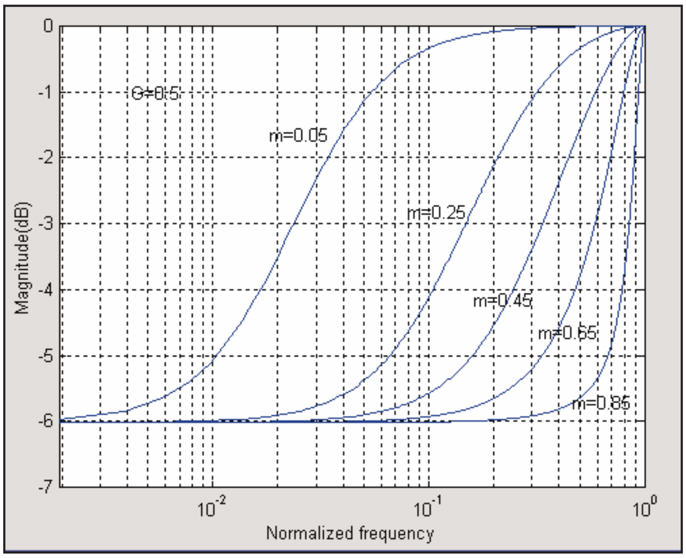


Figure 6. The effects (cut) of varying m with G = 0.5 on the gain response for a low-frequency shelving WDF.

Conversely, for the high-frequency shelving filter, the parameter G controls the amount of boost or cut at high frequencies while the adaptor coefficient m controls the boost or cut bandwidth. Figures 7-9 show the gain responses for the high-frequency shelving filter for different values of G and m.

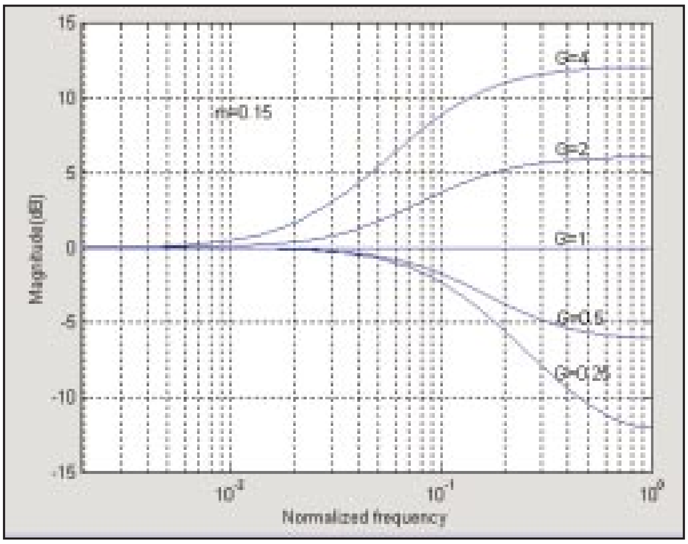


Figure 7. The effects of varying G with m = 0.15 on the gain responses of a high-frequency shelving WDF.

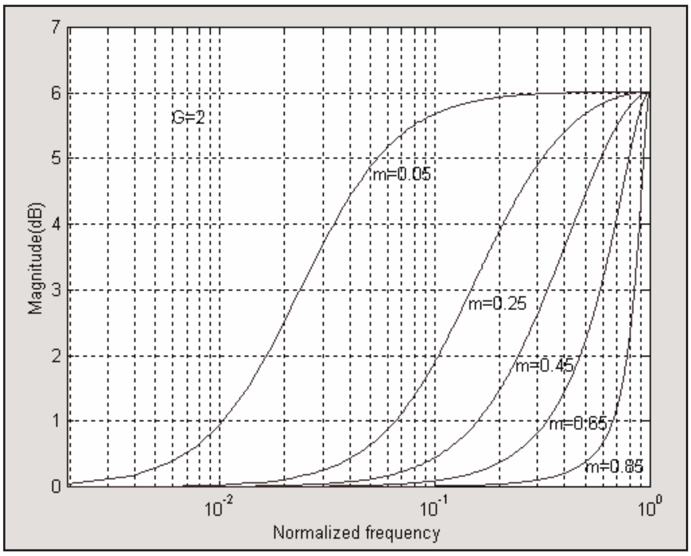


Figure 8. The effects (boost) of varying m with G = 2 on the gain response of a high-frequency shelving WDF.

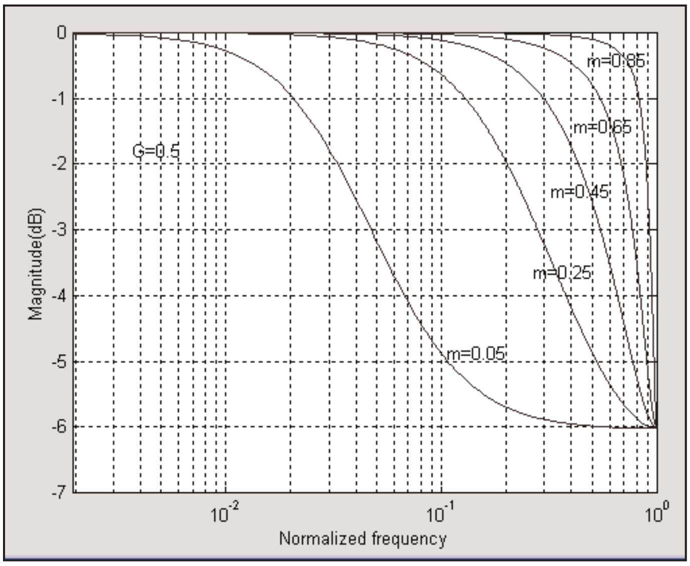


Figure 9. The effects (cut) of varying m with G = 0.5 on the gain response for a high-frequency shelving WDF.

5. Ladder WDF Equalizing Filters

A lowpass to bandpass transformation can be used to obtain a bandpass analog filter from a lowpass analog filter [15]. A second-order Butterworth analog filter obtained from lowpass to bandpass transformation is shown in Figure 10.

The values the resistors are $R_1 = R_2 = 1\Omega$, while the reactive elements are

$$L = \frac{2}{B} \tag{41}$$

and

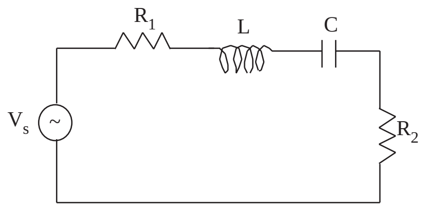


Figure 10. Second-order analog filter.

$$C = \frac{B}{2\Omega_0^2} \tag{42}$$

where B is the 3-dB bandwidth defined as

$$B = \Omega_H - \Omega_L \tag{43}$$

while Ω_0 is the center frequency defined as

$$\Omega_0 = \Omega_H \Omega_L \tag{44}$$

Ω_H and Ω_L are the higher and lower 3-dB passband edge frequencies, respectively.

Applying the WDF principles to the analog components will result in several possible configurations. Consider the realization of the ladder WDF consisting of a 3-port series adaptor and a 2-port adaptor as shown in Figure 11. In this realization, 2 multipliers are required for the WDF structure, one for each adaptor.

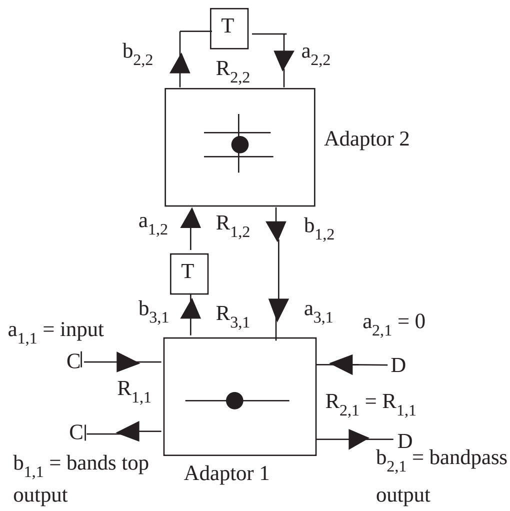


Figure 11. Second-order WDF with 3-port series and 2-port adaptors.

The 3-port adaptor connects the resistive components to the 2-port adaptor representing the series resonance. The 3-port adaptor resistance values are

$$\begin{aligned} R_{1,1} &= R_{2,1} = 1 \\ R_{3,1} &= R_L + R_C = \tfrac{2(1+\Omega_0^2)}{B} \end{aligned} \tag{45}$$

Using (18), the coefficient of this adaptor is therefore

$$m_1 = \frac{B}{B + 1 + \Omega_0^2} \tag{46}$$

with its range given by (30).

The 2-port adaptor is used to connect the series resonant elements to the first adaptor. As such, the port resistance values are [1]

$$\begin{array}{l} R_{1,2} = R_{3,1} = \frac{2(1+\Omega_0^2)}{B} \\ R_{2,2} = \frac{R_C}{R_L}(R_{1,2}) = \frac{2\Omega_0^2(1+\Omega_0^2)}{B} \end{array} \tag{47}$$

Using (26), the coefficient of the 2-port adaptor is

$$m_2 = \frac{1 - \Omega_0^2}{1 + \Omega_0^2} \tag{48}$$

with its range given by (27).

In terms of the digital domain parameters, the coefficient m_1, using (13), (43), (44) and (46), can be shown to be equal to

$$m_1 = \frac{\tan(\frac{\Delta\omega_{3dB}}{2})}{\tan(\frac{\Delta\omega_{3dB}}{2}) + 1} \tag{49}$$

where $\Delta\omega_{3dB}$ is the 3-dB bandwidth of the WDF defined as

$$\Delta\omega_{3dB} = \omega_H - \omega_L \tag{50}$$

ω_H and ω_L are the higher and lower 3-dB digital passband edge frequencies, respectively.

Using (13) and (48), it can be shown that the center frequency ω_0 of the WDF is related to m_2 according to

$$m_2 = \frac{1 - \tan^2(\frac{\omega_0}{2})}{1 + \tan^2(\frac{\omega_0}{2})} \tag{51}$$

Hence the WDF coefficients control the bandwidth and the center frequency of the bandpass filter. As shown in Figure 11, the WDF has an input and a bandpass output. In addition, a stopband response, H_{BS}, that is power complementary to the bandpass response, H_{BP}, is realized simultaneously as indicated in the figure. The responses are related as

$$|H_{BP}|^2 + |H_{BS}|^2 = 1 \tag{52}$$

The transfer function of the bandpass WDF is obtained from the analog filter transfer function

$$H_{BP}(s) = \frac{Bs}{s^2 + Bs + \Omega_0^2} \tag{53}$$

From (12), the digital domain transfer function is thus

$$H_{BP}(z) = \frac{\left(\frac{B}{1+B+\Omega_0^2}\right)(1 - z^{-2})}{1 + 2\left(\frac{\Omega_0^2 - 1}{1+B+\Omega_0^2}\right)z^{-1} + \left(\frac{1-B+\Omega_0^2}{1+B+\Omega_0^2}\right)z^{-2}} \tag{54}$$

Using (46) and (48), the transfer function can be written in terms of m_1 and m_2 as

$$H_{BP}(z) = \frac{m_1(1 - z^{-2})}{1 + 2m_2(m_1 - 1)z^{-1} + (1 - 2m_1)z^{-2}} \tag{55}$$

The bandstop transfer function is obtained from H_{BP} using the power complementary property as

$$H_{BS}(z) = \frac{(1 - m_1) + 2m_2(m_1 - 1)z^{-1} + (1 - m_1)z^{-2}}{1 + 2m_2(m_1 - 1)z^{-1} + (1 - 2m_1)z^{-2}} \tag{56}$$

The transfer function for the ladder equalizing WDF, from (eq55, 56) and (7), is

$$H_{EQ}(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{57}$$

where

$$\begin{aligned} b_0 &= 1 + (G - 1)m_1 \\ b_1 &= 2m_2(m_1 - 1) \\ b_2 &= 1 - m_1(1 + G) \\ a_1 &= b_1 \\ a_2 &= 1 - 2m_1 \end{aligned} \tag{58}$$

The realization of the ladder equalizing WDF is shown in Figure 12.

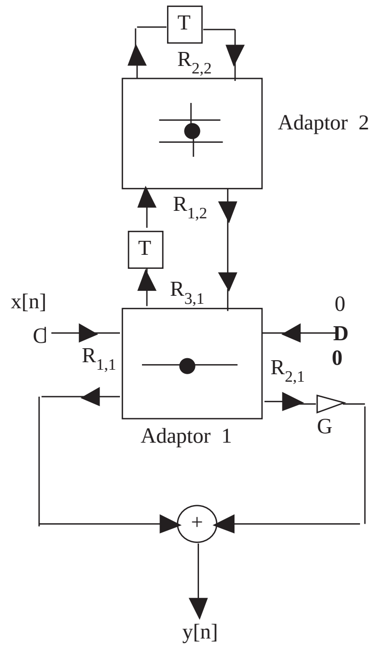


Figure 12. Ladder parametric WDF.

The peak or dip of the gain response occurs at ω_0 and is controlled by m_2, while the 3-dB bandwidth is controlled by m_1. In addition, the height of the peak or the dip of the gain response is controlled by G. Figures 13-15 show the effects on the gain response of the ladder equalizing WDF for various values of G, m_1 and m_2.

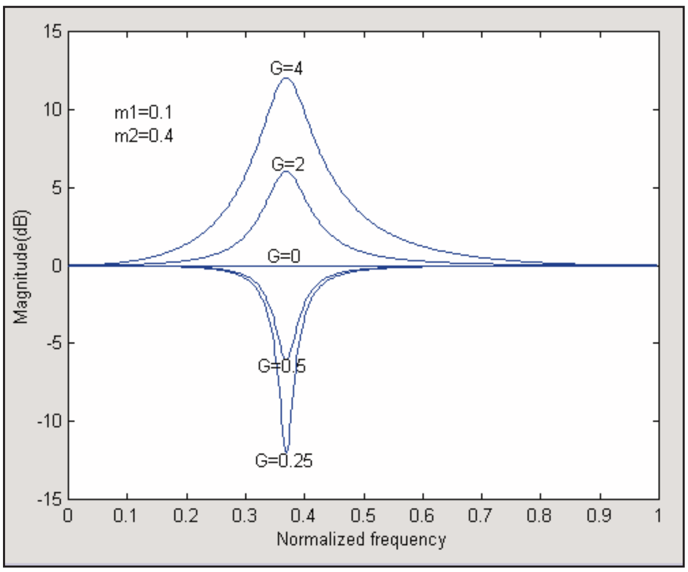


Figure 13. The effects of varying G with $m_1 = 0.1$ and $m_2 = 0.4$ on the gain responses of a second-order equalizing WDF.

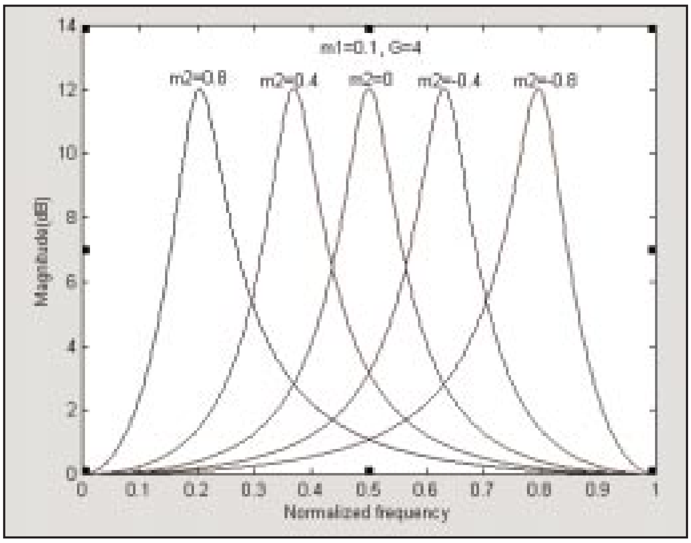


Figure 14. The effects of varying m_2 with $m_1 = 0.1$ and $G = 4$ on the gain responses of a second-order equalizing WDF.

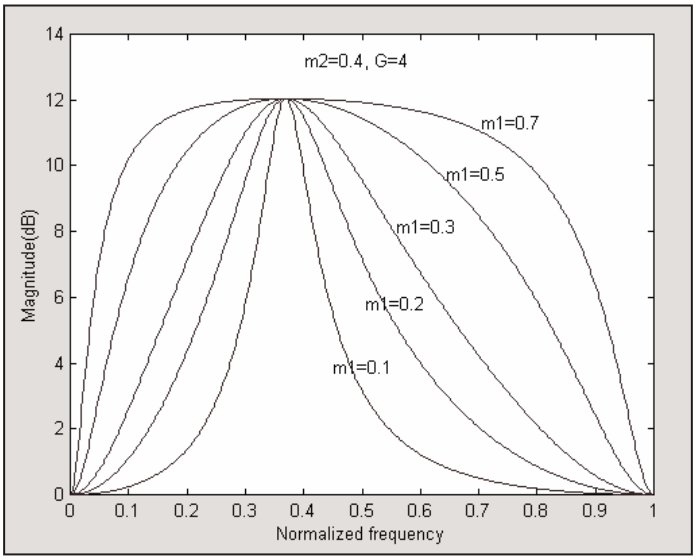


Figure 15. The effects of varying m_1 with $m_2 = 0.4$ and $G = 4$ on the gain responses of a second-order equalizing WDF.

6. Conclusion

This paper has shown the design of parametrically tunable audio shelving and equalizing ladder WDF networks. The power complementary property of WDFs is used to obtain efficient shelving and equalizing filters. From a first order ladder WDF, the simultaneously available lowpass and highpass responses are used to obtain the shelving filters. The responses of these WDFs are either low-pass or high-pass shelving depending on the values of the coefficients. In addition, the coefficients tune the 3-dB cut-off frequency and shelving filter gain. From a second-order ladder WDF, the equalizing filter is obtained by using the simultaneously available bandpass and bandstop responses. The center frequency, the 3-dB bandwidth and gain are the tunable parameters of the equalizing filter. The derivations of the WDF transfer functions in terms of the tunable coefficients have been shown.

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