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Scalar Wave Diffraction by Perfectly Soft Thin Circular Cylinder of Finite Length; Analytical Regularization Method

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Abstract

A new mathematically rigorous and numerically effective method for solving the boundary value problem of scalar wave diffraction by an infinitely thin circular cylindrical screen of finite length is proposed. The method is based on the combination of the Orthogonal Polynomials Approach, and on the ideas of the methods of analytical regularization.

As a result of the suggested regularization procedure, the initial boundary value problem is equivalently reduced to the infinite system of the linear algebraic equations of the second kind, i.e. to an equation of the type $(I+H)x = b$ in the space ℓ_2 of square summable sequences. This equation can be solved numerically by means of truncation with, in principle, any required accuracy.

1. Introduction

The solutions for diffraction problems concerning open screens are to interest of many scientists who design modern antennas, for example, Fresnel zone plates [1-3]. The efforts for solving such finite length geometry problems are usually connected with solutions for finite length strips as well [4]. Most of these methods concern boundary integral equations of the first kind. Consequently, the linear algebraic system of equations obtained as a reduction of that integral equation is of the first kind. Equations of the first kind have well-known instable features [5]. This paper presents a method that is a combination of the the conventional Orthogonal Polynomials Technique [6-7], and Analytical Regularization [5-8-9] aimed at a final linear algebraic system of equations of the second kind. Equations of the second kind have well-known benefits in stability and reliability in terms of numerical solutions [5].

The method suggested is a new mathematically rigorous and numerically efficient method for solving the problem of scalar wave diffraction by an infinitely thin circular cylinder of finite length in the case when scalar waves satisfy the Helmholtz equation with a boundary condition of the first kind, i.e. the Dirichlet problem. This work is a direct extension of a previous study [9]. That is, the initial boundary value problem is reduced to an infinite algebraic system of the second kind,

$$(I + H)y = c, \qquad y, c \in l_2, \tag{1}$$

which gives the relevant basis to construct an efficient numerical method.

The plan of the paper is follows. At first, in section 2, we briefly pose the problem. Section 3 is devoted to the axially symmetric properties of the problem considered and corresponding reduction of the integral equation, the final form of the considered integral equation, the canonical one, which has the solution procedure constructed in [9].

Finally, in section 4, the stability and reliability of the algorithm is demonstrated with the aid of the numerical results.

2. Posing of the Diffraction Problem and Its Reduction to the Integral Equation of the First Kind

Let us consider an infinitely thin circular cylindrical screen, i.e. surface S of finite length $2L$ and radius a (see Figure 1), and suppose that scalar wave $u^i(p)$, which satisfies the homogeneous Helmholtz equation in a vicinity of contour S, incident on the cylinder. It is necessary to find the scattering field, i.e. function $u^s(p)$ that satisfies the homogeneous Helmholtz equation:

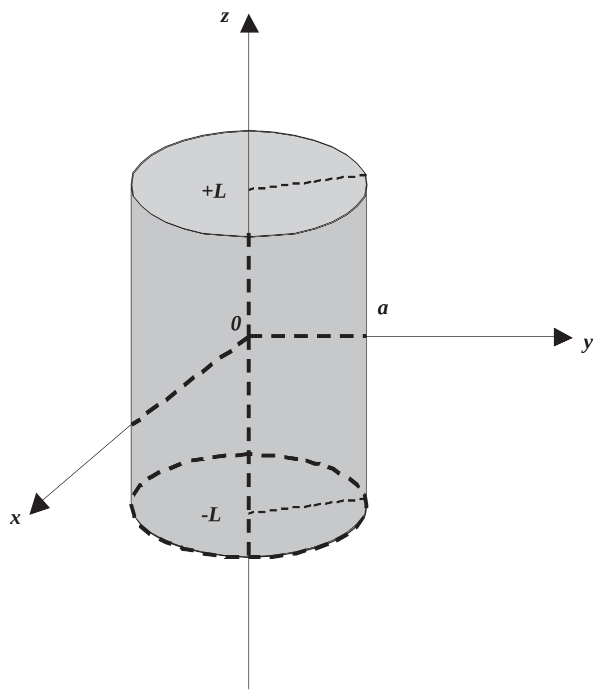

Figure 1. Geometry of the considered problem.

$$\left(\Delta + k^2\right) u^s(p) = 0, \quad p \in R^3 \backslash S, \tag{2}$$

under Sommerfeld radiation conditions and the Dirichlet boundary condition [10]. The resulting integral equation is

$$\int_S G(q,p) Z(p)\, ds_p = -u^i(q), \quad q \in S \tag{3}$$

where $G\ (q,p)$ is the Green's function of free space, $Z(p)$ is the unknown function of the kind $Z(p) = \left[dist(p,\partial S^{(+)}) \cdot dist(p,\partial S^{(-)})\right]^{-1/2} h(p),\ p \in S$, where $h(p)$ is a function, which belongs to Hölder class on $\overline{S}$ in the same sense as explained in [9], and S is the boundary surface written in the following form:

$$S = \{\ (\rho,\varphi,z): \quad \rho = a, \quad \varphi \in [-\pi,\pi], \quad z \in (-L,L)\ \}; \tag{4}$$

$$\partial S^{(\pm)} = \{\ (\rho,\varphi,z), \quad \rho = a, \quad \varphi \in [-\pi,\pi], \quad z = \pm L\}, \tag{5}$$

where (ρ,φ,z) are the coordinates of the points in the corresponding cylindrical coordinate system, and $\partial S^{(\pm)}$ are upper and lower contours that form the boundary of the manifold S (see Figure 1).

The existence and uniqueness of the solution to the diffraction problem considered can be proved to be similar to [9]. Thus the solution of integral equation (3), density $Z(p)$, which satisfies corresponding Meixner condition [10], is unique, if it exists (see [9]).

3. Reduction of Two-Dimensional Integral Equation to the Infinite Set of One-Dimensional Ones

Let us rewrite equation (3) in the above-mentioned cylindrical coordinates (ρ,φ,z). For this goal, we define the following functions

$$R(z,\varphi) = \left\{z^2 + 4a^2 \sin^2\frac{\varphi}{2}\right\}^{1/2}, \quad \varphi \in [-2\pi, 2\pi], \quad z \in [-2L, 2L]; \tag{6}$$

$$\tilde{G}(z,\varphi) = -\frac{1}{4\pi}\frac{e^{ikR(z,\varphi)}}{R(z,\varphi)}; \tag{7}$$

$$\tilde{g}(z_q,\varphi_q) = -u^i(q), \quad q = (a,\varphi_q,z_q) \in S; \tag{8}$$

$$\tilde{Z}(z_p,\varphi_p) = Z(p), \quad p = (a,\varphi_p,z_p) \in S. \tag{9}$$

The distance $|q-p|$ between two points $q,p \in S$, where $q = (a,\varphi_q,z_q)$, and $p = (a,\varphi_p,z_p)$, in the cylindrical coordinate system is

$$|q-p| = R(z_q - z_p, \varphi_q - \varphi_p). \tag{10}$$

Due to this, integral equation (3) can be easily rewritten as follows:

$$a\int_{-L}^{L}\int_{-\pi}^{\pi}\tilde{Z}\left(z_p,\varphi_p\right)\cdot\tilde{G}\left(z_q-z_p,\varphi_q-\varphi_p\right)d\varphi_p dz_p=\tilde{g}\left(z_q,\varphi_q\right),\quad z_q\in[-L,L],\ \ \varphi_q\in[-\pi,\pi] \tag{11}$$

with unknown function $\tilde{Z}\left(z_p,\varphi_p\right)$.

By means of new variables $u=z_q/L$ and v$=z_p/L$ equation (11) can be rewritten again as

$$aL\int_{-1}^{1}\int_{-\pi}^{\pi}\hat{Z}\left(v,\varphi_p\right)\cdot\hat{G}\left(u-v,\varphi_q-\varphi_p\right)d\varphi_p dv=\hat{g}\left(u,\varphi_q\right),\quad u\in[-1,1],\quad \varphi_p\in[-\pi,\pi], \tag{12}$$

where

$$\hat{G}\left(t,\varphi\right)=\tilde{G}\left(tL,\varphi\right),\quad t\in[-2,2]; \tag{13}$$

$$\hat{Z}\left(v,\varphi_p\right)=\tilde{Z}\left(vL,\varphi\right),\quad v\in[-1,1]; \tag{14}$$

$$\hat{g}\left(u,\varphi_q\right)=\tilde{g}\left(uL,\varphi_q\right),\quad u\in[-1,1]. \tag{15}$$

Functions $\hat{G}\left(t,\varphi\right),\quad \hat{Z}\left(v,\varphi\right)$ and $\hat{g}\left(u,\varphi\right)$ can be represented as their Fourier series:

$$\hat{G}\left(t,\varphi\right)=\sum_{m=-\infty}^{\infty}G_m\left(t\right)e^{im\varphi},\quad \varphi\in[-\pi,\pi],\quad t\in[-2,2]; \tag{16}$$

$$\hat{Z}\left(v,\varphi\right)=\sum_{m=-\infty}^{\infty}Z_m\left(v\right)e^{im\varphi},\quad \varphi\in[-\pi,\pi],\quad v\in[-1,1]; \tag{17}$$

$$\hat{g}\left(u,\varphi\right)=\sum_{m=-\infty}^{\infty}g_m\left(u\right)e^{im\varphi},\quad \varphi\in[-\pi,\pi],\quad u\in[-1,1], \tag{18}$$

where functions $G_m\left(t\right),\ Z_m\left(v\right)$ and $g_m\left(u\right)$ are equal to

$$G_m\left(t\right)=\frac{1}{2\pi}\int_{-\pi}^{\pi}e^{-im\varphi}\hat{G}\left(t,\varphi\right)d\varphi,\quad t\in[-2,2]; \tag{19}$$

$$Z_m\left(v\right)=\frac{1}{2\pi}\int_{-\pi}^{\pi}e^{-im\varphi}\hat{Z}\left(v,\varphi\right)d\varphi,\quad v\in[-1,1] \tag{20}$$

$$g_m(u) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-im\varphi} \hat{g}(u,\varphi)\, d\varphi, \quad u \in [-1,1] \tag{21}$$

The substitution of formulas (19)-(21) into formula (11) gives the equality of the form

$$aL \int_{-1}^{1} \int_{-\pi}^{\pi} \sum_{n=-\infty}^{\infty} Z_n(v)\, e^{in\varphi_p} \sum_{m=-\infty}^{\infty} G_m(u-v)\, e^{im(\varphi_q-\varphi_p)} d\varphi_p dv = \sum_{m=-\infty}^{\infty} g_m(u)\, e^{im\varphi_q},$$

$$u \in [-1,1], \varphi_q \in [-\pi,\pi] \tag{22}$$

After changing the order of integration and term-by-term integration of the series with respect to φ_p and using orthogonal properties of the functions system $\left\{e^{im\varphi}\right\}_{m=-\infty}^{\infty}$, one obtains the equality of two Fourier series and, consequently, the equality of the correspondent Fourier coefficients, i.e.

$$2\pi La \int_{-1}^{1} G_m(u-v)\, Z_m(v)\, dv = g_m(u), \quad u \in [-1,1] \quad m = 0, \pm 1, \pm 2, \pm 3, ... \tag{23}$$

Thus, integral equation (3) has been reduced to the infinite set of non-inter-connected integral equation (23) of the first kind with kernels defined by formulas (6), (7), (13) and (21) with unknown functions $Z_m(v)$, which evidently represents the form satisfying edge condition (see formulas (14), (17), (20) and [9])

$$Z_m(v) = \left(1-v^2\right)^{-1/2} h_m(v), \quad v \in (-1,1), \tag{24}$$

where $h_m(v)$ are functions of the Hölder class [10] on the interval $[-1,1]$.

Before constructing a solving procedure for equation (23), we have to understand the singular structure and properties of smoothness of the kernel $G_m(t)$. It is clear that $G_m(t)$ is an infinitely smooth function in any point $t \neq 0$, and only the behavior of the function $G_m(t)$ in the vicinity of the point $t = 0$ requires detailed investigation. The analysis of the integral (19) with function $\hat{G}(t,\varphi)$ shows that the function $G_m(t)$ has the representation of the form

$$G_m(t) = \frac{1}{4\pi^2 a} \ln|t| \left\{1 - t^2 \mathrm{A}_2^m - t^4 \mathrm{A}_4^m + t^5 \psi_0(t)\right\} + \psi_1(t), \tag{25}$$

where $\psi_0(t)$ and $\psi_1(t)$ are at least five times continuously differentiable functions, A_2^m and A_4^m are some constants and

$$\mathrm{A}_2^m = \frac{(kL)^2}{4} - \frac{m^2 - 1/4}{4} \left(\frac{kL}{ka}\right)^2. \tag{26}$$

Equation (25) defines the singular behavior of the function $G_m(t)$. Let us define functions $\mathrm{K}_m(t)$ by means of the following equality

$$-4\pi a G_m(t) = -\frac{1}{\pi}\ln|t| + K_m(t), \quad m = 0, \pm 1, \pm 2, \pm 3, ... \tag{27}$$

From Equations (25)-(27) those functions $\mathrm{K}_m(t)$ have the following representation

$$\mathrm{K}_m(t) = t^2 \ln|t| \cdot \left\{ \mathrm{A}_2^m + t^2 \mathrm{A}_4^m + t^3 \psi_0(t) \right\} - 4\pi a \psi_1(t) \tag{28}$$

with the same coefficients, A_2^m, A_4^m, and functions, $\psi_0(t)$, $\psi_1(t)$, as in formula (25).

Using formula (27) and multiplying equation (23) by (-2), one can transform this equation to the following one easily.

$$\int_{-1}^{1} \left\{ -\frac{1}{\pi}\ln|u-v| + \mathrm{K}_m(u-v) \right\} \hat{Z}_m(v)\, dv = -2g_m(u), \quad u \in [-1,1], \quad m = 0, \pm 1, \pm 2, ... \tag{29}$$

with unknown function

$$\hat{Z}_m(v) = L Z_m(v) \quad , \tag{30}$$

which, of course, has the representation of the kind (24). It is noteworthy that kernel K_m (u-v) depends on the difference (u-v) only. This somewhat simplifies the numerical implementation of the algorithm described below.

Thus, integral equation (3) has been equivalently reduced to the infinite set of non-interconnected integral equation of the first kind of the type (29). Each kernel of these equations is the sum of special logarithmic singularity and correspondent smooth function. Any solution of this equation has to have the representation of the form (24).

Reduction of the canonical integral equation (29) to a linear algebraic equation of the first kind followed by a transformation to a linear algebraic equation of the second kind of the form

$$y_n + \sum_{s=0}^{\infty} \hat{k}_{ns} y_s = \hat{b}_n, \quad n = 0, 1, 2, ... \tag{31}$$

is done by means of the analytical regularization [5] given in [9] to obtain a functional equation of the form

$$\left(I + \hat{K}\right) y = \hat{b}, \quad y, \hat{b} \in \ell_2 \tag{32}$$

provided that

$$\sum_{n=0}^{\infty}\sum_{s=0}^{\infty} (1+n)(1+s)\left|\hat{k}_{ns}\right|^2 < \infty \tag{33}$$

and, consequently, with compact and Hilbert-Schmidt operator [10] $\hat{K}$. The reader can find the expressions connecting y_n, $\hat{k}_{ns}$ and $\hat{b}_n$ in (31) with $Z_m(v)$, $K_m(u-v)$ and $g_m(u)$ in our paper [9].

3.1. Analytical approximation for kL << ka << 1, m = 0, and normally incident plane wave

It is interesting to see to which end the analytical solution will be available. In this subsection, using the corresponding approximate expressions for the special case mentioned in the subsection title, we give the analytical expressions for the unknown Fourier-Chebyshev coefficients $\{y_n\}$ in (31), with accuracy $O((ka)^2)$ and $O((L/a)^2)$, where values of the kind $O((ka)^3)$ and $O((L/a)^3)$ are neglected:

$$y_0 = \frac{2\sqrt{\pi}}{\ln 2 + \pi\left(\frac{1}{\pi}\ln\frac{8a}{L} + ika\right)} + \quad O((ka)^2) + O((L/a)^2) \tag{34}$$

$$y_1 = i2kL\sqrt{\pi/2} + \quad O((ka)^2) + O((L/a)^2) \tag{35}$$

$$y_n = O\left((ka)^2\right) + O\left((L/a)^2\right), \qquad n \geq 2 \tag{36}$$

Scattering field $u^s(q)$ at an arbitrary point q can be found by means of the equation

$$u^s(q) = \int_{-\pi}^{\pi}\int_{-L}^{L} Z_D(z_p,\varphi_p)G(z_p,a,\varphi_p;z_q,r_q,\varphi_q)\,a\,d\varphi_p\,dz_p. \tag{37}$$

With the considered approximation, field on the surface S can be obtained as

$$u^s(q) = -1 - ikz_q + O((ka)^2) + O((L/a)^2), \qquad q = (z_q, a) \in S, \tag{38}$$

which is a negated normally incident ($\theta = 0$) plane wave with the same accuracy. Therefore, the Dirichlet boundary condition is satisfied in terms of taken accuracy:

$$u^s(q) + u^i(q) = O((ka)^2) + O((L/a)^2). \tag{39}$$

Consequently, solution with the approximate coefficients (34), (35) of the unknown function $Z_D(z_p, \varphi_p)$ in (37) leads to the solution of the Dirichlet diffraction problem with accuracy $O((ka)^2)+O((L/a)^2)$ uniformly in all space R^3 in the very special case considered.

4. Numerical Testing and Results

In this section, numerical results will be used in order to verify the algorithm and its efficiency and possibilities.

A few samples will be used for this purpose. However, we would like to state that the physical meaning is outside the scope of this paper it will be presented in our next publications.

4.1. Verification of the algorithm by the low frequency analytical approximation

Below is the comparison of approximately (by means of formulas (34), (35)) and exactly (i.e. by means of the suggested algorithm) calculated coefficients for various parameters given in the Table. When ka = 0.3, kL = 0.1 in the last sample, the relative error between analytical approximation (34), (35) and numerical data modulus does not exceed 5% , which is noteworthy. The numerically given modulus of the coefficient y_2 shows that it is negligible compared to both y_0 and y_1 .

Table. Comparison of approximated analytical solution of the linear algebraical system considered, with the algorithm proposed, i.e. exact one.

ka	kL	y_0	Approx.	y_0	Exact	y_1	Approx.	y_1	Exact	y_2 Exact.
		Modulus	Arg (rad)	Modulus	Arg (rad)	Modulus	Arg (rad)	Modulus	Arg (rad)	Modulus
0.01	0.001	0.6984	−0.04255	0.6987	−0.04247	0.02506	1.5708	0.02502	1.5712	7.9 x 10^{-5}
0.05	0.001	0.5301	−0.02349	0.5305	−0.02345	0.02506	1.5708	0.02506	1.5711	3.8 x 10^{-5}
0.1	0.001	0.48005	−0.04256	0.48136	−0.04246	0.02506	1.5708	0.02506	1.5713	9.7 x 10^{-6}
0.1	0.005	0.6136	−0.05441	0.6158	−0.05433	0.01253	1.5708	0.01253	1.5713	4.7 x 10^{-7}
0.1	0.01	0.6971	−0.06182	0.7001	−0.06178	0.02506	1.5708	0.02502	1.5713	7.1 x 10^{-4}
0.3	0.1	0.8897	−0.2388	0.9341	−0.2432	0.02506	1.5708	0.02458	1.5706	2.6 x 10^{-3}

Figures 2a, 2b and 2c show the resulting current density (modulus, phase) on the cylinder, near (total) field and far (scattered) field for the last case considered in the Table. The far field pattern is almost the some as one for a point source because the dimensions of the cylinder are very small compared to wavelength, and the singularity of the current density (Figure 3a) determines the scattered far field, which follows from (37).

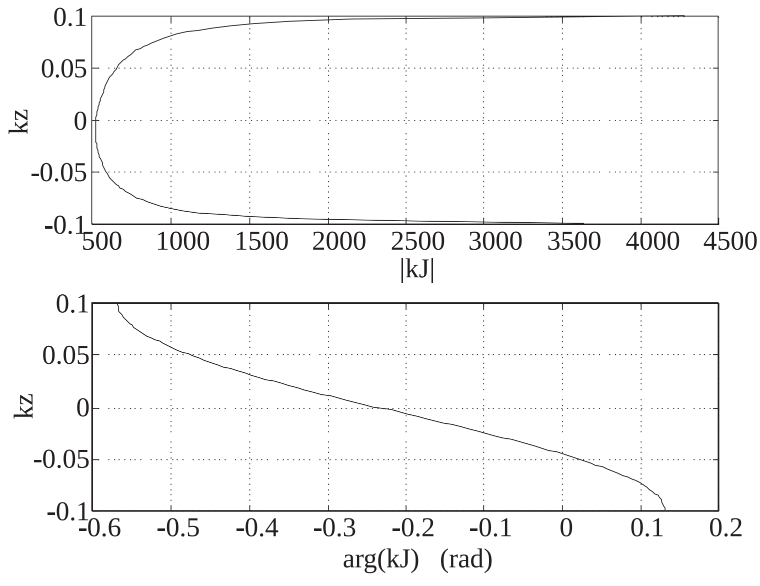

Figure 2a. Distribution of modulus and phase of current density on the cylinder surface. ka = 0.3, kL = 0.1.

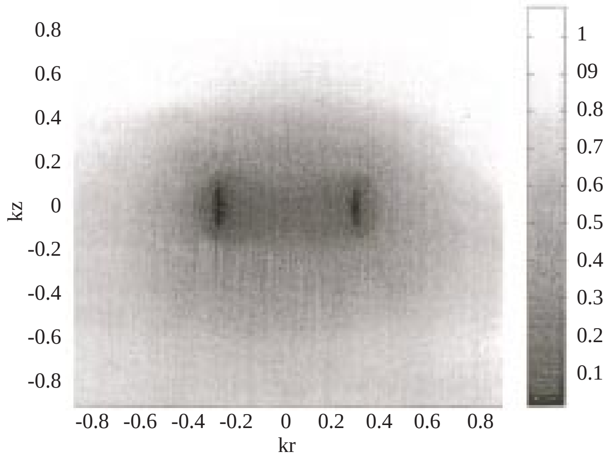

Figure 2b. Total near field. ka = 0.3, kL = 0.1. Plane wave propagates from up to down in the picture.

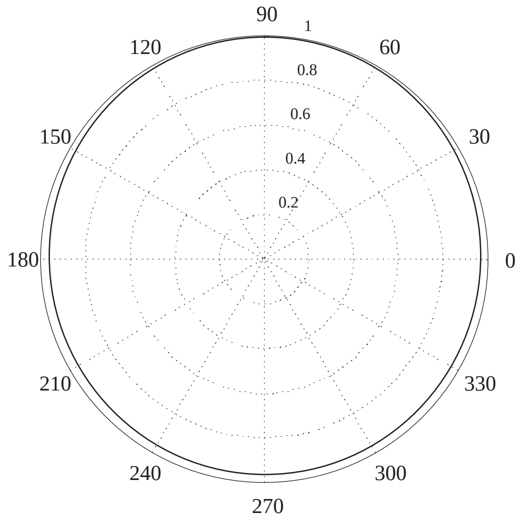

Figure 2c. Far field pattern for normally incident plane wave. ka = 0.3, kL = 0.1.

4.2. Stability of the solution, condition number

The corresponding condition number to the matrix operator in (32) of the linear algebraic system of the second kind given in (31) tends to a reasonably small, constant value. That is just the reason why the solution can be obtained in any pre-determined accuracy. In Figure 2 the condition number for the kernel matrix $\left(I+\hat{K}\right)$ of the linear algebraic equation system (32) is depicted. It is clear that condition number of truncated systems is uniformly bounded.

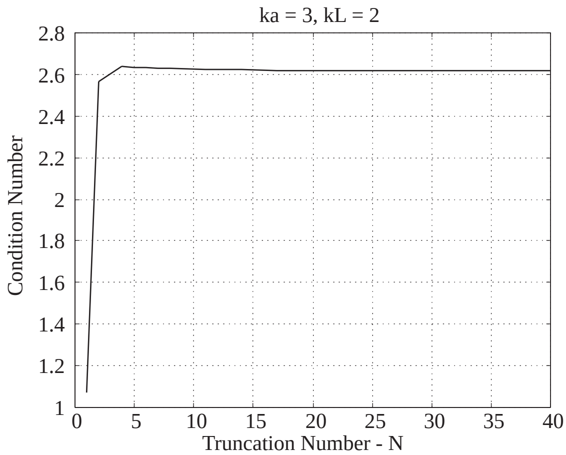

Figure 3. Typical condition number behaviour for an algebraic system of the second kind ka = 3, kL = 2.

Figures 4 a-b-c and 5 a-b-c show the numerical results for the cylinders of parameters kL = 1, ka = 1 and kL = 1, ka = 5, current density-near field-far field pattern respectively.

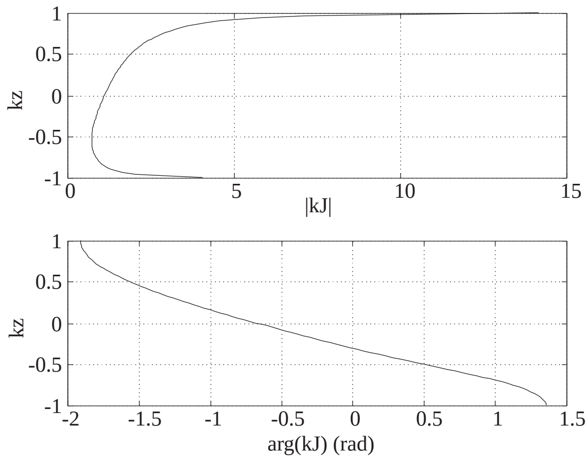

Figure 4a. Current density distributions on the cylinder surface: ka = 1, kL = 1.

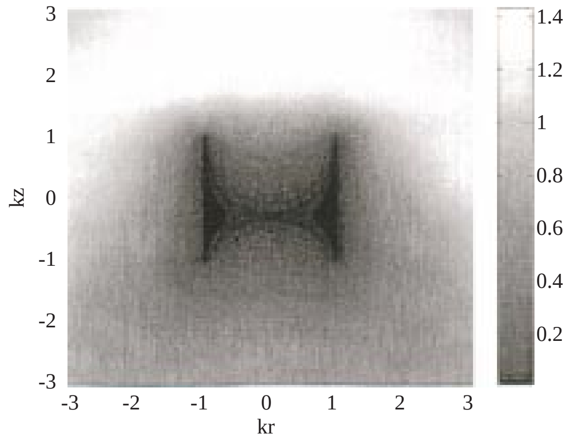

Figure 4b. Total near field. ka = 1, kL = 1. Normal plane wave incidence.

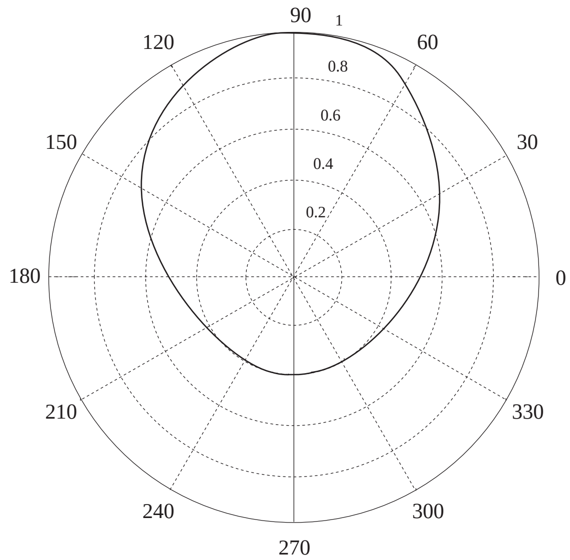

Figure 4c. Far field pattern for normally incident plane wave. ka = 1, kL = 1.

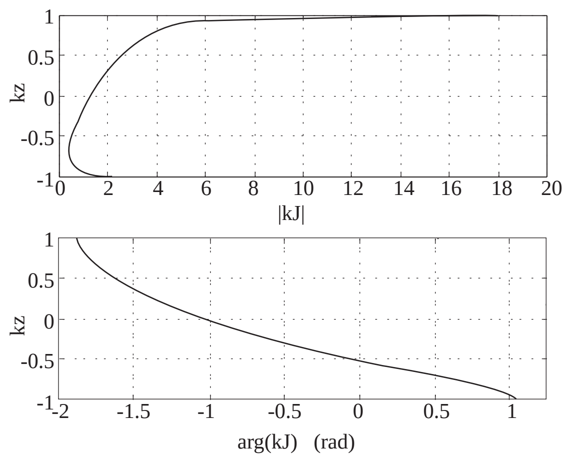

Figure 5a. Distribution of modulus and phase of current density on the cylinder surface. ka = 5, kL = 1.

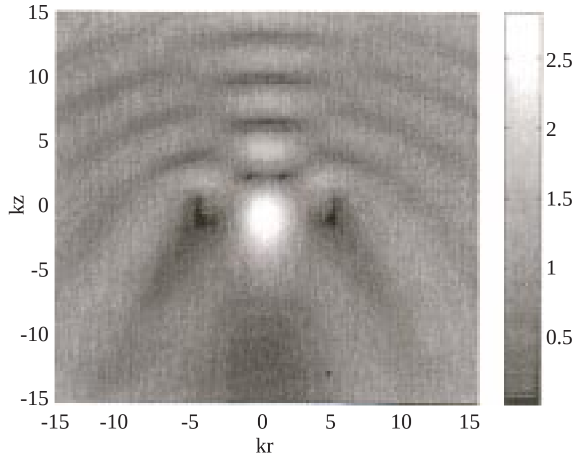

Figure 5b. Total near field. ka = 5, kL = 1. Plane wave propagates from up to down in the picture.

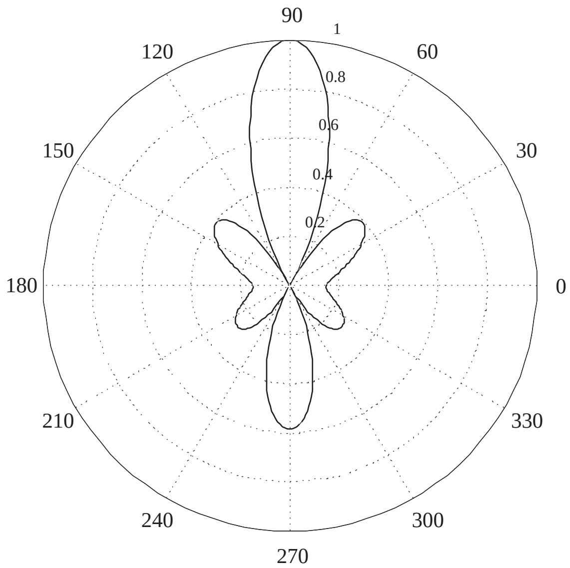

Figure 5c. Far field pattern for normally incident plane wave. ka = 5, kL = 1.

Figure 6 is an extreme case where length of the cylinder can be considered in some respects as infinity, i.e. like a wave-guide with open ends. In Figure 6b. the maximum value of the total field inside the cylinder can be recognized as the mode inside a circular wave-guide.

Current density distributions in all sample figures denoted by "a" explicitly demonstrate the edge singularity of the hollow cylinder. This singular property naturally follows from the qualitative properties of the integral equation for a scattering field written for each point in the entire space.

Sample figures denoted by "b" demonstrate the total near fields at a (ρ, z) cross section either for parts $\rho < a$, or for $\rho \geq a$. This gives clear understanding of the wave diffraction phenomena around the cylinder.

The normalized far field patterns in all sample figures denoted by "c" are distinctly varying, which opens possibilities for the optimization of such structures for different purposes: A high-directivity pattern is seen in Figure 6c., and a very wide pattern in Figure 4c., while different multi-lobe patterns are obtained in Figure 5c.

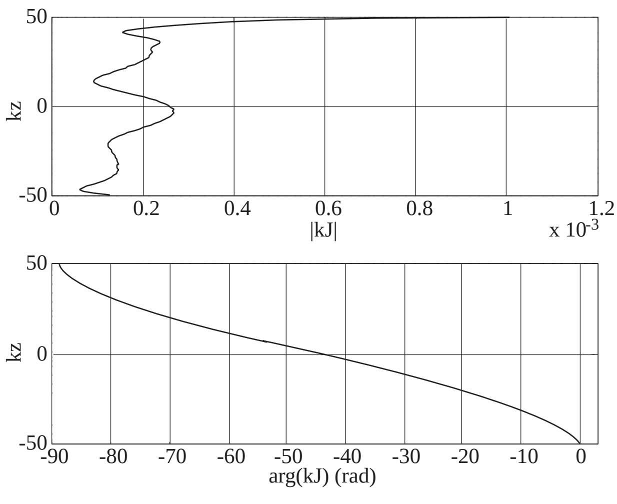

Figure 6a. Current density distributions on the long cylinder surface. ka = 5, kL = 50.

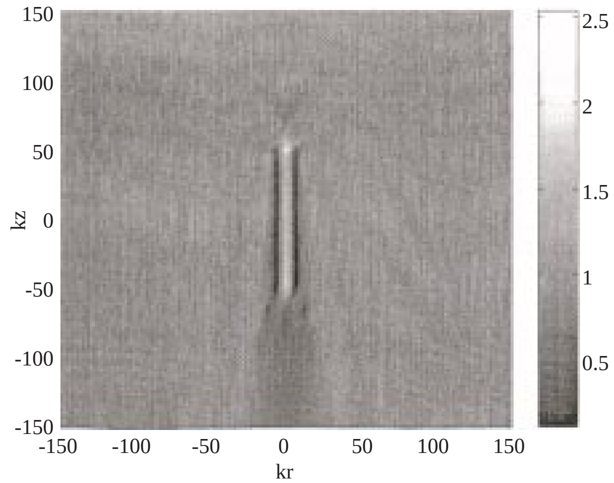

Figure 6b. Total near field of long cylinder. ka = 5, kL = 50. Plane wave propagates from up to down in the picture.

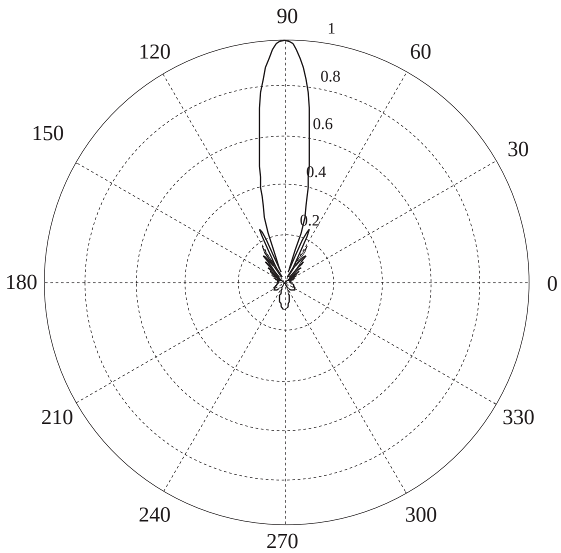

Figure 6c. Far field pattern for long cylinder. Normally incident plane wave. ka = 5, kL = 50.

5. Conclusion and Perspective

The Dirichlet boundary value problem of scalar wave diffraction by infinitely thin system of circular cylinder of finite length has been solved, i.e. it has been equivalently in a mathematical sense reduced to the infinite set of functional equation of the second kind in space ℓ_2 with compact in ℓ_2 operators. Every such equation, which is an infinite system of algebraic equations, has only one solution. This solution can be obtained by means of the truncation procedure for the algebraic system. Thus, the numerical solution is stable and can be obtained with any necessary accuracy. The infinite set of solutions of these functional equations produces the unique solution of the boundary value problem.

The model considered is for scalar waves scattering from perfectly soft screens, i.e. the Dirichlet problem. Scattering from perfectly hard screens, as rigid boundaries in acoustics, requires solving the Neumann problem. This can be done in a very similar manner using the combination of Orthogonal

Polynomials method with Analytical Regularization. Orthogonal polynomials used in the Neumann problem can be Chebyshev polynomials of the second kind, which have orthogonality weight fitting to the edge condition. This will be the subject of our future research.

For electromagnetic waves, extension of the method can be done using the representation of such waves by corresponding scalar potentials (Hertz, Debye, Lorentz). The solution for those scalar potentials can be obtained by means of a vector integral equation that is reducible to the scalar integral equations, just as the one rigorously considered in this paper or as one obtained for the Neumann problem as mentioned above.

The results obtained by means of the suggested method reveal a powerful tool for theoretical study and numerical solving of a rather wide class of diffraction problems. One is the extension of the geometry to an axially symmetrical system consisting of the same kind of cylinders as in this paper, with different dimensions. Such an extension may enable us, in our future research, to consider Fresnel zone antennas and lenses upon the proper focal location of the point source to obtain a high directivity radiation pattern. For this goal, the accuracy of the Kirchhoff approximation was used in [1-3] only.

References

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