

## A new approach to linear displacement measurements based on Hall effect sensors

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**Abstract:** Since displacement is a vital variable to be considered in many industrial applications, displacement sensing devices have been extensively studied both theoretically and experimentally. There have been also many studies on Hall effect-based displacement measurement, but for many systems linearity still remains a problem. This paper discusses different approaches to calculate the magnetic field due to a cylindrical permanent magnet and proposes a new set-up geometry with 2-Hall effect sensors and a permanent magnet between them to overcome the linearity problems. Furthermore, theoretical and experimental studies of the discussed displacement sensor were presented by focusing on the linear range and the sensitivity of the system. These parameters were investigated for different sensor-to-sensor distances. As this value decreases, it is found that sensitivity increases, and yet the linear range decreases as well. For the 25.5 mm sensor-to-sensor distance linear range was determined as around 3 mm, and the sensitivity and maximum error in displacement measurement was calculated as 351 mV/mm and 0.029 mm, respectively. We believe that by using the results of our study, it is possible to develop Hall sensor-based approaches that can be used for sensitive displacement measurements in a certain range and can meet the needs of researchers and industry in many applications

**Key words:** Displacement measurement, Hall effect, linearity, magnetic sensors

### 1. Introduction

Displacement sensors are widely used in many industrial applications including automotive and petrochemical industries, robotics, aviation, and aerospace engineering, monitoring systems and are crucially important for the work done [1–8]. For this reason, the design of displacement sensing devices based on different working principles (magnet combinations, geometries...), and the improvements of existing designs are intensively studied both theoretically and experimentally, and it continues to attract the attention of researchers [1, 5, 9–15]. In practice, there are many displacement sensors based on optical, capacitance-inductance-resistance measurement, photoelectric, ultrasonic, eddy current, and magnetic approaches [1, 3, 9, 16].

Each of these known methods has disadvantages in some ways. Of these, the potentiometric resistive sensors are thermally affected, have analog nature, and have short life span due to their wear and tear prone structure [17]. Capacitive sensors have limited ranges, are affected by environmental factors, require additional circuitry for efficient use [18, 19]. Optical sensors can be very expensive and sensitive measuring devices, they are not well suited for use in vibrating, dusty, humid, and harsh environments [18, 20, 21]. Ultrasonic sensors need to compensate the studied data, their range are limited with the wave frequency. Moreover, the output signal is dependent on the refractive index of medium and thermally affected [17].

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In particular, Hall effect-based magnetic displacement sensors are highly preferred due to considerable advantages including being simple, contactless, small in size, robust, low-cost, high repeatable, reliable, not sensitive to harsh, humid, and polluted environmental conditions. They also require low power (0.1–0.2 W), possess long life span, are versatile, and can be used in a wide variety of fixed or variable field measurements [3, 16, 22, 23]. Due to these advantages, they are preferred by researchers in many applications.

By using a different number of Hall effect sensors and magnetic materials with different configurations and geometries, it is possible to design and develop many sensor applications such as radial and displacement sensors [2, 12, 13, 23–26]. The basic working principle for most of these applications is described by measuring the magnetic field formed on the Hall sensor due to the change in the motion or state of the magnetic material. In Wenying et al., an axial inductive displacement sensor was designed with 2.63 V/mm sensitivity and 0.46% linearity [15]. Takuya Yano et al. proposed a sensor for position control using Hall sensor and a magnetic sheet [12]. Nicolas Dupre et al. developed a displacement sensor design including Hall elements and obtained 1% error over 15 mm linear range [11]. A variable reluctance-Hall combined approach was used by Sandra et al. to measure displacement [6, 18]. 2 Hall sensors, a permanent magnet, an ‘E’ core, and soft ferromagnetic blocks were used for this design. Sankalp and Sujana fabricated 2 different pressure sensor design; 1 Hall-1Magnet and 2 Hall-2 magnet combinations with 0.243 mV/mbar and 0.03mV/mbar sensitivities [13]. In 2018, Addabbo et al. designed a structural crack monitoring system by using only one Hall and one permanent magnet [4].

There are some limitations in magnetic displacement sensors: accuracy by offset, noise, temperature dependence, and aging. Furthermore, it is known that the measurements are getting nonlinear as you move away from the magnetic source and change proportionally with  $1/r^3$  for a distance  $r$  [3]. For all these reasons, they are not the first devices that come to mind in linear sensing applications [3]. Many studies are carried out to overcome these problems completely or to minimize the undesired effects [1, 10, 24–26]. Blache and Lemarquand proposed magnetic systems by placing different numbers of magnets in different combinations, to construct linear input-output relations and more sensitive devices at certain intervals ( $< 1mm$ ) [27]. By Frazier et al., 0.01–0.02 T/m and 0.75 T/mm sensitivity values were reported for a single magnet and two oppositely oriented magnets respectively [16]. Anoop and Bobby developed a linearizing converter that can be used for any nonlinear system [26]. However, it is very difficult to design a sensor that is both sensitive and linear over a wide range [16]. The linear range-sensitivity-error relation is another challenging in this kind of applications.

With system configurations where a sensor is placed between two radially different magnets or magnets with opposite poles, close to linear results can be obtained in a certain range [22, 28]. Although there are a limited number of studies showing that close to linear results can be obtained by using one or two axes Hall sensors, only measurement results or simulations are shared in these studies, and in few of them theoretical analysis or supplementary work has been made [3, 6]. Sreekantan and George performed radial angle measurement by referring only to the linear feature of the sensors with the polynomial expansion [29]. Öztürk and Yarıçı performed the tilt measurement by the serial expansion of the sum and difference magnetic fields as an exponential function [30]. However, as far as we know, there are not many studies that present theoretical and experimental results together. In addition, there are a few studies that examine the linear region deeply and indicate the relationship between linear region-sensitivity-error sizes. The existing studies mostly have special complicated designs and/or need fabrication processes. Different from the other given Hall-based displacement sensor systems, the proposed system uses easy-to-find and commercially available cheap elements to develop a linear, sensitive device.

In this paper, a new magnetic displacement measurement set-up is developed and the linearity of this system is studied both theoretically and experimentally. To compensate linearity problem, firstly, the use of a single Hall sensor was examined in terms of known functions, then a new approach to linear displacement measurement based on 2-Hall effect sensors was proposed. The proposed system including 2 symmetrical Hall sensors and a permanent magnet was presented theoretically and experimentally. Over 25.5 mm sensor-to-sensor distance for the obtained linear range (about 3 mm) the sensitivity and error was achieved as 351 mV/mm and 0.029 mm, respectively. The maximum linear range measured/calculated as 5.82 mm for 40.5 mm sensor-to-sensor distance. The results show that, as the Hall-to-Hall distance increases, the displacement measurement sensitivity decreases, conversely, the linear range and the maximum error at that range increase. This study will contribute to the understanding and enhancement of the linear region in Hall effect-based displacement sensors.

## 2. Magnetic field of the magnet on the axes

It is important to characterize the magnetic field vectors of a magnet to calculate or predict the Hall Effect sensors-magnet position. To be able to evaluate the proposed setup with a uniformly magnetized solid cylindrical magnet placed between two Hall sensors facing each other, it is needed to calculate the magnetic field of the magnet on its axis  $z$ . The experimental set-up and schematic of designed system is shown in Figure 1. The Lorentzian approach is one of the most common methods to calculate the magnetic field of uniformly magnetized magnets. The magnetic field of a solid cylindrical magnet (as shown in Figure 1) with a length of  $2l$  and radius of  $r$  can be calculated on its axis  $z$  by Equation 1.

$$B_z = \frac{\mu_0 M}{2} \left( \frac{z+l}{\sqrt{r^2 + ((r+l)^2)}} - \frac{z-l}{\sqrt{r^2 + (r-l)^2}} \right) \tag{1}$$

where  $\mu_0$  is the permeability of free space and  $M$  is the magnetization ( $\vec{M} = M\hat{z}$ ) of the magnet at  $z=0$  in Figure 1.

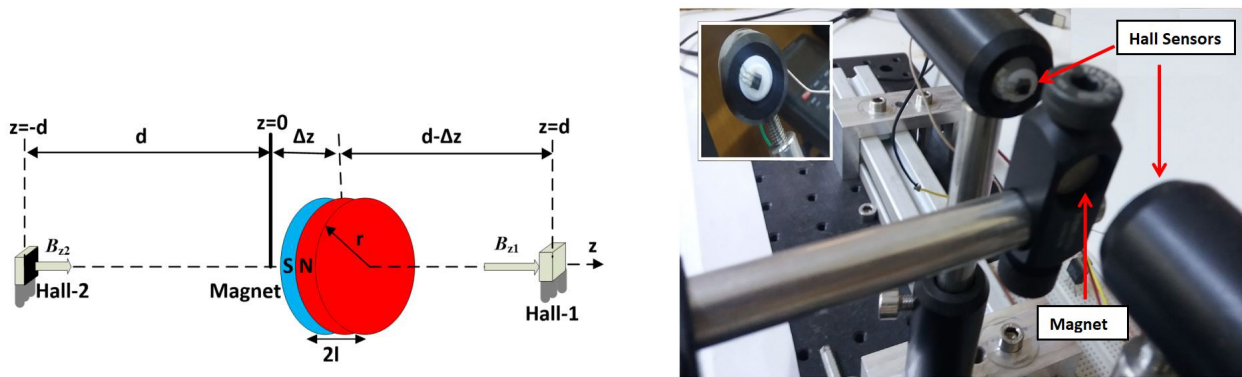


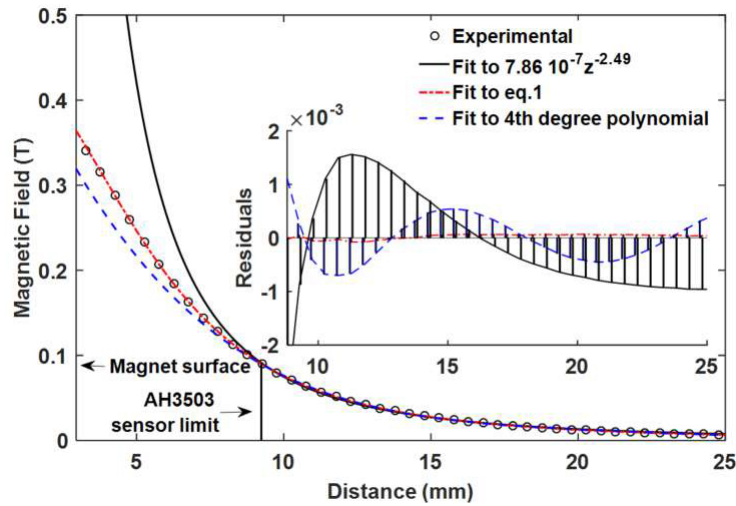
Figure 1. a-Schematic of Hall sensors and magnet. b-Experiment set-up.

Approximate solutions of Equation 1 were obtained or used in several studies. In these studies, the magnetic field is used as it is proportional to the power law of distance,  $B = B_0 z^{-n}$  when  $r \ll z$  [31, 32]. In addition, in our previous study, the Taylor series expansion of power law equation of magnetic field was used to model a tilt sensor [30]. Before starting two-Hall-sensor-based set up 3 different approaches (Equation 1, 4<sup>th</sup>

degree polynomial, and power-law) were fitted to experimental data to show the compatibility of these functions with experimental data and efficiency of using only one sensor.

Experimental data were first taken by a commercial Lakeshore 455-DSP model gaussmeter with an HMMA-2502-VF model Hall probe and the magnet with a length of 5.9 mm and a radius of 6 mm. The distance between the sensor and the magnet was tuned with Thorlabs PT1 linear translation stage. The functions were fitted to the obtained experimental data in the range of the Hall sensor linear limit which is less than  $\pm 0.1$  T. This limit is shown in Figure 2 and was obtained when the sensor was 8.8 mm away from the center of the magnet. Therefore, Hall sensors are always placed further than this distance so that the sensors do not saturate and operate in a linear regime.

Best fitting to the experimental results is obtained by Equation 1. By fitting Equation 1 to experimental data, the saturation magnetization of the magnet is determined to be around  $826.1 \pm 0.4$  kA/m. The obtained saturation magnetization value is used in following calculations. It is seen that the residual values (difference between experimental data and fitted function) are less than 2 mT for all functions, less than 0.7 mT for the 4<sup>th</sup> degree polynomial and  $71 \mu\text{T}$  for Equation 1. These results show that the most suitable solutions to determine the position using a single Hall sensor are Equations 1 and 4<sup>th</sup> degree polynomial. 6.2 mT residual value is observed in the second order polynomial.



**Figure 2.** Magnetic field of the magnet with respect to distance along its symmetry axis ( $z$  axis). It is focused on the measurement region and for that region the results agreed well with the fitting.

### 3. Experimental and theoretical studies

In this study, symmetrically placed two Hall sensors and a cylindrical magnet are planned to be used for the displacement measurement as shown in Figure 1. The Hall effect sensor provides a voltage output proportional to the applied magnetic field which can be either positive or negative. The used sensors have a positive null output voltage at zero magnetic field. This value increases when a positive magnetic field is sensed and decreases under a negative magnetic field. The Hall sensors were operated by a DC supply voltage output of a 7805 voltage regulator connected to a 9 V battery. The output of 7805 and so input of Hall sensor was measured to be around 5.01 V. Each sensor output is proportional to both the applied magnetic field on their sensing surface and their sensitivity in their linear range.

The sensor output voltages were measured with Protek 506 model multimeters and each has a null voltage output of around half of the supply DC voltage (around 2.5 V) when the system is in balance. The linear ranges of the Hall sensors are around  $\pm 90$  mT. Each Hall sensor was calibrated with a commercial Lakeshore 455-DSP model gaussmeter with HGT-1020 Hall probe and their sensitivity ratio was determined as around 1.06. The magnet and sensors were placed as shown in Figure 1 and the measurements were taken with 0.25 mm steps by translating the magnet by using Thorlabs PT1/M model manual translation stage. The magnetic field value increases on the Hall sensors when the mobile magnet is put close up to any one of them and vice versa. As a result of this magnetic field change, the magnet position and/or displacement can be determined by using the proposed system.

Equation 1 can be used to calculate magnetic field intensities on two symmetrically placed sensors by assuming the field is uniform on the detecting Hall sensor surfaces. The magnetic fields on each Hall sensor are named as  $B_{z1}$  and  $B_{z2}$ , and Taylor expansions of these magnetic fields around  $\Delta z=0$  (Maclaurin series) are expressed as follows (Equations 2 and 3).

$$B_{z1}(z) = B_z(d - \Delta z) = \frac{\mu_0 M}{2} \left( \frac{d - \Delta z + l}{\sqrt{r^2 + ((d - \Delta z + l)^2)}} - \frac{d - \Delta z - l}{\sqrt{r^2 + (d - \Delta z - l)^2}} \right) \quad (2)$$

$$= a_0 + a_1 \Delta z + a_2 \Delta z^2 + a_3 \Delta z^3 + a_4 \Delta z^4 + \dots$$

$$B_{z2}(z) = B_z(-d - \Delta z) = B_z(d + \Delta z) = \frac{\mu_0 M}{2} \left( \frac{-d - \Delta z + l}{\sqrt{r^2 + ((-d - \Delta z + l)^2)}} - \frac{-d - \Delta z - l}{\sqrt{r^2 + (-d - \Delta z - l)^2}} \right) \quad (3)$$

$$= a_0 + a_1 \Delta z + a_2 \Delta z^2 + a_3 \Delta z^3 + a_4 \Delta z^4 + \dots$$

In these equations (Equations 2 and 3), Taylor series coefficients ( $a_n$ ) depend on the magnetic and geometric properties of the system. The voltages of each ideal and identical linear Hall sensors can be calculated as shown in Equations 4 and 5.

$$V_1(\Delta z) = 2.5 + k(B_{z1} + B_{SF}) \quad (4)$$

$$V_2(\Delta z) = 2.5 - k(B_{z2} + B_{SF}) \quad (5)$$

where 2.5 V is null voltage and  $k$  is Hall sensor sensitivity. The  $B_{SF}$  field is an external field perpendicular to the Hall sensor surface where it is constant like the earth's magnetic field. By using Equations 4 and 5, the sum and difference of the Hall sensor output voltages can be obtained as shown in Equations 6 and 7.

$$\sum V = V_1 + V_2 = 5 + k(B_{z1} - B_{z2}) = 5 + k[2a_1 \Delta z + 2a_3 \Delta z^3 + \dots] \quad (6)$$

$$\Delta V = V_1 - V_2 = k(B_{z1} + B_{z2} + 2B_{SF}) = k[2a_0 + 2a_2 \Delta z^2 + 2a_4 \Delta z^4 + \dots] + 2k B_{SF} \quad (7)$$

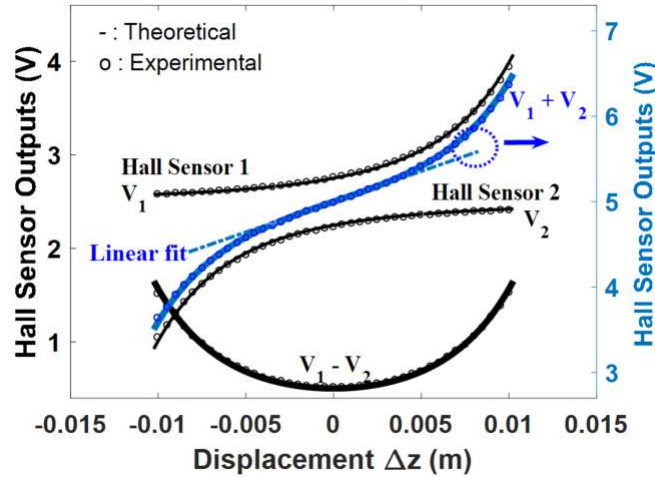
As can be seen from Equation 6, when uniform and constant fields are neglected, it is possible to obtain a linear relationship between the magnetic fields and the measured voltages. The same linear relation can be obtained between displacement and output voltage by ignoring the 3rd and higher-order terms of the sum of

the sensor outputs ( $\sum V$ ). However, the difference of the sensor outputs ( $\Delta V$ ) will be a function of even orders of  $z$  and it can be reduced to quadratic form for small  $\Delta z$  values as shown in Equation 7. The difference  $\Delta V$  is also affected by the external  $B_{SF}$  field. As an important remark, these calculations can be used in any symmetric nonlinear sensor systems to obtain linear output in certain ranges.

In Figure 3, the obtained voltage values versus displacement of each sensor around  $\Delta z=0$  are shown both separately and in the form of their sum and differences.  $Bz1$  and  $Bz2$  values were determined using Equation 1,  $V1$  and  $V2$  voltages were calculated by substituting these values in Equations 4 and 5. In calculations, the determined parameters ( $M_o = 826.1kA/m$ ,  $l=5.9$  mm,  $r=6.0$  mm, the distance between two Hall sensors= $40$  mm) were used. Both theoretical results (Equations 6 and 7) and the obtained experimental data (shown in Figure 3) indicate that in certain regions, the sum of the two sensor outputs behaves linearly, while the difference follows a quadratic regime. Using two symmetrical sensors compared to one also ensures higher sensitivity. The sensitivity of the system for the ( $\sum V$ ) can be calculated by using Taylor series expansion at  $\Delta z = 0$  as shown in Equations 8 and 9, where the system is assumed to be linear.

$$Linear\ sensitivity = 2ka_1 = 2k \frac{\mu_0 M}{2} \left| \frac{\partial \left( \frac{d-\Delta z+l}{\sqrt{r^2+(d-\Delta z+l)^2}} - \frac{d-\Delta z-l}{\sqrt{r^2+(d-\Delta z-l)^2}} \right)}{\partial \Delta z} \right|_{\Delta z=0} \quad (8)$$

$$Linear\ sensitivity = 2ka_1 = k\mu_0 M r^2 \left( \frac{1}{(r^2 + (d-l)^2)^{3/2}} - \frac{1}{(r^2 + (d+l)^2)^{3/2}} \right) \quad (9)$$



**Figure 3.** Left axis: calculated Hall sensor voltage outputs ( $V1$ ,  $V2$ ), their difference ( $V1-V2$ ), and right axis: sum ( $V1+V2$ ) linear fit line with respect to magnet displacement from equilibrium ( $\Delta z$ ).

The sensitivity is linearly dependent on magnet magnetization ( $M$ ) and sensor sensitivity ( $k$ ) as expected. And also sensitivity is a nonlinear function of the some other parameters such as magnet radius ( $r$ ), equally important magnet length ( $l$ ), and the sensor-to-magnet distance ( $d$ ) as shown in Equations 8 and 9. In this study, only the effects of the  $d$  parameter on the linear sensitivity and linear range of the system were investigated.

4. Linearity studies and discussion

This part of the article focuses on the linear region of the experimentally obtained results, which are also theoretically confirmed. Experiments were conducted using the system shown in Figure 2 and measurements were taken for different Hall to Hall distance (2d) values. Firstly, the measurements were taken for 2d=40.50 mm. The experimental and theoretical values of the sensor output for each sensor, their sum ( $\sum V$ ), and their difference  $\Delta V$  are presented in Figure 3. Theoretical data were calculated by using Equation 1. The output of each sensor has nonlinear characteristics with respect to  $\Delta z$  and each sensor sensitivity increases while the magnet gets closer and decreases as the magnet moves further. The difference  $\Delta V$  fits a nearly quadratic function of  $\Delta z$  and has relatively low sensitivity (the tangent of the curve) around  $\Delta z = 0$  compared to ( $\sum V$ ). In the same region, however, the sum of sensor outputs ( $\sum V$ ) has an approximately linear relation with the  $\Delta z$ . Moreover, in accordance with Equation 8 by using 2 Hall sensors, about 2 times more sensitive systems than a single Hall sensor systems can be achieved. An increase of displacement  $\Delta z$  results in an increase in third and higher terms as given in Equation 6 and  $\sum V$  deviates from the linear curve.

For the fixed sensor-to-sensor distance of 40.5 mm, the obtained theoretical results (by using Equation 1), the experimental results, and the obtained linear fit for ( $\sum V$ ) are given in Figure 4. The region where the linearity error is smaller than 10 mV is determined as linear range. The linear range (in terms of  $\Delta z$ ) is determined as  $2.81 \cdot 2 = 5.62$  mm and  $2 \cdot 2.85 = 5.70$  mm, theoretically and experimentally, respectively. The tangent of the linear fit, linear sensitivity, is calculated to be around 70.41 V/m. Linearity error is determined as the difference between ( $\sum V$ ) experimental and linear fit, the results are presented in Figure 4. In this region due to the higher-order terms, there is a small error for  $|\Delta z| < 1$  and around midpoint ( $\Delta z = 0$ ). The first maximum linearity error in the figure is named El1, that is used to investigate the position error around the midpoint. The El1 is relatively small and around  $86 \mu V$  for sensor spacing 40.5 mm. This could theoretically cause a positional error of  $1.2 \mu m$  that is a negligible change in 0.45 mm. Since this error value is out of multimeter ranges used in the experiments, for this range experimental error cannot be calculated.

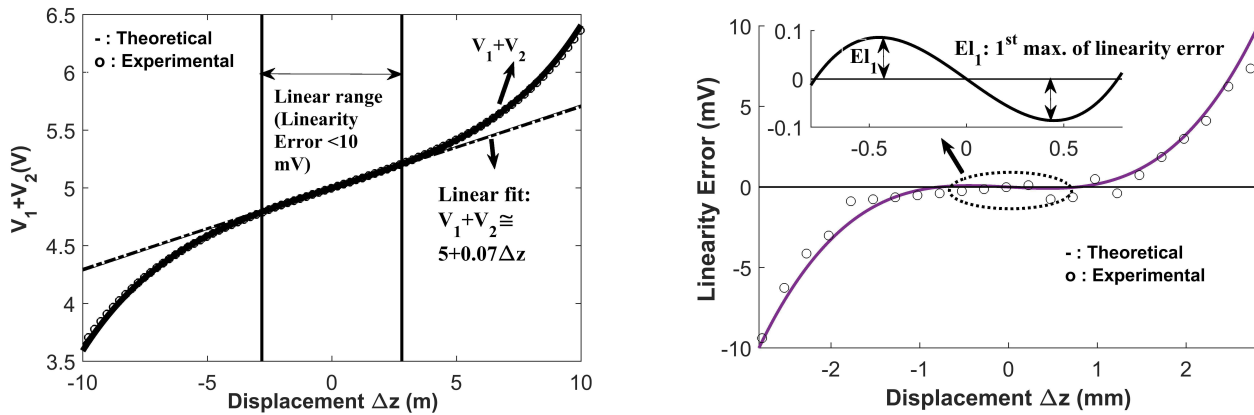
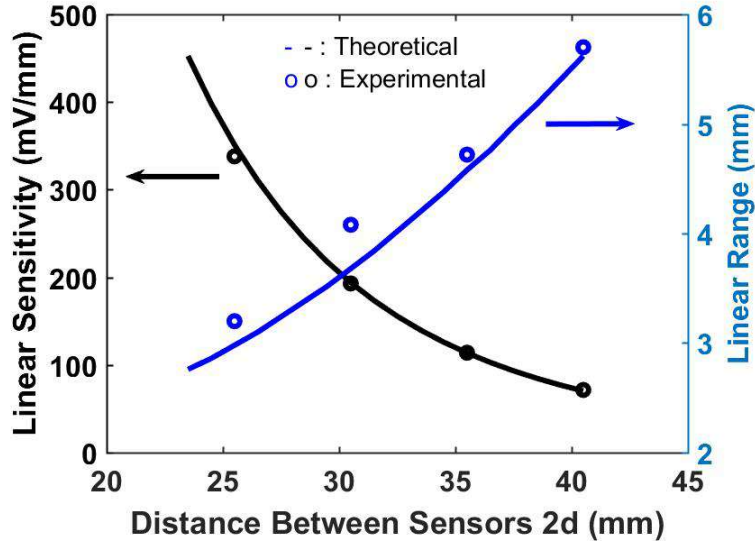


Figure 4. a-Experimental results of V1+V2 with linear fit. b-Linearity error versus displacement  $\Delta z$ .

Linear sensitivity and linear range depending on magnet spacing are given in Figure 5. It is seen that the obtained experimental data and calculated results by Equation 1 are compatible with each other. When the distance between the magnets and the sensor increases, the magnetic field change due to the displacement of the magnet decreases, and this results in a decrease in its sensitivity. Conversely, the linear range increases

under the same conditions. As discussed earlier, since two Hall sensors are used in this study, linear sensitivity increases while linear range decreases. Inevitably, a local max, local min, and optimum point cannot be found in the system (as the distance of the sensors) and it follows that the factor limiting sensor spacing may be accepted as the sensor saturation value. Correspondingly, the limiting factor of the sensor range will also be the sensor saturation value, and the sensitivity will increase as the measured magnetic field approaches this value. However, near the saturation value, sensor linearity and measurement accuracy will be deteriorated. Hall sensor pairs with high saturation value and high magnetic field sensitivity should be used for better system sensitivity.



**Figure 5.** Linear sensitivity and linear range with respect to distance between Hall sensors.

Both theoretically and experimentally obtained sensitivity, linear range, and El1 (1st max of linearity error) results of the designed sensor system for different  $2d$  values are listed in Table 1. Although the obtained results show that the calculated values are in agreement with the experimental data, this agreement deteriorates with the increasing sensor spacing and the obtained results diverge from each other. This is valid for both precision and linear range. This difference may exist due to inconsistency between the sensor sensitivity values used in system calibration and calculations. The discussed inconsistency may be explained by the nonuniformity of the magnets, possible errors in the alignment of the sensors, using only the magnetic field value on the magnet axis in the calculations, and/or not existing completely homogeneous magnetic field in the sensor surface area.

**Table 1.** The theoretical and experimental results of sensor system for different sensor spacings (T: Theoretical, E: Experimental).

Distance between sensors [mm]	Sensitivity [mV/mm]		Linear range [mm]		1st max of linearity error [mV] (at $\Delta z = \pm 0.45$ mm)	Theoretical max. error at linear range limit [mm]
	T	E	T	E		
40.5	70.41	71.5	5.62	5.7	$\pm 0.086$	0.14
35.5	113.77	114	4.58	4.72	$\pm 0.170$	0.088
30.5	194.00	193	3.68	4.08	$\pm 0.353$	0.052
25.5	351.13	338	2.98	3.20	$\pm 0.755$	0.029



Theoretically (also experimentally shown in Figure 4), it has been observed that there is a local maximum value of linearity error at 0.45 mm from the equilibrium position of the magnet. The calculations made at this point indicate that the linearity error increases as the distance between sensors decrease. Negligible errors were obtained in the calculations as 0.086 mV (0.0012 mm), 0.170 mV (0.0015 mm), 0.353 mV (0.0018 mm), and 0.755 mV (0.0022 mm), for 40.5 mm, 35.5 mm, 30.5 mm, and 25.5 mm sensor spacing, respectively. However, these errors can be a limiting factor for very sensitive measurements based on a linear approach where the sensor spacing is too small.

Another limiting factor of single sensor-based systems is that the sensor reads 10 times more than the earth's magnetic field at very large sensor spacing values. In the proposed system, since the broad surfaces of the sensors are facing each other, the homogeneous external magnetic field creates opposite values in these sensors and  $\sum V$  is independent of the external magnetic field. This also shows another advantage of the proposed system over the systems using two magnets and one sensor [22, 28]. This study and the existing different types of displacement sensors available in the literature are given in Table 2 for comparison. Sensitivity of our system is close to the recent study of Wenyin Li [15], but it is worth to mention that it possible to increase sensitivity by using a simple amplifier circuit or more sensitive Hall sensors. The proposed design has a comparable sensitivity, linear range, and close nonlinearity error among the presented studies. There are many displacement sensors which use designed magnet [15, 33], with commercially available [34] or designed Hall sensors [24]. The proposed system has the advantage of easy-to-find and commercially available magnet and Hall sensor configuration in comparison to the other studies presented in Table 2.

**Table 2.** Performance comparison of the proposed sensor and different types of displacement sensors.

	Type	Commercially available	Range (mm)	Sensitivity (mV/mm)	Resolution ( $\mu m$ )	Nonlinearity %
[15]	Inductive	No/Magnet and coil design	$\sim 0.4$	263	4.5	0.46
[33]	Variable inductance	No/Coil and core design	70	NA	2	0.40
[24]	CMOS double Hall device	No/Hall sensor design	3	1000	NA	NA
[27]	Single Hall	No/Magnet design	0.5	1400	NA	NA
[16]	Single Hall	No/Magnetic material design	11	630	NA	0.4
[34]	Double Hall	Yes	5	250	NA	NA
[11]	Four Hall	No/Chip design	15	NA	NA	NA
This study	Double Hall	Yes	2.98	351	NA	0.97
			5.62	70		2.49

## 5. Conclusion

In this article, a system consisting of 2 Hall effect magnetic field sensors and a magnet between them was designed with the advantages of using commercially available and easy-to-find elements, high sensitivity and low nonlinearity error in the defined linear range. The Lorentzian approach, Taylor series expansion of power law equation of magnetic field and 4<sup>th</sup> degree polynomial approaches were examined to characterize the magnetic field vector, and the Lorentzian method (Equation 1) is determined as the best approach. By using Equation 1, the voltages were obtained from each Hall sensor placed at ends, and then the sum and difference of obtained

voltages were examined both experimentally and theoretically. It has been observed that the sum of the voltages eliminates the external factors and the system becomes linear in a certain region. This region depends on the Hall-to-Hall distance and was found as 5.62 mm and 2.98 mm for 40.5 mm and 25.5 mm, respectively. Furthermore, it was observed that by this new approach both the system sensitivity is increased and the margin of error is decreased. The best sensitivity is measured as about 350 mV/mm with a negligible error 0.029 mm. The proposed system can be used as a linear magnetic displacement sensor in a certain range and the theory used can contribute to other existing studies in the literature.

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