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# Exclusive Disjunctions in Indefinite and Maybe Information in Relational Databases\*

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## Abstract

*Incorporating indefinite information into databases has been studied extensively. In this paper, we propose a structure called an E-table to represent maybe information and inclusive and exclusive disjunctions. We define the type of redundancies in E-tables and show how to eliminate them. Also we present an extended relational algebra to operate on E-tables. In this paper we expand the concepts and operations defined by Liu and Sunderraman [1] in order to accommodate exclusive disjunctions in relational databases.*

## 1. Introduction

Although the relational model [2] did not originally allow incomplete information, the real world modeling requirements stimulated a considerable amount of research on extending the relational model to include incomplete information. Various kinds of incomplete information have been considered [3-6]. There has been a thorough treatment of indefinite and maybe kinds of incomplete information in relational databases by Liu and Sunderraman [1].

A data structure called an I-table is defined [1]. An I-table has three components: definite, indefinite and maybe. For example the I-table in Figure 1 represents the following facts:

TEACHES			
Teacher	Course		
John	CS 101	}	definite part
Mac	CS 102		
Mac	CS 104	}	indefinite part
Robin	CS 500		
			maybe part
a)			
TEACHES			
Teacher	Course		
John	CS 101	}	definite part
Mac	CS 102		
Mac	CS 104	}	inclusive part
Lisa	CS 200		
Lisa	CS 400	}	exclusive part
Robin	CS 500		
			maybe part
b)			

Figure 1.a) An I-table, b) An E-table.

- (1) John teaches CS 101.
- (2) Mac teaches CS 102 or CS 104.
- (3) Robin maybe teaches CS 500.

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The indefinite part of an I-table consists of a set, S, of set of tuples (i.e.  $S = \{w_1, w_2, \dots, w_j, \dots, w_k\}$ , where  $w_i = \{t_1, t_2, \dots, t_i, \dots, t_j\}$  and where  $t_i$  is a tuple). Each set  $w_i$  can be viewed as a statement  $R(t_1)$  or  $R(t_2) \dots$  or  $R(t_j)$ . In Figure 1. a), the I-table contains in its indefinite part just one set with two tuples in it.

In this paper we extend the I-table so that it includes exclusive disjunctions as well as inclusive disjunctions. In other words, the indefinite part of a table is divided into two parts: inclusive and exclusive. We call such an extended I-table an E-table and explain it formally in the next section. The exclusive part of an E-table is also a set of tuples. However here a set  $w_v = \{t_1, t_2, \dots, t_z\}$  is viewed as a statement  $R(t_1)$  xor  $R(t_2) \dots$  xor  $R(t_z)$ . In Figure 1.b), in the E-table we represent not only all the information in the I-table in Figure 1.a) but also the information that Lisa teaches either CS 200 or CS 400 but not both. This example may suggest exclusive disjunctions are needed for representing an important class of incomplete information and are often present in real life applications.

In the next section, we redefine the relational algebra operations for the E-table and also define an operation called ELIMRID to reduce the redundancies in an E-table.

## 2. E-tables, Redundancies and Relational Algebra Operations

The E-table is an extension of the table representing a relation in the relational model. The table allows the existence of disjunctive facts as proposed by the definition of I-tables [1]. The difference between an I-table and an E-table is that the latter is capable of representing not only inclusive disjunctions, but exclusive disjunctions as well.

An E-table scheme is an ordered list of attribute names  $R = \langle A_1, \dots, A_n \rangle$ . Associated with each attribute name,  $A_i$ , is a domain  $D_i$ . Then,  $T = \langle T_D, T_I, T_E, T_M \rangle$  is an E-table over the scheme R where  $T_D \subseteq D_1 X \dots X D_n$

$$T_I \subseteq 2^{D_1 X \dots X D_n} - \{\{\theta\} \cup \{t\} | t \in D_1 X \dots X D_n\}$$

$$T_E \subseteq 2^{D_1 X \dots X D_n} - \{\{\theta\} \cup \{t\} | t \in D_1 X \dots X D_n\}$$

$$T_M \subseteq 2^{D_1 X \dots X D_n} - \{\theta\}$$

$T_D$  is the set of tuples named definite tuples.  $T_I$  is the set of sets of tuples called inclusive indefinite tuple sets.  $T_E$  is the set of sets of tuples called exclusive indefinite sets.  $T_M$  is the set of sets of tuples called maybe tuple sets. The symbols T,  $T_1, \dots$  are used for E-tables,  $t, t_1, \dots$ , for tuples,  $w_1, w_2, \dots, q_1, q_2, \dots$  for tuple sets,  $r, r_1, \dots$  for relations. We use 'or' for the logical or operator and 'xor' for the logical exclusive or operator.

Suppose that we have an E-table for storing information about a predicate P. The way to interpret the content of an E-table can be explained as follows:

- i)  $t \in T_D$  is interpreted as  $P(t)$ .
- ii)  $w \in T_I$  and  $w = \{t_1, \dots, t_n\}$  is interpreted as  $P(t_1)$  or  $\dots$  or  $P(t_n)$
- iii)  $w \in T_E$  and  $w = \{t_1, \dots, t_n\}$  is interpreted as  $P(t_1)$  xor  $\dots$  xor  $P(t_n)$
- iv)  $w \in T_M$  and  $w = \{t\}$  is interpreted as  $P(t)$  xor  $\neg P(t)$

$w \in T_M$  and  $w = \{t_1, \dots, t_n\}$  is interpreted as  $(P(t_1) \text{ xor } \dots \text{ xor } P(t_n)) \text{ xor } (\neg P(t_1) \text{ and } \dots \text{ and } \neg P(t_n))$

In Lipski's [7] words,  $T_M$  provides us "the set of objects for which we cannot rule out the possibility that they belong to an external interpretation of a query". The need to store the elements of  $T_M$  arises from i) the user's desire to store such information, ii) the system's generation of such tuples as a result of the application of the algebra operators on  $T_I, T_E$  and  $T_M$ .

Introduction of redundant information may be incurred through the application of the extended relational algebra operators to E-tables or after an insert/modify operation. Possible sources of redundancy in E-tables and the way to eliminate these redundancies can be summarized as follows:

1)  $t \in T_D, w \in T_I$  and  $t \in w$ . Remedy: Delete  $w$  from  $T_I$ . Insert in  $T_m$  each of the tuples in  $w - \{t\}$ .

2)  $w_1 \in T_I, w_2 \in T_I$  and  $w_2 \supset w_1$ .

Remedy: Delete  $w_2$  from  $T_I$ . Insert in  $T_m$  each of the tuples in  $w_2 - w_1$ .

3)  $w \in T_M, t \in T_D$  and  $t \in w$ . Remedy: Delete  $w$  from  $T_m$ .

4)  $w_1 \in T_M, |w_1| = 1$  and  $w_2 \in T_I$  and  $w_2 \supset w_1$ . Remedy: Delete  $t$  from  $T_m$ .

5)  $t \in T_D, w \in T_E$  and  $t \in w$ . Remedy: Delete  $w$  from  $T_E$ .

6)  $w_1 \in T_E, w_2 \in T_E$  and  $w_2 \supset w_1$ . Remedy: Delete  $w_2$  from  $T_E$ .

7)  $w_1 \in T_E, w_2 \in T_M$  and  $w_1 \supset w_2$ . Remedy: Delete  $w_2$  from  $T_m$ .

8)  $w_1 \in T_E, w_2 \in T_M$  and  $w_2 \supset w_1$ . Remedy: Delete  $w_1$  from  $T_E$ .

9)  $w_1 \in T_E, w_2 \in T_I$  and  $w_2 \supset w_1$ .

Remedy: Delete  $w_2$  from  $T_I$ . Insert each element of  $w_2 - w_1$  to  $T_M$ .

10)  $w_1 \in T_E, w_2 \in T_I$  and  $w_1 \supset w_2$ .

Remedy: Delete  $w_1$  from  $T_I$ . Delete  $w_2$  from  $T_E$ . Insert  $w_1$  to  $T_E$ .

In the light of the different types of redundancies listed above, we can define an operator, ELIMRED, that takes in as input an E-table and returns the E-table with all the redundancies eliminated.

Definition 2.1 ELIMRED :  $\Psi_{R-} \rightarrow \Psi_R$  is a mapping such that  $\text{ELIMRED}(T) = T^0$ , where  $T^0$  is defined as follows:

$$\begin{aligned}
 T_D^0 &= \{t \mid t \in T_D\} \\
 T_I^0 &= \{w \mid w \in T_I \wedge \neg(\exists t)(t \in T_D \wedge t \in w) \wedge \neg(\exists w_1)(w_1 \in T_I \wedge w \supset w_1) \\
 &\quad \wedge \neg(\exists w_2)(w_2 \in T_E \wedge w \supset w_2) \wedge \neg(\exists w_3)(w_3 \in T_E \wedge w_3 \supset w)\} \\
 T_E^0 &= \{w \mid w \in S_1 \wedge \neg(\exists t_1)(t_1 \in T_D \wedge t_1 \in w) \wedge \neg(\exists w_1)(w_1 \in T_E \wedge w \supset w_1) \\
 &\quad \wedge \neg(\exists w_2)(w_2 \in T_M \wedge w_2 \supset w) \wedge \neg(\exists w_3)(w_3 \in T_I \wedge w \supset w_3)\} \\
 &\quad \text{where } S_1 = \{w \mid w \in T_E \vee (\exists w_1)(\exists w_2)(w_1 \in T_E \wedge w_2 \in T_I \wedge w_1 \supset w_2 \wedge w = w_1)\} \\
 T_M^0 &= \{w \mid w \in S_2 \wedge (\forall t)(t \in w \rightarrow \neg(t \in T_D^0)) \wedge \neg(\exists w_1)(w_1 \in T_I^0 \wedge |w| = 1 \wedge w_1 \supset w) \\
 &\quad \wedge \neg(\exists w_2)(w_2 \in T_E^0 \wedge w \supset w_2)\} \\
 &\quad \text{where } S_2 = \{w \mid w \in T_M \vee (\exists w_1)(\exists w_2)(w_1 \in T_I \wedge w_2 \in T_I \wedge w_2 \supset w_1 \wedge (\forall t)(t \in w_2 - w_1 \rightarrow \\
 &\quad w = \{t\}) \vee (\exists w_3)(\exists w_4)(w_3 \in T_E \wedge w_4 \in T_I \wedge w_4 \supset w_3 \wedge (\forall t)(t \in w_4 - w_3 \rightarrow w = \{t\}) \\
 &\quad \vee (\exists t_1)(\exists w_5)(t_1 \in T_D \wedge w_5 \in T_I \wedge t_1 \in w_5 \wedge (\forall t)(t \in w_5 - \{t_1\} \rightarrow w = \{t\}))\}
 \end{aligned}$$

Now, we define our extended algebra operators. The definitions of  $T_D$  and  $T_I$  are the same as those in [1] for the union, selection, cartesian product, projection and intersection operators. Note that  $\Psi_R : \{T \mid T: \text{E-Table over } R\}$

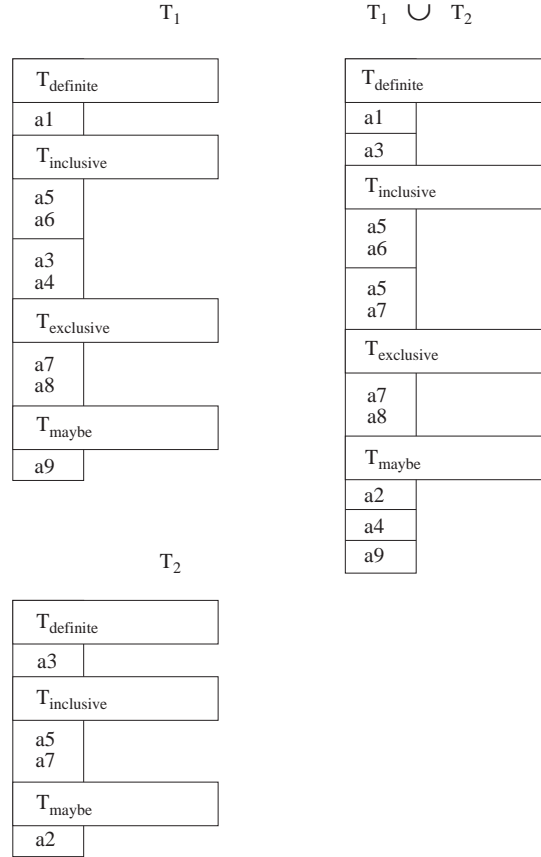
*Definition 2.2* Union on  $\Psi_R$  is a mapping  $U: \Psi_R \times \Psi_{R-} \rightarrow \Psi_R$ .

Let  $T_1$  and  $T_2$  be two domain compatible E-tables. Then,  $T_1 U T_2 = \text{ELIMRED}(T)$ , where  $T$  is defined

as follows:

$$\begin{aligned}
 T_D &= \{t | t \in T_D^1 \vee t \in T_D^2\} \\
 T_I &= \{w | w \in T_I^1 \vee w \in T_I^2\} \\
 T_E &= \{w | w \in T_E^1 \vee w \in T_E^2\} \\
 T_M &= \{w | w \in T_M^1 \vee w \in T_M^2\}
 \end{aligned}$$

An example of the onion operation is given in Figure 2.



**Figure 2.** An example of the extended union operation.

*Definition 2.3* Selection on elements of  $\Psi_R$  is a mapping  $\sigma : \Psi_R \rightarrow \Psi_R$ .

Let  $T_1$  be an E-table and  $F$  be a formula involving operands that are constants or attribute numbers and arithmetic comparison operators:  $< . =, >$ , and logical operators  $\vee, \wedge, \neg$ . Then,  $\sigma_F(T_1) = \text{ELIMRED}(T)$ , where  $T$  is defined as follows:

$$\begin{aligned}
 T_D &= \{t | t \in T_D^1 \vee F(t)\} \\
 T_I &= \{w | w \in T_I^1 \wedge (\forall t)(t \in w \rightarrow F(t))\} \\
 T_E &= \{w | w \in T_E^1 \wedge (\forall t)(t \in w \rightarrow F(t))\} \\
 T_M &= \{w | (w \in T_M^1 \wedge (\forall t)(t \in w \rightarrow F(t))) \\
 &\quad \vee (\exists w_1)(w_1 \in T_I^1 \wedge t \in w_1 \wedge F(t) \wedge w = t) \\
 &\quad \vee (\exists w_2)(w_2 \in T_E^1 \wedge w_2 \supset w \wedge (t \in w \rightarrow F(t)))\}
 \end{aligned}$$

An example of the selection operation is given in Figure 3.

T	$\sigma_{I="a3"}(T)$																																																								
<table style="width: 100%; border-collapse: collapse;"> <tr><td colspan="2" style="text-align: center;">T<sub>definite</sub></td></tr> <tr><td style="width: 50%;">a3</td><td style="width: 50%;">b3</td></tr> <tr><td>a2</td><td>b2</td></tr> <tr><td colspan="2" style="text-align: center;">T<sub>inclusive</sub></td></tr> <tr><td>a3</td><td>b1</td></tr> <tr><td>a3</td><td>b2</td></tr> <tr><td>a4</td><td>b2</td></tr> <tr><td>a3</td><td>b4</td></tr> <tr><td colspan="2" style="text-align: center;">T<sub>exclusive</sub></td></tr> <tr><td>a4</td><td>b1</td></tr> <tr><td>a4</td><td>b4</td></tr> <tr><td>a3</td><td>b5</td></tr> <tr><td>a4</td><td>b6</td></tr> <tr><td>a3</td><td>b8</td></tr> <tr><td colspan="2" style="text-align: center;">T<sub>maybe</sub></td></tr> <tr><td>a3</td><td>b7</td></tr> <tr><td>a3</td><td>b6</td></tr> </table>	T <sub>definite</sub>		a3	b3	a2	b2	T <sub>inclusive</sub>		a3	b1	a3	b2	a4	b2	a3	b4	T <sub>exclusive</sub>		a4	b1	a4	b4	a3	b5	a4	b6	a3	b8	T <sub>maybe</sub>		a3	b7	a3	b6	<table style="width: 100%; border-collapse: collapse;"> <tr><td colspan="2" style="text-align: center;">T<sub>definite</sub></td></tr> <tr><td style="width: 50%;">a3</td><td style="width: 50%;">b3</td></tr> <tr><td colspan="2" style="text-align: center;">T<sub>inclusive</sub></td></tr> <tr><td>a3</td><td>b1</td></tr> <tr><td>a3</td><td>b2</td></tr> <tr><td colspan="2" style="text-align: center;">T<sub>maybe</sub></td></tr> <tr><td>a3</td><td>b4</td></tr> <tr><td>a3</td><td>b5</td></tr> <tr><td>a3</td><td>b8</td></tr> <tr><td>a3</td><td>b7</td></tr> <tr><td>a3</td><td>b6</td></tr> </table>	T <sub>definite</sub>		a3	b3	T <sub>inclusive</sub>		a3	b1	a3	b2	T <sub>maybe</sub>		a3	b4	a3	b5	a3	b8	a3	b7	a3	b6
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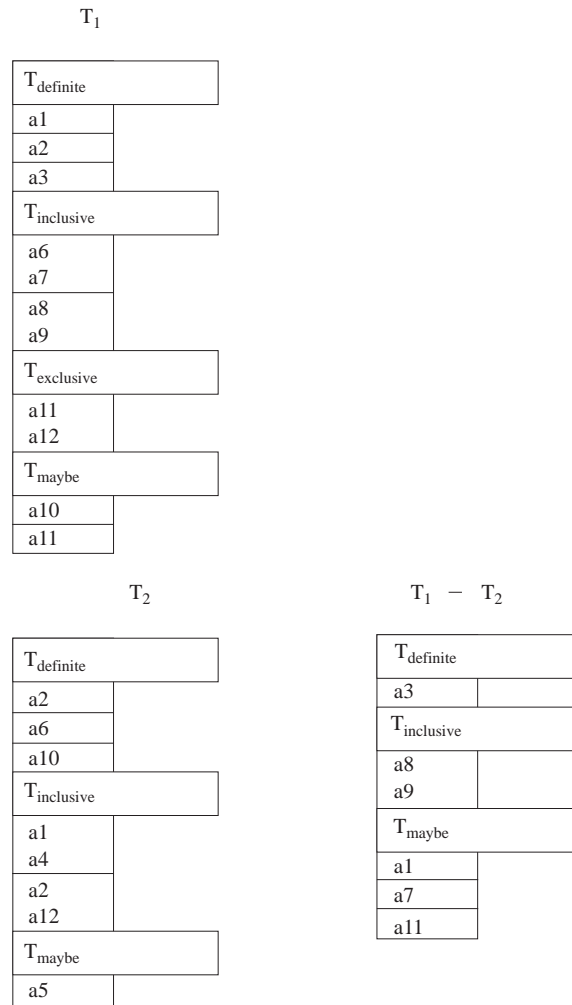
**Figure 3.** An example of the extended selection operation.

*Definition 2.4* Difference on  $\Psi_R$  is a mapping  $-: \Psi_R \times \Psi_R \rightarrow \Psi_R$ .

Let  $T_1$  and  $T_2$  be two domain-compatible E-tables. Then,  $T_1 - T_2 = \text{ELIMRED}(T)$ , where T is defined as follows:

$$\begin{aligned}
 T_D &= \{t \mid (t \in T_D^1) \wedge \neg(t \in T_D^2) \wedge \neg(\exists w)((w \in T_I^2 \wedge t \in w) \vee (w \in T_E^2 \wedge t \in w) \vee (w \in T_M^2 \wedge t \in w))\} \\
 T_I &= \{w \mid (w \in T_I^1) \wedge \neg(\exists t)(t \in T_D^2 \wedge t \in w \wedge \neg(\exists w_1)((w_1 \in T_I^2 \wedge \neg(w \cap w_1 = \phi)) \\
 &\quad \vee (w_1 \in T_E^2 \wedge \neg(w \cap w_1 = \phi)) \vee (w_1 \in T_M^2 \wedge \neg(w \cap w_1 = \phi)))\} \\
 T_E &= \{w \mid (w \in T_E^1) \wedge \neg(\exists t)(t \in T_D^2 \wedge t \in w) \\
 &\quad \wedge \neg(\exists w_1)((w_1 \in T_I^2 \wedge \neg(w \cap w_1 = \phi)) \\
 &\quad \vee (w_1 \in T_E^2 \wedge \neg(w \cap w_1 = \phi)) \\
 &\quad \vee (w_1 \in T_M^2 \wedge \neg(w \cap w_1 = \phi)))\} \\
 T_M &= \{w \mid (w \in T_M^1) \\
 &\quad \vee (\exists w_1)(\exists t)(t \in T_D^1 \wedge w_1 \in T_I^2 \wedge t \in w_1 \wedge w = \{t\}) \\
 &\quad \vee (\exists w_1)(\exists t)(t \in T_D^1 \wedge w_1 \in T_E^2 \wedge t \in w_1 \wedge w = \{t\}) \\
 &\quad \vee (\exists w_1)(\exists t)(t \in T_D^1 \wedge w_1 \in T_M^2 \wedge t \in w_1 \wedge w = \{t\}) \\
 &\quad \vee (\exists w_1)(\exists t_1)(w_1 \in T_I^1 \wedge t_1 \in T_D^2 \wedge t_1 \in w_1 \wedge (\forall t_2)(t_2 \in w_1 - \{t_1\} \rightarrow w = \{t_2\})) \\
 &\quad \vee (\exists w_1)(\exists w_2)(w_1 \in T_I^1 \wedge w_2 \in T_I^2 \wedge \neg(w_1 \cap w_2 = \phi) \wedge (\forall t_1)(t_1 \in w_1 \rightarrow w = \{t_1\})) \\
 &\quad \vee (\exists w_1)(\exists w_2)(w_1 \in T_I^1 \wedge w_2 \in T_M^2 \wedge \neg(w_1 \cap w_2 = \phi) \wedge (\forall t_1)(t_1 \in w_1 \rightarrow w = \{t_1\})) \\
 &\quad \vee (\exists w_1)(\exists w_2)(w_1 \in T_I^1 \wedge w_2 \in T_E^2 \wedge \neg(w_1 \cap w_2 = \phi) \wedge (\forall t_1)(t_1 \in w_1 \rightarrow w = \{t_1\})) \\
 &\quad \vee (\exists w_1)(\exists w_2)(w_1 \in T_E^1 \wedge w_2 \in T_I^2 \wedge \neg(w_1 \cap w_2 = \phi) \wedge (\forall t_1)(t_1 \in w_1 \rightarrow w = \{t_1\}))\}
 \end{aligned}$$

An example of the difference operation is given in Figure 4.



**Figure 4.** An example of the extended difference operation.

*Definition 2.5* Projection on elements of  $\Psi_R$  is a mapping  $\prod : \Psi_R \rightarrow \Psi_R$

Let  $T_1$  be an E-table and A be a list of attribute numbers. Then,  $\prod_A(T_1) = \text{ELIMRED}(T)$ , where T is defined as follows:

$$\begin{aligned}
 T_D &= \{t \mid (\exists t_1)(t_1 \in T_D^1 \wedge t[A] = t_1[A]) \vee (\exists w_1)(w_1 \in T_I^1 \wedge (\forall t_1)(t_1 \in w_1 \rightarrow t[A] = t_1[A])) \\
 &\quad \vee (\exists w_2)(w_2 \in T_I^1 \wedge (\forall t_2)(t_2 \in w_2 \rightarrow t[A] = t_2[A]))\} \\
 T_I &= \{w \mid (\exists w_1)(w_1 \in T_I^1 \wedge w = \prod_A(w_1) \wedge |w| > 1)\} \\
 T_E &= \{w \mid (\exists w_1)(w_1 \in T_E^1 \wedge w = \prod_A(w_1) \wedge |w| > 1)\} \\
 T_M &= \{w \mid (\exists w_1)(w_1 \in T_M^1 \wedge w = \prod_A(w_1))\}
 \end{aligned}$$

An example of the difference operation is given in Figure 5.

*Definition 2.6* Cartesian product of elements of  $\Psi_{R1}$  and  $\Psi_{R2}$  is a mapping  $x : \Psi_{R1} \times \Psi_{R2} \rightarrow \Psi_{R1.R2}$

Let  $T_1$  and  $T_2$  be I-tables such that  $T_1^1 = \{w_1^1, \dots, w_m^1\}$  and  $T_2^2 = \{w_1^2, \dots, w_n^2\}$ .

Let  $E = \{\{t_1, \dots, t_m\} \mid (\forall i)(1 \leq i \leq m \rightarrow t_i \in w_i^1)\}$  and  $F = \{\{t_1, \dots, t_n\} \mid (\forall i)(1 \leq i \leq n \rightarrow t_i \in w_i^2)\}$ .

Let the elements of E be  $E_1, \dots, E_e$  and those of F be  $F_1, \dots, F_f$ . Let

$$A_{ij} = \{t \mid (\exists t_1)(\exists t_2)(t_1 \in T_D^1 \wedge t_2 \in F_I \wedge t = t_1.t_2)$$

$$V(\exists t_1)(\exists t_2)(t_1 \in E_k \wedge t_2 \in T_D^2 \wedge t = t_1.t_2)$$

$$V(\exists t_1)(\exists t_2)(t_1 \in E_k \wedge t_2 \in F_I \wedge t = t_1.t_2)\}$$

T		
T <sub>definite</sub>		
a3	b3	
a2	b2	
T <sub>inclusive</sub>		
a3	b1	
a3	b2	
a4	b2	
a3	b4	
T <sub>exclusive</sub>		
a4	b1	
a4	b4	
a3	b5	
a4	b6	
a3	b8	
T <sub>maybe</sub>		
a3	b7	
a3	b6	

Π <sub>1</sub> (T)	
T <sub>definite</sub>	
a2	
a3	
a4	
T <sub>maybe</sub>	
a5	

**Figure 5.** An example of the extended projection operation.

where  $1 \leq k \leq e, 1 \leq l \leq f$ ;  $i=k$  if  $\neg(e=0)$  and  $i=0$  otherwise;

$j=l$  if  $\neg(f=0)$  and  $j=0$  otherwise.

Let  $A_1, \dots, A_g$  be the distinct  $A_{ijs}$ .

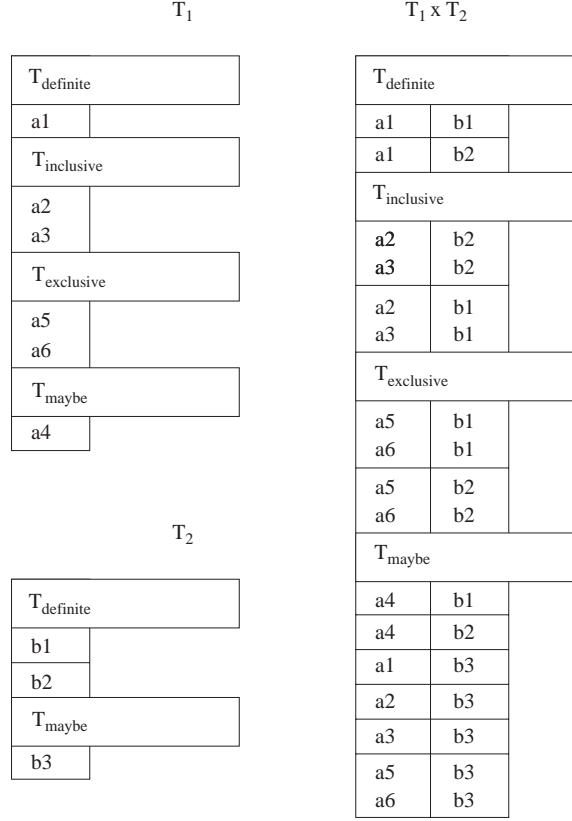
Then,  $T_1 \times T_2 = \text{ELIMRED}(T)$ , where  $T$  is defined as follows

$$\begin{aligned}
 T_D &= \{t \mid (\exists t_1)(\exists t_2)(t \in T_D^1 \wedge t_2 \in T_D^2 \wedge t = t_1.t_2)\}, \\
 T_I &= \{w \mid (\exists t_1) \dots (\exists t_g)(t_1 \in A_1 \wedge \dots \wedge t_g \in A_g \wedge w = \{t_1, \dots, t_g\})\}, \\
 T_E &= \{w \mid (\exists t_1)(\exists w_1)(t_1 \in T_D^1 \wedge w_1 = \{t_{11}, t_{12}, \dots, t_{1k}\} \in T_E^2 \wedge w = \{t_1.t_{11}, t_1.t_{12}, \dots, t_1.t_{1k}\}) \\
 &\quad V(\exists w_1)(\exists w_2)(w_1 = \{t_{11}, t_{12}, \dots, t_{1k}\} \in T_E^1 \wedge w_2 = \{t_{21}, t_{22}, \dots, t_{2m}\} \in T_E^2 \\
 &\quad \wedge w = \{t_{11}.t_{21}, \dots, t_{1k}.t_{21}, t_{11}.t_{22}, \dots, t_{1k}.t_{22}, \dots, t_{1k}.t_{2m}\})\} \\
 T_M &= \{w \mid (\exists t_1)(\exists w_1)(t_1 \in T_D^1 \wedge w_1 = \{t_{11}, t_{12}, \dots, t_{1k}\} \in T_M^2 \wedge w = \{t_1.t_{11}, t_1.t_{12}, \dots, t_1.t_{1k}\}) \\
 &\quad V(\exists t_1)(\exists w_1)(t_1 \in T_D^2 \wedge w_1 = \{t_{11}, t_{12}, \dots, t_{1k}\} \in T_M^1 \wedge w = \{t_{11}.t_1, t_{22}.t_1 \dots t_{1k}.t_1\}) \\
 &\quad V(\exists w_1)(\exists w_2)(w_1 = \{t_{11}, t_{12}, \dots, t_{1k}\} \in T_I^1 \wedge w_2 = \{t_{21}, t_{22}, \dots, t_{2m}\} \in T_M^2 \\
 &\quad \wedge (w = \{t_{11}.t_{21}, t_{11}.t_{22}, \dots, t_{11}.t_{2m}\} \\
 &\quad Vw = \{t_{12}.t_{21}, t_{12}.t_{22}, \dots, t_{12}.t_{2m}\} \\
 &\quad \dots \dots \\
 &\quad \dots \dots \\
 &\quad Vw = \{t_{1k}.t_{21}, t_{1k}.t_{22}, \dots, t_{1k}.t_{2m}\}) \\
 &\quad V(\exists w_1)(\exists w_2)(w_1 = \{t_{11}, t_{12}, \dots, t_{1k}\} \in T_I^2 \wedge w_2 = \{t_{21}, t_{22}, \dots, t_{2m}\} \in T_M^1 \\
 &\quad \wedge (w = \{t_{21}.t_{11}, t_{22}.t_{11}, \dots, t_{2m}.t_{11}\} \\
 &\quad Vw = \{t_{21}.t_{12}, t_{22}.t_{12}, \dots, t_{2m}.t_{12}\} \\
 &\quad \dots \dots \\
 &\quad \dots \dots \\
 &\quad Vw = \{t_{21}.t_{1k}, t_{22}.t_{1k}, \dots, t_{2m}.t_{1k}\}) \\
 &\quad V(\exists w_1)(\exists w_2)(w_1 = \{t_{11}, t_{12}, \dots, t_{1k}\} \in T_E^1 \wedge w_2 = \{t_{21}, t_{22}, \dots, t_{2m}\} \in T_M^2 \\
 &\quad \wedge w = \{t_{11}.t_{21}, \dots, t_{1k}.t_{21}, t_{11}.t_{22}, \dots, t_{1k}.t_{22}, \dots, t_{1k}.t_{2m}\})
 \end{aligned}$$



$$\begin{aligned}
 &V(\exists w_1)(\exists w_2)(w_1 = \{t_{11}, t_{12}, \dots, t_{1k}\} \in T_M^1 \wedge w_2 = \{t_{21}, t_{22}, \dots, t_{2m}\} \in T_E^2 \\
 &\wedge w = \{t_{11}.t_{21}, \dots, t_{1k}.t_{21}, t_{11}.t_{22}, \dots, t_{1k}.t_{22}, \dots, t_{1k}.t_{2m}\}) \\
 &V(\exists w_1)(\exists w_2)(w_1 = \{t_{11}, t_{12}, \dots, t_{1k}\} \in T_M^1 \wedge w_2 = \{t_{21}, t_{22}, \dots, t_{2m}\} \in T_M^2 \\
 &\wedge w = \{t_{11}.t_{21}, \dots, t_{1k}.t_{21}, t_{11}.t_{22}, \dots, t_{1k}.t_{22}, \dots, t_{1k}.t_{2m}\})
 \end{aligned}$$

An example of the cartesian product operation is given in Figure 6.



**Figure 6.** An example of the extended cartesian product operation.

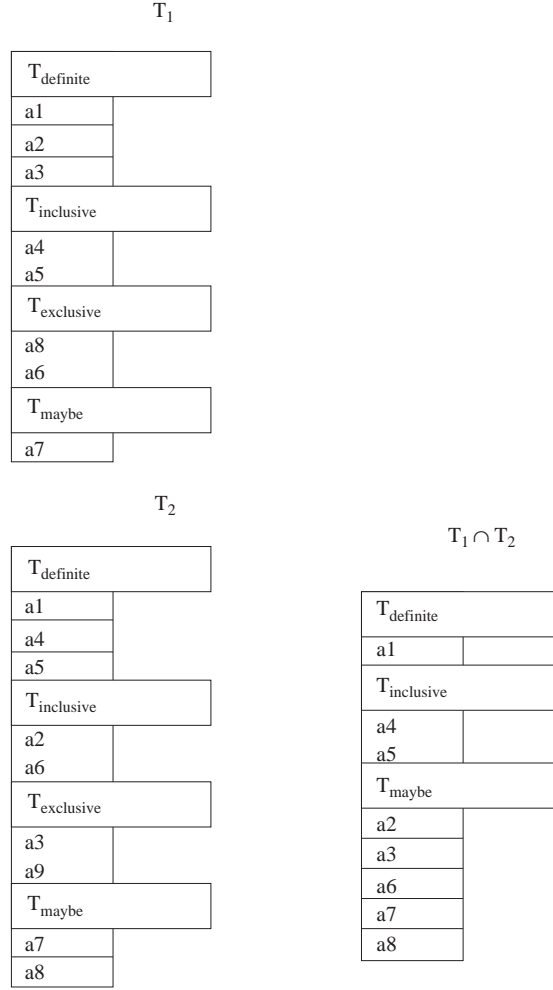
Since  $T_1 \cap T_2 \neq T_1 - (T_1 - T_2)$  in our extended algebra, we also define the intersection operator.

*Definition 2.7* Intersection on  $\Psi_R$  is a mapping  $\cap : \Psi_R \times \Psi_R \rightarrow \Psi_R$

Let  $T_1$  and  $T_2$  be two domain-compatible E-tables. Then,  $T_1 \cap T_2 = \text{ELIMRED}(T)$ , where  $T$  is defined as follows:

$$\begin{aligned}
 T_D &= \{t | t \in T_D^1 \wedge t \in T_D^2\} \\
 T_I &= \{w | (w \in T_I^1 \wedge T_D^2 \supset w) \wedge (w \in T_I^2 \wedge T_D^1 \supset w)\} \\
 T_E &= \{w | (w \in T_E^1 \wedge T_D^2 \supset w) \wedge (w \in T_E^2 \wedge T_D^1 \supset w)\} \\
 T_M &= \{w | (\exists w_1)(\exists t)(w_1 \in T_M^1 \wedge t \in T_D^2 \wedge t \in w_1 \wedge w = \{t\}) \\
 &\quad V(\exists w_1)(\exists t)(w_1 \in T_M^2 \wedge t \in T_D^1 \wedge t \in w_1 \wedge w = \{t\}) \\
 &\quad V(\exists w_1)(\exists t)(w_1 \in T_E^1 \wedge t \in T_D^2 \wedge t \in w_1 \wedge w = \{t\}) \\
 &\quad V(\exists w_1)(\exists t)(w_1 \in T_E^2 \wedge t \in T_D^1 \wedge t \in w_1 \wedge w = \{t\}) \\
 &\quad V(\exists w_1)(\exists w_2)(w_1 \in T_I^1 \wedge w_2 \in T_I^2 \wedge t \in w_1 \cap w_2 \wedge w = \{t\}) \\
 &\quad V(\exists w_1)(\exists w_2)(\exists t)(w_1 \in T_I^1 \wedge t \in T_D^2 \wedge w = \{t\}) \\
 &\quad V(w_1 \in T_E^1 \wedge w_2 \in T_I^2 \wedge w = w_1 \cap w_2) V(w_1 \in T_E^2 \wedge w_2 \in T_I^1 \wedge w = w_1 \cap w_2) \\
 &\quad V(w_1 \in T_E^1 \wedge w_2 \in T_M^2 \wedge w = w_1 \cap w_2) V(w_1 \in T_E^2 \wedge w_2 \in T_M^1 \wedge w = w_1 \cap w_2) \\
 &\quad V(w_1 \in T_I^1 \wedge w_2 \in T_M^2 \wedge w = w_1 \cap w_2) V(w_1 \in T_I^2 \wedge w_2 \in T_M^1 \wedge w = w_1 \cap w_2) \\
 &\quad V(w_1 \in T_E^1 \wedge w_2 \in T_E^2 \wedge w = w_1 \cap w_2) V(w_1 \in T_M^2 \wedge w_2 \in T_M^1 \wedge w = w_1 \cap w_2)\}
 \end{aligned}$$

An example of the cartesian product operation is given in Figure 7.



**Figure 7.** An example of the extended intersection operation.

The use of the new relational algebra in our model can be demonstrated with an example.

*Example 2.1:* Consider the following query about Mary’s teachers:

$$\prod_3(\sigma_{2=4}((\sigma_{1='mary'} \text{takes}) \times \text{teaches})).$$

Assume that we have the following stored E-tables for the relations *takes* (*student*, *course*) and *teaches* (*teacher*, *course*).

$\text{takes}_D = \{ \langle \text{john}, \text{CS421} \rangle, \langle \text{mary}, \text{CS321} \rangle, \langle \text{mary}, \text{CS335} \rangle \}$  (John takes CS421. Mary takes CS 321. Mary takes CS 335.)

$\text{takes}_E = \{ \{ \langle \text{john}, \text{CS415} \rangle, \langle \text{john}, \text{CS495} \rangle \} \}$  (John takes either CS 421 or CS 495 but not both.)

$\text{takes}_I = \{ \{ \langle \text{mary}, \text{CS 364} \rangle, \langle \text{mary}, \text{MATH310} \rangle \} \}$  (Mary takes CS 364 or MATH 310 or both.)

$\text{takes}_M = \{ \{ \langle \text{mary}, \text{ART201} \rangle, \langle \text{mary}, \text{ART301} \rangle \} \}$  (Mary may take only one of ART 201 and ART 301 or neither.)

$\text{teaches}_D = \{ \langle \text{ann}, \text{CS 321} \rangle, \langle \text{jim}, \text{CS 364} \rangle \}$  (Ann teaches CS 321. Jim teaches CS 364.)

$\text{teaches}_E = \{ \{ \langle \text{dick}, \text{CS 335} \rangle, \langle \text{ed}, \text{CS 335} \rangle \} \}$  (Either Dick or Ed teaches CS335, but not both.)

$teaches_I = \{ \}$

$teaches_M = \{ \{ \langle tom, ART201 \rangle, \langle tom, ART301 \rangle \} \}$  (Tom may teach only one of ART201 or ART301 or neither.)

The result of the query about Mary's teachers is an E-table Q as:

$Q_D = \{ \langle ann \rangle \}$  (Ann is definitely Mary's teacher.)

$Q_E = \{ \{ \langle dick \rangle, \langle ed \rangle \} \}$  (Either Dick or Ed is Mary's teacher, but not both.)

$Q_I = \{ \}$

$Q_M = \{ \{ \langle tom \rangle, \langle jim \rangle \} \}$  (Tom may be Mary's teacher. Jim may be Mary's teacher.)

### 3. Conclusions

In this paper we presented a structure called an E-table which makes it possible to store exclusive and inclusive disjunctions and maybe information in relational databases. We also redefined the relational algebra operations to include exclusive disjunctions and defined an operator to get rid of redundancies in E-tables. The main contribution of this work is to make possible the storing of exclusive disjunctions, an often needed type of incomplete information. The concepts presented in this paper have been applied to LOGOB[8], a deductive data model with predicates representing object sets.

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