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An Analytical Model for Calculating Bounds on Call Blocking Probabilities in LEO Satellite Networks

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Abstract

We present an analytical model for calculating upper and lower bounds for call-blocking probabilities in a LEO satellite network that carries voice calls. The method is especially useful for large systems where calculation of call-blocking probabilities are too expensive. Our method calculates upper and lower bounds very easily and produces fairly accurate results.

1. Introduction

Recent advances in satellite communications make it possible to use satellites as an alternative to wireless telephone and wireless networks. The 20th century witnessed the development of satellite communication systems aiming at providing mobile telephony and data transmission services. These services are globally available and independent terrestrial networks. Satellite systems are location-insensitive, and can be used to extend the reach of networks and applications to anywhere on earth with a fixed constellation cost.

A low (or medium) earth orbit (LEO or MEO) satellite system is a set of identical satellites, launched in several orbital planes with the orbits having the same altitude. The satellites move in a synchronized manner in trajectories relative to the earth. Such a set of satellites is referred to as a *constellation* of satellites. The position of all the satellites in relation to the earth at some instance of time repeats itself after a predetermined period, called the *system period*, which is usually several days, while a satellite within an orbit also comes to the same point in the sky relative to the earth after a certain time, called the *orbit period*, which is approximately 100 minutes for LEO systems.

If satellites are equipped with advanced on-board processing, they can communicate directly with each other by line of sight using inter-satellite links (ISL). If the ISL is between satellites on the same orbit, it is called or intra-plane ISL, and if it is between satellites in adjacent planes it is called an inter-plane ISL. The use of ISLs permits routing in the sky, and therefore increases the flexibility of the system. Although ISLs require complex call management functions due to the dynamic nature of the constellation, they move the burden of the network from ground to space since they permit two users in different footprints to communicate without the need for a terrestrial system.

Depending on the antenna technology used, satellite constellations can provide one of two types of coverage. If the satellite antenna is fixed as the satellite moves along its orbit, then the coverage is called *satellite-fixed*. In this case, the footprint area moves along with the satellite. In *earth-fixed coverage*, the earth's surface is divided into cells, as in a terrestrial cellular system, and a cell is serviced continuously by the same beam during the entire time that the cell is within the footprint area of the satellite. This type of coverage requires an antenna that tracks the cell area.

The performance of satellite systems has been studied by several authors. In general, most studies rely on simple queueing models to evaluate call blocking probabilities, and focus on devising methods for improving the performance of calls during hand-offs (e.g., by assigning higher priority to hand-off calls, using guard channels, or making reservations ahead of a hand-off instant). In [1], Ganz *et al.* expressed the system performance in terms of the *distribution of the number of hand-offs* occurring during a single transaction time and the *average call-drop probability.* In their work, each cell is modeled as an M/M/K/K queue where K denotes the number of channels per cell, assuming that the number of hand-off calls entering a cell is equal to the number of hand-off calls leaving the cell. Del Re *et al.*, in [2] and [3], proposed an analytical model to analyze hand-off queueing strategies under fixed channel allocation. Their method is designed for satellite-fixed cell coverage. In [4], Pennoni and Ferroni described an algorithm to improve the performance of hand-offs in LEO systems. They defined two queues for each cell, one for new calls and one for hand-off calls. The calls are held in these two queues for a maximum allowed waiting time. The hand-off queue has higher priority than the new calls queue. In [5], Dosiere *et al.* used the same model to calculate the hand-off traffic rate over a street-of-coverage. Once the hand-off arrival rate has been calculated, as in [4], the total arrival rate is computed as the summation of the new call arrival rate and the hand-off arrival rate. In [6], Ruiz *et al.* used a similar technique to the one used in [4]. However, this time they used some guard channels for hand-off calls and distinguished between the new arrival rate and the hand-off attempt rate. In [7], Respero and Maral defined a guaranteed hand-off mechanism for LEO satellite systems with satellite-fixed cell configuration. In this method, channel reservation is performed according to the location of the user. The advantage of this method is that the reservation is performed only on the next satellite instead of the whole call path. With this approach, the amount of redundant circuitry is minimized and the hand-off success rate is as high as in the static reservation technique. In [8], Wan *et al.* defined a channel reservation algorithm for hand-off calls. In this algorithm, they keep three queues, one for hand-off requests, one for new call requests and one for available channels. Each request comes with the information indicating the position of the user within the footprint area. The position information is then used to calculate the time of the next hand-off. The aim of the algorithm is to match the available channels with the hand-off and new call request queues according to the time criteria. A similar approach is proposed by Obradovic and Cigoj in [9]. They proposed a dynamic channel reservation scheme. Hand-off management is performed with two queues, one for hand-off requests and one for new call requests. Available channels are also divided into two subgroups, reserved and non-reserved. Reserved channels have priority over non-reserved channel during the assignment.

In [10], Zaim *et al.* proposed an approximation method for calculating call-blocking probabilities in a group of LEO/MEO satellites arranged in a single orbit. Both satellite-fixed and earth-fixed types of coverage with hand-offs were considered. In the model, it was assumed that each satellite has a single beam and that the arrival process is Poisson with a rate independent of the geographic area. The model was analyzed using decomposition. Specifically, the entire orbit is decomposed into sub-systems, each consisting of a small number of satellites. Each sub-system is analyzed exactly, by observing that its steady-state probability distribution has a product-form solution. An efficient algorithm was proposed to calculate the normalizing constant associated with this product-form solution. The results obtained from each sub-system are combined in an iterative manner in order to solve the entire orbit.

In [11], the authors generalized the above algorithm to an entire constellation of LEO/MEO satellites involving multiple orbits. They considered both satellite-fixed and earth-fixed constellations with inter-orbit links and hand-offs. They assumed that each satellite employs a single beam and that calls arrive in a Poisson fashion with a fixed arrival rate independent of the geographical area. They presented an approximate decomposition algorithm for the calculation of the call-blocking probabilities in LEO/MEO satellite constellations. Specifically, the entire constellation is decomposed into sub-systems, and each sub-system is analyzed exactly like a Markov process using the solution technique presented in [10]. This approach lead to an iterative scheme, where the individual sub-systems are solved successively until a convergence criterion is satisfied.

In this paper, we derive upper and lower bounds on the link-blocking probabilities. These bounds are computed efficiently, and can be useful for large satellite constellations when each satellite employs multiple beams.

The paper is organized as follows: in Section 2 we derive efficient upper and lower bounds on the call-blocking probabilities. We present numerical results in Section 3, and in Section 4 we conclude the paper.

2. Bounds on the Call-Blocking Probabilities

In [11], we extended the algorithm presented in [10] to systems with multiple orbits. This permitted us to analyze a whole LEO satellite constellation without a seam. LEO satellite constellations with a seam can also be analyzed by adjusting the routing paths according to the location of the seam. However, we have so far assumed systems with a single beam per satellite. In order to remove this assumption, we need to treat each beam spot as a single satellite. That is, for a LEO constellation with 16 satellites and 10 beams per satellite, we need to analyze a system with 160 satellites. This system can be analyzed using the decomposition algorithm described in [11]. However, due to the large number of satellites, the complexity of the algorithm will be significantly increased, especially when dealing with hand-offs. In this paper, we present an upper and lower bound on the call-blocking probabilities. These bounds permit us to calculated blocking probabilities in a large system with multiple orbits and multiple beams per satellite. In order to obtain the lower bound, we treated each link independently. That is, we calculate the blocking probability at each link using only the constraint at that link. Breaking the dependency among links causes the blocking probability to decrease. This method is explained in the next section with the help of a truncated process.

2.1. The Upper and Lower Bounds: the Two-Satellite System

In this section, we describe a method for calculating an upper and lower bound on the call-blocking probabilities using the two-satellite system shown in Figure 1. In this system, there are two intersatellite links, ISL1 and ISL2. In addition each satellite has an up-and-down link (UDL).

The Markov process for this system is

$$\underline{n} = (n_{11}, n_{12}, n_{22}) \tag{1}$$

where n_{11} is the number of calls using the up-and-down link of satellite 1, n_{12} is the number of calls using ISL1 or ISL2, and n_{22} is the number of calls using the up-and-down link of satellite 2.

The constraints on UDL on satellite 1, UDL on satellite 2 and ISL are

$$2n_{11} + n_{12} \quad \leq \quad C_{UDL} \tag{2}$$

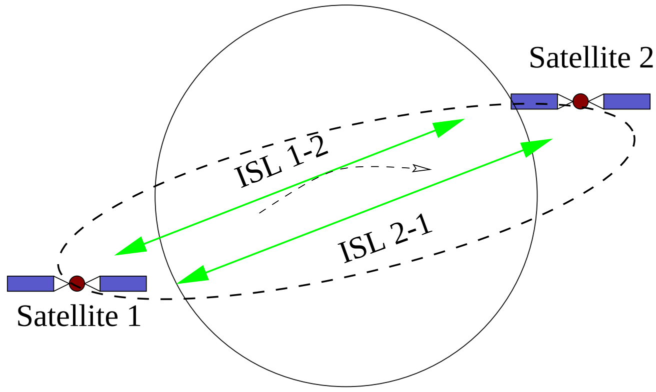

Figure 1. The Simplest Satellite System with 2 Satellites

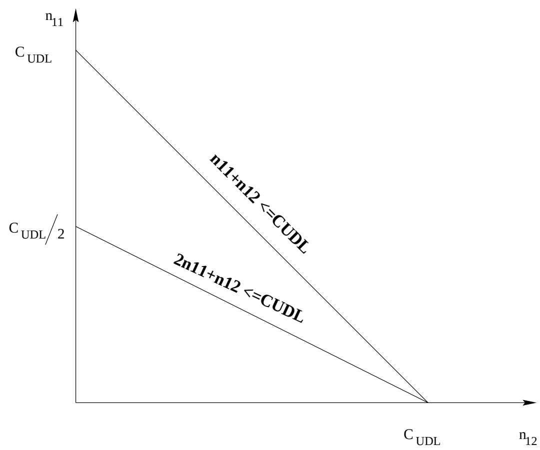

Figure 2. The state spaces for $\underline{n}_{UDL1}$: original and relaxed

$$n_{12} + 2n_{22} \quad \leq \quad C_{UDL} \tag{3}$$

$$n_{12} \quad \leq \quad C_{ISL} \tag{4}$$

In order to obtain the upper and lower bound, we define a truncated Markov process of $\underline{n}$ for each link in the satellite system. For instance, for the up-and-down link of satellite 1, we define the truncated Markov process $\underline{n}_{UDL1} = (n_{11}, n_{12})$, where $2n_{11} + n_{12} \leq C_{UDL}$. This process is obtained from $\underline{n}$ by simply setting to zero all random variables not related to the up-and-down link of satellite 1. In this case, only variable $n_{22} = 0$.

Likewise, for the up-and-down link of n_{UDL2} satellite 2, we define the truncated process of $\underline{n}$, by setting $n_{11} = 0$. We have $n_{UDL2} = (n_{12}, n_{22})$ where $2n_{22} + n_{12} \leq C_{UDL}$. Finally, we define $n_{ISL} = (n_{12})$ by zeroing n_{11} and n_{22} where $n_{12} \leq C_{ISL}$.

Now, let us analyze the first Markov process. In order to calculate the blocking probability, we need to find the normalizing constant G_{UDL1}. For the given Markov process, the normalizing constant G_{UDL1} can be computed as follows:

$$G_{UDL1} = \sum_{0 \leq 2n_{11} + n_{12} \leq C_{UDL}} \frac{\rho_{11}^{n_{11}} \rho_{12}^{n_{12}}}{n_{11}! n_{22}!} \tag{5}$$

If we multiply the right-hand side of the above expression by $C_{UDL}!/C_{UDL}!$, we obtain

$$G = \frac{1}{C_{UDL}!} \sum_{0 \leq 2n_{11}+n_{12} \leq C_{UDL}} C_{UDL}! \frac{\rho_{11}^{n_{11}} \rho_{12}^{n_{12}}}{n_{11}! n_{22}!} \tag{6}$$

Now, instead of summing up over the state space defined by constraint 2, we sum up over the state space defined by the following constraint:

$$n_{11} + n_{12} \quad \leq \quad C_{UDL} \tag{7}$$

The resulting state space is shown in Figure 2. The area under the line marked $2n_{11} + n_{12} \leq C_{UDL}$ is the state space of the truncated process $\underline{n}_{UDL1}$, whereas the area under the line marked $n_{11} + n_{12} \leq C_{UDL}$ is the new state space.

Relaxing the constraint permits us to calculate the normalizing constant more easily. We have

$$G_{UDL1} = \frac{1}{C_{UDL}!} \sum_{0 \leq n_{11}+n_{12} \leq C_{UDL}} C_{UDL}! \frac{\rho_{11}^{n_{11}} \rho_{12}^{n_{12}}}{n_{11}! n_{22}!} \tag{8}$$

For each set of values for which $n_{11} + n_{12} = K$, where $K \leq C_{UDL}$, we can write (8) as follows:

$$S_{n_{11}+n_{12}=K} = \frac{1}{(n_{11}+n_{12}=K)!} \sum_{n_{11}+n_{12}=K} (n_{11}+n_{12}=K)! \frac{\rho_{11}^{n_{11}} \rho_{12}^{n_{12}}}{n_{11}! n_{22}!} \tag{9}$$

We observe that (9) is in fact a multinomial distribution where $n_{11} + n_{12} = K$. Therefore, we can rewrite this equation as follows:

$$S_{n_{11}+n_{12}=K} = \frac{1}{K!} (\rho_{11} + \rho_{12})^K \tag{10}$$

In view of this, the normalizing constant can be calculated as follows:

$$G_{UDL_1} = \sum_{0 \leq K \leq C_{UDL}} \frac{1}{K!} (\rho_{11} + \rho_{12})^K \tag{11}$$

The blocking probability on UDL 1 is given by the expression

$$P_{UDL1} = \frac{\sum_{n_{11}+n_{12}=C_{UDL}} \frac{\rho_{11}^{n_{11}} \rho_{12}^{n_{12}}}{n_{11}! n_{12}!}}{\sum_{0 \leq K \leq C_{UDL}} \frac{1}{K!} (\rho_{11} + \rho_{12})^K} \tag{12}$$

Multiplying the numerator by $C_{UDL}!/C_{UDL}!$ gives

$$P_{UDL1} = \frac{\frac{1}{C_{UDL}!} \sum_{n_{11}+n_{12}=C_{UDL}} C_{UDL}! \frac{\rho_{11}^{n_{11}} \rho_{12}^{n_{12}}}{n_{11}! n_{12}!}}{\sum_{0 \leq K \leq C_{UDL}} \frac{1}{K!} (\rho_{11} + \rho_{12})^K} \tag{13}$$

or

$$P_{UDL1} = \frac{\frac{1}{C_{UDL}!} (\rho_{11} + \rho_{12})^{C_{UDL}}}{\sum_{0 \leq K \leq C_{UDL}} \frac{1}{K!} (\rho_{11} + \rho_{12})^K} \tag{14}$$

This blocking probability can be calculated easily using a recursive algorithm as shown in Figure 3.

A Recursive Algorithm to Calculate P_{UDL1}

1. Begin
2. Enter C_{UDL}
3. Enter ρ_{ij} s
4. $\rho_T = \rho_{11} + \rho_{12}$
5. $P_{UDL1} = 1$ // initialization step
6. For n=1 to C_{UDL}
   $P_{UDL1} = \rho_T * P_{UDL1}/(n + \rho_T * P_{UDL1})$
7. End of the algorithm

Figure 3. A Recursive Algorithm to Calculate Blocking Probabilities

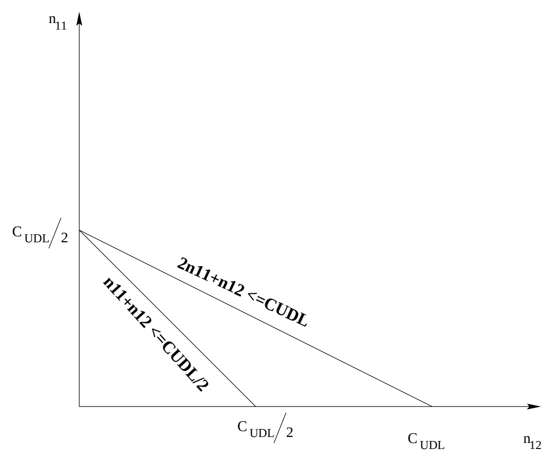

Figure 4. The state spaces for $\underline{n}_{UDL1}$: original and tightened

We can apply the same method to calculate the blocking probabilities P_{UDL2} and P_{ISL} using the Markov process $\underline{n}_{UDL2}$ and $\underline{n}_{ISL}$ respectively (the calculation of P_{ISL} is in fact trivial). The blocking probability between satellite 1 and satellite 2 is

$$P_{1-2} = 1 - (1 - P_{UDL_1})(1 - P_{ISL})(1 - P_{UDL_2}) \tag{15}$$

Since we relaxed the original constraints, the resulting blocking probability is a lower bound for the blocking probability between satellite 1 and satellite 2. A similar approach can be used to calculate upper bounds. This time, we need to tighten the constraints instead of relaxing them. That means that we will solve $\underline{n}_{UDL1}$ using the smaller state space defined by the constraint $n_{11} + n_{12} \leq C_{UDL}$ as shown in Figure 4. The choice of $C_{UDL}/2$ is based on the transformation of the formulation into multinomial distribution. That is, although it is possible to use other ratios, relaxing C_{UDL} to $C_{UDL}/2$ gives us the chance to use multinomial distribution in our calculation.

The resulting blocking probability on UDL 1 is given as follows:

$$P_{UDL_1} = \frac{\frac{1}{(C_{UDL}/2)!}(\rho_{11} + \rho_{12})^{(C_{UDL})/2}}{\sum_{0 \leq K \leq (C_{UDL}/2)} \frac{1}{K!}(\rho_{11} + \rho_{12})^K} \tag{16}$$

End-to-end blocking probabilities are calculated similarly with lower bounds.

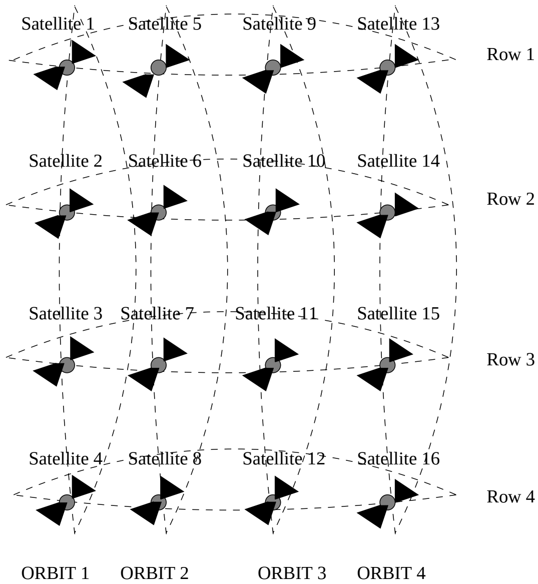

Figure 5. Original 16-satellite constellation

We can easily verify that the lower bound described in this section is lower than the original blocking probability, which is higher than the truncated process defined by constraint 2. For $C_{UDL} = 2$, we can show algebraically that 17 holds [1].

$$\frac{\sum_{n_{11}+n_{12}=C_{UDL}} \frac{\rho_{11}^{n_{11}} \rho_{12}^{n_{12}}}{n_{11}! n_{12}!}}{\sum_{n_{11}+n_{12}\leq C_{UDL}} \frac{\rho_{11}^{n_{11}} \rho_{12}^{n_{12}}}{n_{11}! n_{12}!}} \leq \frac{\sum_{2n_{11}+n_{12}=C_{UDL}} \frac{\rho_{11}^{n_{11}} \rho_{12}^{n_{12}}}{n_{11}! n_{12}!}}{\sum_{2n_{11}+n_{12}\leq C_{UDL}} \frac{\rho_{11}^{n_{11}} \rho_{12}^{n_{12}}}{n_{11}! n_{12}!}} \leq$$

$$\frac{\sum_{2n_{11}+n_{12}=C_{UDL}, n_{12}+2n_{22}\leq C_{UDL}, n_{12}\leq C_{ISL}} \frac{\rho_{11}^{n_{11}} \rho_{12}^{n_{12}} \rho_{22}^{n_{22}}}{n_{11}! n_{12}! n_{22}!}}{\sum_{2n_{11}+n_{12}\leq C_{UDL}, n_{12}+2n_{22}\leq C_{UDL}, n_{12}\leq C_{ISL}} \frac{\rho_{11}^{n_{11}} \rho_{12}^{n_{12}} \rho_{22}^{n_{22}}}{n_{11}! n_{12}! n_{22}!}} \tag{17}$$

This can be generalized to any value of C_{UDL}. The details are not given here. Interested readers may refer to [11].

2.2. The Upper and Lower Bound for Any Satellite System

In Section 2, we described how to calculate an upper and a lower bound for the simplest system consisting of two satellites. In this section, we show how the upper and lower bound can be calculated for a satellite system with any number of satellites. For the sake of presentation, we will use the example satellite system of 16 satellites as shown in Figure 5. Below, we describe how we calculate the bounds for the blocking probability of the up-and-down link on satellite 1, the up-and-down link on satellite 2, and the link between satellites 1 and 2.

For the up-and-down link of satellite 1, we defined a similar truncated Markov process

$$\underline{n}_{UDL1} = (n_{11}, n_{12}, n_{13}, n_{14}, ...) \tag{18}$$

[1] For $C_{UDL} = 2$, Original blocking probability (0.3043) $>$ Truncated process' blocking probability (0.2941) $>$ lower bound (0.2). Upper bound (0.5) is higher than all three blocking probabilities.

by zeroing the remaining random variables, where

$$2n_{11} + n_{12} + n_{13} + n_{14} + n_{15} + n_{16} + n_{17} + n_{18} + n_{19} + n_{1,10} + n_{1,11} + n_{1,12} + n_{1,13} + n_{1,14} + n_{1,15} + n_{1,16} \leq C_{UDL} \quad (19)$$

Likewise, we define $\underline{n}_{UDL2}$, where

$$n_{12} + 2n_{22} + n_{23} + n_{24} + n_{25} + n_{26} + n_{27} + n_{28} + n_{29} + n_{2,10} + n_{2,11} + n_{2,12} + n_{2,13} + n_{2,14} + n_{2,15} + n_{2,16} \leq C_{UDL} \quad (20)$$

Finally, we define $\underline{n}_{ISL}$, where

$$n_{12} + n_{13} + n_{16} + n_{17} + n_{110} + n_{111} + n_{114} + n_{115} \leq C_{ISL} \quad (21)$$

The normalizing constant for the lower bound of the up-and-down blocking probabilities of satellite 1 can be calculated as follows:

$$G = \sum_{0 \leq K \leq C_{UDL}} \frac{1}{K!} (\rho_{11} + \rho_{12} + \rho_{13} + \rho_{14} + \rho_{15} + \rho_{16} + \rho_{17} + \rho_{18} + \rho_{19} + \rho_{110} + \rho_{111} + \rho_{112} + \rho_{113} + \rho_{114} + \rho_{115} + \rho_{116})^K \quad (22)$$

where K is defined as

$$K = \sum_{1 \leq j \leq 16} n_{1j} \quad (23)$$

The blocking probability on UDL of satellite 1 is as follows:

$$P_{UDL_1} = \frac{\frac{1}{C_{UDL}!} (\sum_{1 \leq j \leq 16} \rho_{1j})^{C_{UDL}}}{\sum_{0 \leq K \leq C_{UDL}} \frac{1}{K!} (\sum_{1 \leq j \leq 16} \rho_{1j})^K} \quad (24)$$

We can use the recursive method described in Figure 3 to calculate blocking probabilities. The lower bound of the up-and-down blocking probabilities is similar to 24 except that we set C_{UDL} equal to $C_{UDL}/2$.

We do not show the calculation for the bounds for the other blocking probabilities. Once these blocking probabilities have been calculated, we can obtain the following expressions for the lower and upper bounds.

$$P_{1-2}^{lower} = 1 - (1 - P_{UDL1}^{lower})(1 - P_{ISL}^{lower})(1 - P_{UDL2}^{lower}) \quad (25)$$

$$P_{1-2}^{upper} = 1 - (1 - P_{UDL1}^{upper})(1 - P_{ISL}^{upper})(1 - P_{UDL2}^{upper}) \quad (26)$$

3. Numerical Results

In this section, we compare the upper and lower bound to simulation and exact analytical results. We used three different traffic models: uniform traffic, locality pattern and multiple community models. In the uniform traffic model, each satellite sends an equal amount of traffic to every other satellite. In the locality pattern, a satellite sends 70% of its outgoing traffic to its neighboring satellites and 30% of its traffic to the rest. In the multi-community model, each satellite is the member of a community and sends 70% of its traffic

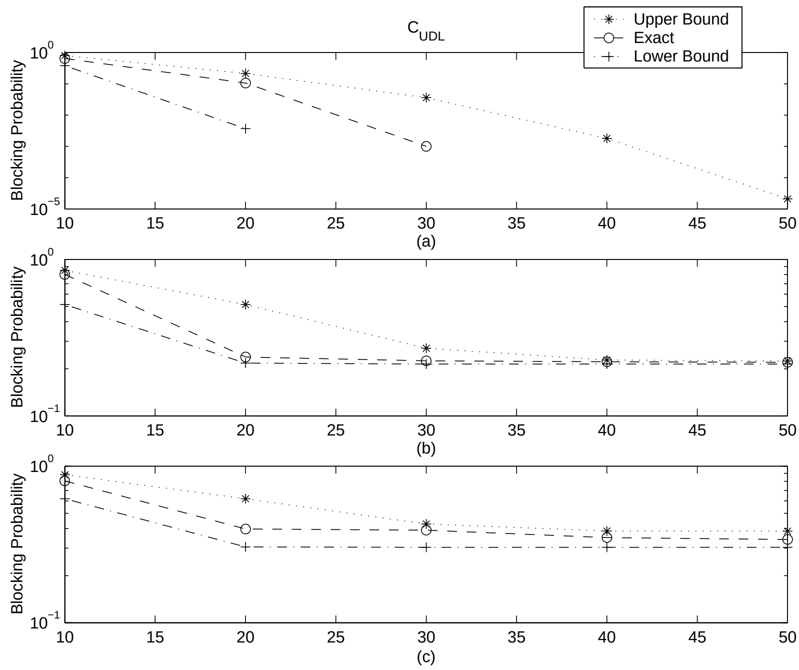

Figure 6. Call-blocking probability, 5-satellite orbit, $\lambda = 10$, $C_{ISL} = 10$, uniform pattern

to the satellites within its own community and 30% of its traffic to satellites in other communities. We ran our analytical and simulation programs on a Sun Sparc 20 machine. For 5 satellites, our analytical solutions take approximately 30 seconds while simulation takes 1 minute. For a 12-satellite system, analytical results can be taken in 2 minutes while it takes 20 minutes for the simulation. Once we increased the number of satellites to 16, we observed a huge difference in running time. Analysis takes 3 minutes while simulation runs approximately 1 hour.

In all these results we used a logarithmic scale as y-axis. Therefore, some of the values do not appear as they go to zero.

3.1. Single-Orbit Case

In this section, we calculated upper and lower bounds for three different traffic patterns for 5 and 12-satellite systems. For the 5-satellite system, we calculated bounds and compared them with exact results. Note that in all those figures, the y-axis is logarithmic and the x-axis shows the UDL Capacity.

Figure 6 plots the upper and lower bounds and the exact results for the blocking probability against the capacity C_{UDL} of up-and-down links, when the arrival rate $\lambda = 10$ and the capacity of inter-satellite links $C_{ISL} = 10$, for the uniform traffic pattern. Three sets of plots are shown: (a) for calls originating and terminating at the same satellite, (b) for calls traveling over a single inter-satellite link, and (c) for calls traveling over two inter-satellite links [2]. Each set consists of three plots, one corresponding to blocking probability values obtained by solving the Markov process, one corresponding to the approximate upper bound and one corresponding to the approximate lower bound.

From this figure, we observe that, as the capacity C_{UDL} of up-and-down link increases, the exact bounds approach the exact values of the blocking probability. In Figure 6-a, the bounds appear to be far

[2] These are the only possible types of calls in a 5-satellite orbit using shortest-path routing.

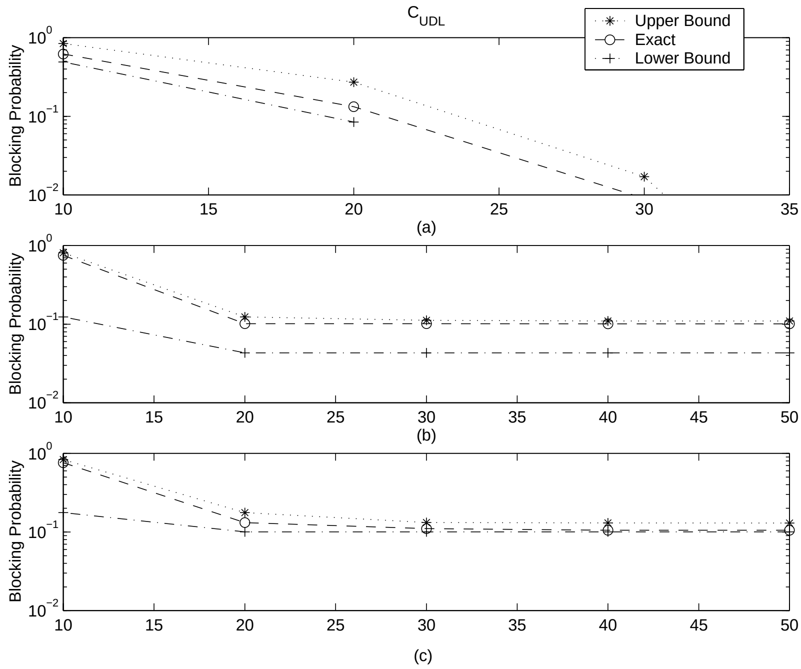

Figure 7. Call-blocking probability, 5-satellite orbit, $\lambda = 10$, $C_{ISL} = 10$, locality pattern

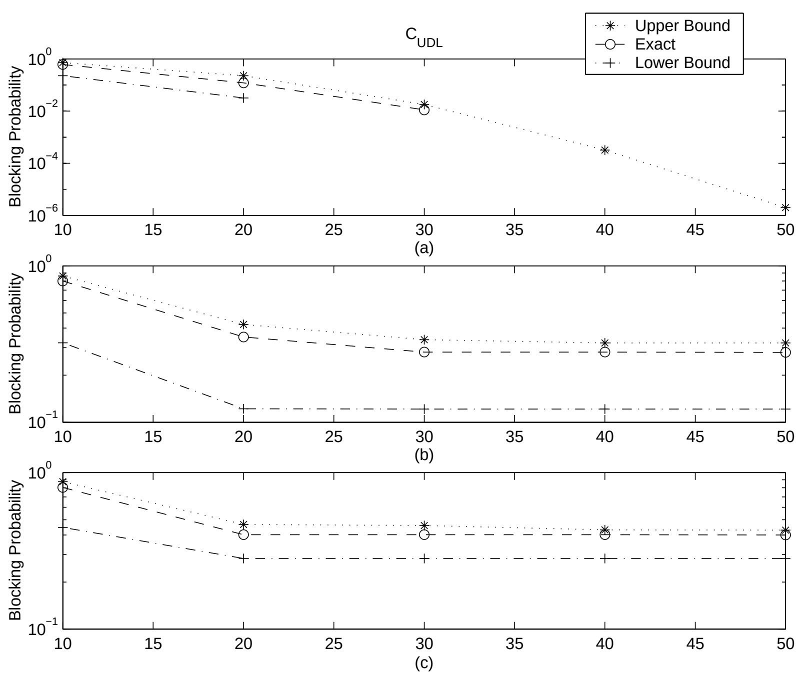

Figure 8. Call-blocking probability, 5-satellite orbit, $\lambda = 10$, $C_{ISL} = 10$, 2-community pattern

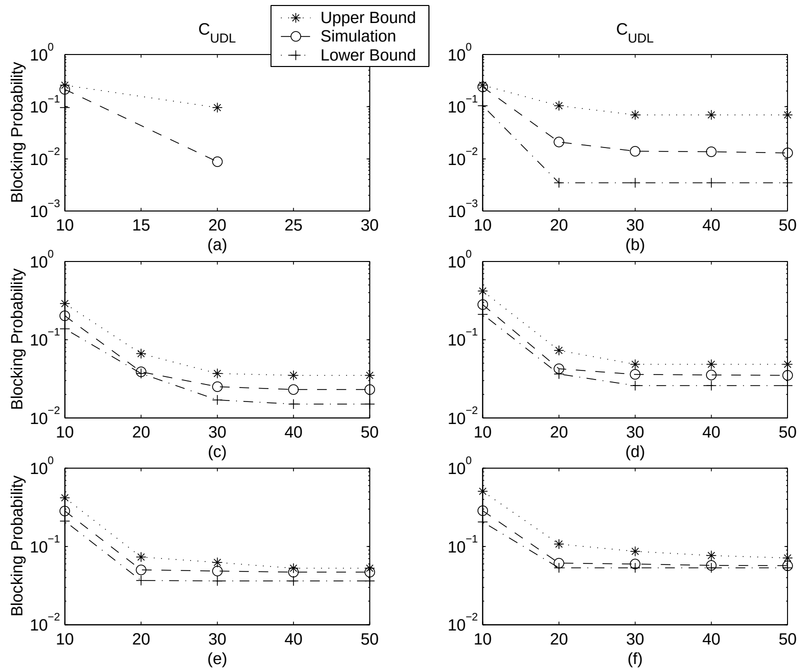

Figure 9. Call-blocking probability, 12-satellite orbit, $\lambda = 5$, $C_{ISL} = 10$, uniform pattern

apart and getting wider. This is because the y-axis is logarithmic and the scale goes down to 10^{-4}. In Figures 6-b and c, the upper and lower bounds converge on each other, and the exact solution stays between them.

Figures 7 and 8 show results for the same parameters as in Figure 6 for the locality and 2-community traffic patterns, respectively. The curves are similar but the actual blocking probability and bound values depend on the traffic pattern used.

Figures 9, 10 and 11 show the results for 12-satellite orbit for uniform, locality and 2-community traffic patterns, respectively. We used the simulation results instead of exact values. Each figure gives 6 sets of plots (a) for local calls only, (b) for calls traversing one ISL, (c) for calls traversing 2 ISLs, (d) for calls traversing 3 ISLs, (e) for calls traversing 4 ISLs, and (f) for calls traversing 5 ISLs. Five ISLs is the maximum number of hops a call can make using the shortest path routing in a single-orbit system with 12 satellites. The parameters are $\lambda = 5$, $C_{ISL} = 10$, and C_{UDL} changes from 10 to 50. The y-axis is again logarithmic and shows the blocking probability. As can be seen in those figures, the upper and lower bounds always stay very close to the simulation results and converge on each other as the UDL capacity increases.

3.2. Multiple-Orbit Case

We used the constellation described in Section 2.2 to validate the accuracy of the bounds in a multiple-orbit system. The results are shown in Figures 12, 13, and 14. The figures illustrate up-and-down link only calls in (a), one ISL hop calls in (b), two ISL hop calls in (c) and three ISL hop calls in (d). As can be seen in the figures, the upper and lower bounds are very close to the simulation values for three different traffic patterns.

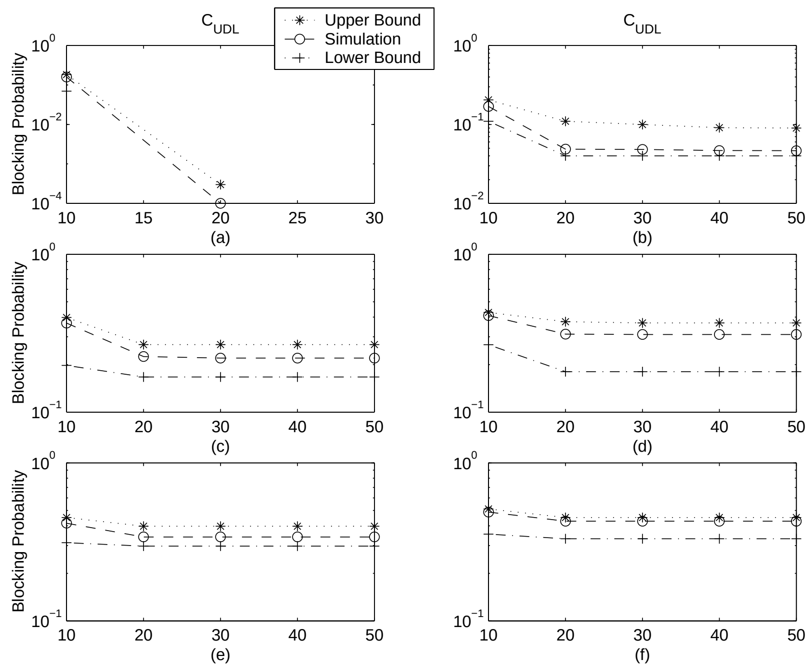

Figure 10. Call-blocking probability, 12-satellite orbit, $\lambda = 5$, $C_{ISL} = 10$, locality pattern

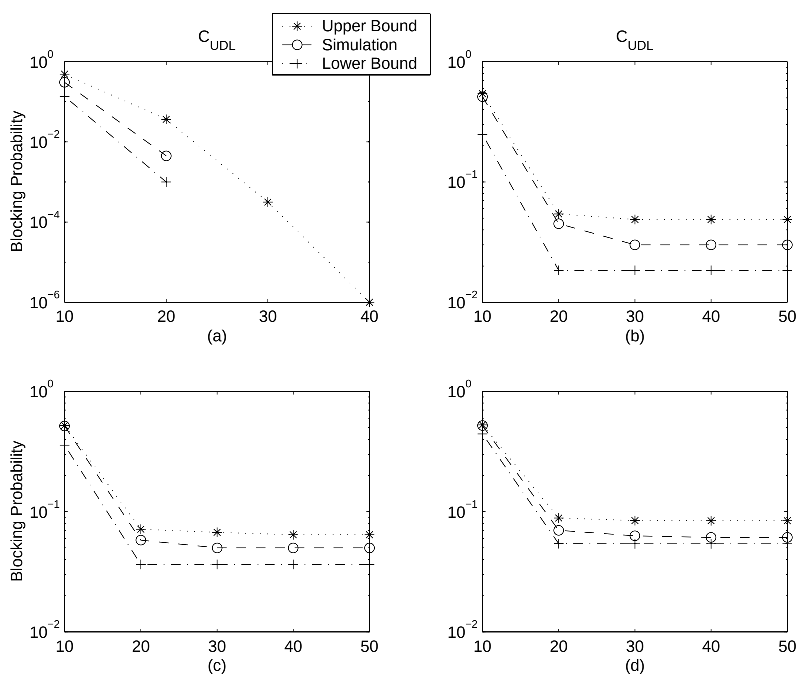

Figure 11. Call-blocking probability, 12-satellite orbit, $\lambda = 5$, $C_{ISL} = 10$, 2-community pattern

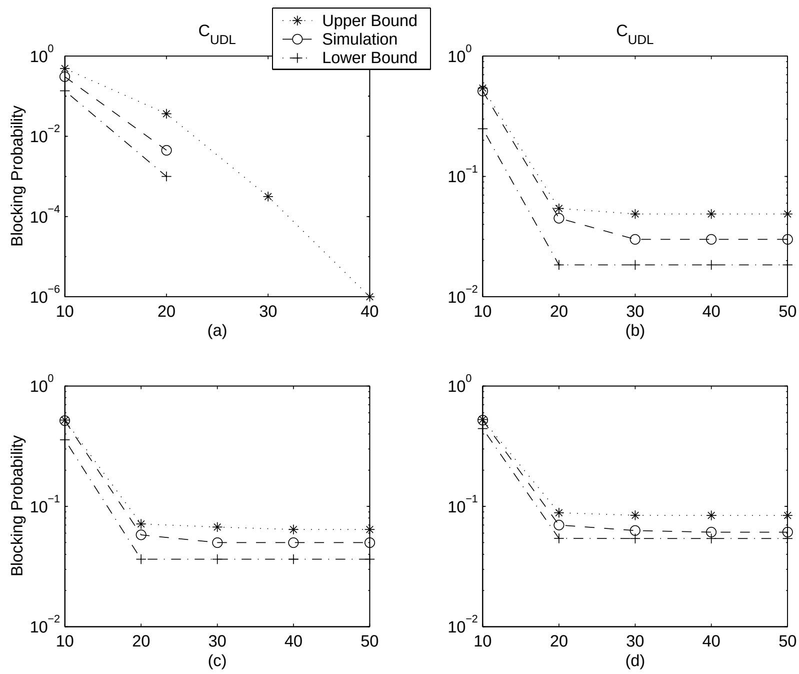

Figure 12. Call-blocking probability, 16-satellites, $\lambda = 5$, $C_{ISL} = 10$, uniform pattern

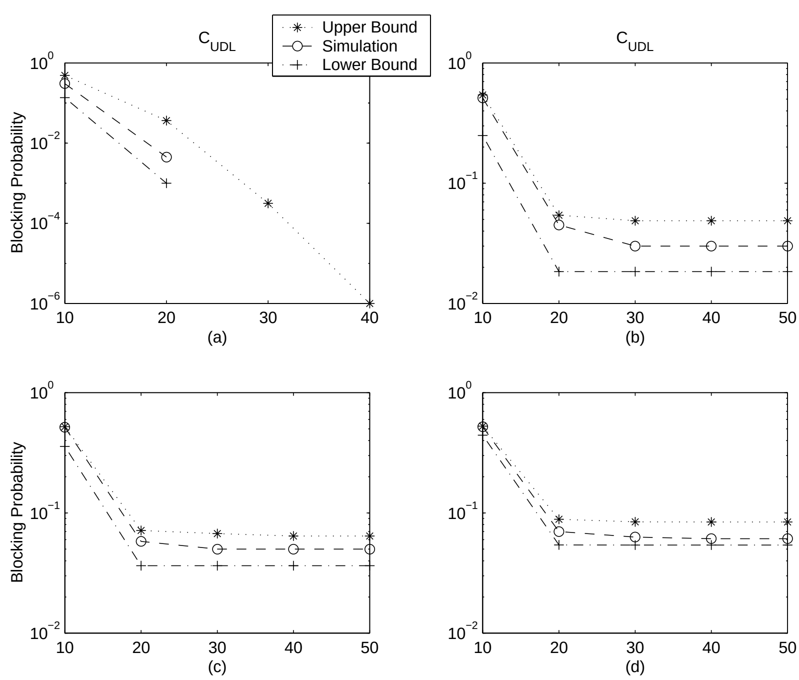

Figure 13. Call-blocking probability, 16-satellites, $\lambda = 5$, $C_{ISL} = 10$, locality pattern

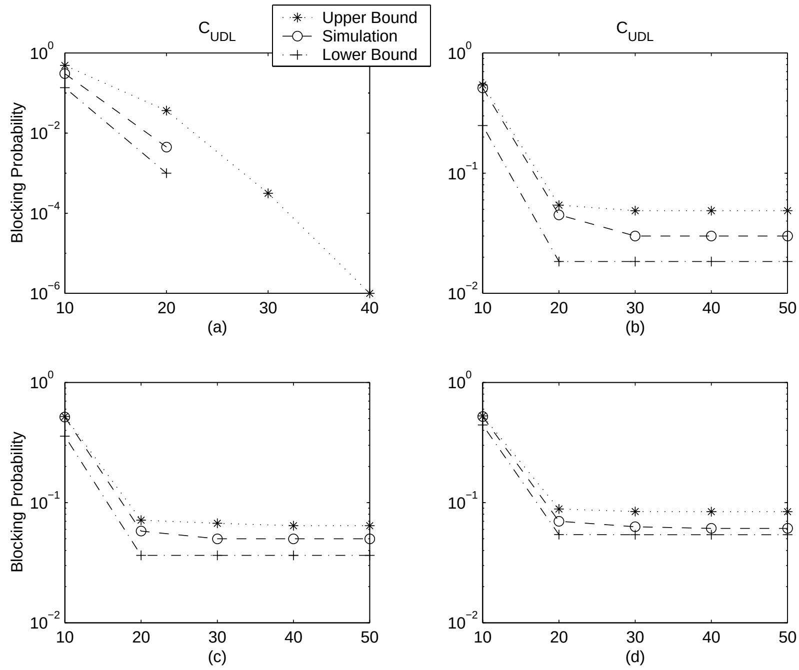

Figure 14. Call-blocking probability, 16-satellites, $\lambda = 5$, $C_{ISL} = 10$, 4-community pattern

4. Concluding Remarks

In this paper, we presented an analytical model for computing upper and lower bounds on blocking probabilities for LEO satellite constellations.

This method can be used to approximate the call-blocking probability on LEO satellite constellations with a large number of satellites and a large number of spot beams per satellite. With the aid of this method, these systems can be analyzed easily, representing each spot beam with a satellite and then using the bounds described in this paper.

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