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T. ENGİN TUNCER

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Optimum and Suboptimum Blind Channel and Symbol Estimation for SISO Channels

T. Engin TUNCER*
Electrical and Electronics Engineering Department,
Middle East Technical University, 06531, Ankara-TURKEY
e-mail: etuncer@metu.edu.tr

Abstract

We present three methods for blind channel and symbol identification from a single or multi-block observation. These methods are deterministic approaches suitable for the identification of quickly changing wireless channels. The first method uses the finite alphabet property and it has good performance even for noisy observations. It requires only a single data frame, which is a unique feature of the method. This method can also be used to identify the channel order. For multi-block observations, we present the maximal ratio combining cross relation (MRCCR) method. It is an optimum approach in terms of instantaneous SNR and is based on the cross relation and maximal ratio combining techniques. The equal gain combining cross relation (EGCCR) is a suboptimum alternative to MRCCR with low computational complexity. MRCCR and EGCCR methods require only two frames for estimation and their performances are considerably better compared to alternative methods when the number of data blocks is small. In addition, they can perform as long as the data block length $M \geq 1$. These three methods are especially well suited for trailing zero or burst transmission schemes.

Key Words: *Blind channel identification, SISO, maximal ratio combining, trailing zeros.*

1. Introduction

Blind channel identification is an efficient and effective alternative to training based systems. Once the channel is identified, one can take advantage of this information in the transmitter side by either preequalizing or precoding the transmitted signal. Such systems have been shown to increase the MSE performance by transmitting the channel information with a feedback channel from the receiver [1]. In mobile communications, the channel should be identified as quickly as possible due to fast changing channel characteristics. In this respect, there is a need for algorithms which can identify the channel from the limited number of observation samples.

In baud rate single-input single-output (SISO) channels, blind identification is a hard problem to solve for several reasons. There is only a limited number of samples from which to extract the information related to the channel and input symbols since the channel varies quickly. Observations are noisy, which limits the

*Corresponding author

performance for low SNR. In addition, the assumptions on the input and channel usually do not hold and statistical approaches can only have limited success due to the fact that there is only a limited number of observed samples. On the other hand, there is a considerable amount of deterministic information at the received signal, thanks to the LTI convolution operation. This information cannot be extracted by second or higher order statistical approaches including subspace methods.

It turns out that if the convolution output is completely available, as in the case of trailing zeros (TZ) or burst transmission schemes, one can find a variety of approaches to extract the information regarding the channel and the symbols. The number of available output data blocks mostly dictates the approach that one should follow. In this respect, we treat the blind identification problem for single and multi-data block cases separately. In the single block case, we assume that there is only a single output block available for the identification and the next block corresponds to another channel and input sequence. For the multi-block case where the number of blocks is greater than one, $N \geq 2$, we have more than one output block corresponding to the same channel with different input sequences.

In this paper, we will consider the single block case and propose a blind channel and symbol identification method by taking advantage of the finite alphabet property of the input sequence. There are different approaches for finding the input sequence when the input is from a finite alphabet. These approaches require long output sequences [2], [3] and many iterations for convergence or several data blocks for estimation. In [4], search space size is about $O(J^R)$ where J is the constellation size and $R \geq JL+1$ with channel order L. In [5], a Viterbi-like algorithm is used to limit the search space. Even though there exist alternative methods with lower computational complexity, their performances are acceptable only for high SNR. This is mainly due to the fact that the error surface is highly nonlinear and any search within a subset of the full search space may converge to a local minimum especially for low SNR. In our case, we propose a method which requires $a^M/2$ point search by using a sign blind least squares error expression. Here a is the number of symbols in the alphabet, which is even for most of the modulation types. In this respect, this is an exhaustive search with high computational complexity. Even though previous works point to such an approach, they always choose a computationally efficient alternative and the performance in this case is yet to be analyzed. It turns out that the proposed approach is robust to noise since noiseless input candidates are used during the evaluations. In addition, it can be used to identify the channel and input sequence as well as the channel order. In fact, this approach can be used to identify the channel order before the application of multi-block the methods proposed in this paper.

In the multi-block case, we take advantage of the fact that the channel remains the same during the channel coherence time and there are a number of output blocks available for identification purposes. In this respect, we modified and adapted the cross relation approach in [6] for SISO systems [7]. We show that only two data blocks are sufficient to estimate both the channel and the input sequences [7]. As long as there are no common zeros of the input sequences, blind identification is always possible. Furthermore, it is sufficient to have the input block length $M \geq 1$. It turns out that when the number of observed output blocks increases, we can obtain a set of input, $\mathbf{x}$, and channel, $\mathbf{h}$, copies each with a different unknown scale factor. It is possible to combine the input and channel copies in an optimum manner by using the maximal ratio combining (MRC) technique [8], [9]. In the original case, MRC optimally combines the received branch signals obtained from a set of antennas in a spatial diversity receiver. In our case, we have to combine the scaled versions of a sequence which are correlated with each other. Fortunately, it is shown that MRC can also be applied for correlated signals [10]. Optimum performance is obtained for such cases as well. In this paper, we propose the maximal ratio combining cross relation (MRCCR) method. This method combines the input, $\mathbf{x}$, and the channel, $\mathbf{h}$, estimates in an optimum manner in order to maximize the instantaneous SNR. MRCCR has certain advantages. It is an effective method for finding both the channel and symbol sequences. It requires only two data blocks for identification. This method has significantly better performance especially for short data blocks, which is an important advantage for wireless channels where the channel varies quickly over time.

We also present a suboptimum alternative of the MRCCR method. The equal gain combining cross relation (EGCCR) method coherently adds the input and channel copies in order to obtain a better estimate and suppress the noise components. This approach trades the MSE performance with computational complexity. Therefore it is a good candidate for channel identification for such cases where a rough estimate is desired with little computation. We will compare the MRCCR and EGCCR approaches with the subspace method (SSM) in [12]. This method is optimum in a least-squares sense but it has certain restrictions. Channel identification can be done when the number of data blocks, N, is greater than $M+L-1$ where M is the length of the data block. Also it is shown that SSM approaches the theoretical limits only when the number of blocks is large [13]. Several examples will be given in order to show the practical performances of the proposed approaches.

Notation: We denote the vectors and matrices as boldface lower and upper case letters respectively. $\mathbf{A}^{\mathbf{H}}$ is the transposed and conjugated matrix $\mathbf{A}$. $\mathbf{A}^{\dagger}$ is the pseudoinverse matrix of $\mathbf{A}$.

2. Single Block Blind Identification

Blind channel identification from a single data block in SISO systems is an important task for wireless communications due to short channel coherence time. This is a highly nonlinear problem which has an irregular error surface. This fact is shown in Figure 1, where the error for each possible input is plotted at SNR=10dB for a BPSK modulated signal. A global minimum is clearly observable, but there are several local minima as well. The solution for the above problem can be found by an exhaustive search for all possible inputs. The input sequence which returns the minimum error (or cost) is picked as the true input and the corresponding channel can be found by the Moore-Penrose pseudoinverse. This approach can be seen as the deterministic counterpart of maximum likelihood approaches. It is possible to find alternative methods to decrease the computational complexity by using iterative or adaptive approaches. However, such methods have a high chance of converging to a local minimum especially for low SNR. In fact, any attempt to decrease the search space might lead to a substantial degradation in performance. As is obvious from Figure 1, a single error in the input sequence might result in a considerably large error. In this respect, the proposed method has either no alternative or an alternative with limited performance for low SNR.

An important issue is how to choose the error function such that a unique solution (up to a scale factor) is always found. In the following part, we will show that this is possible for the noiseless case and its extension for a more general case can be easily done. Let us assume that $x_i(n)$ is a length M input sequence taken from a finite alphabet A (for example $A=\{1,-1\}$ for two level signals as in the BPSK modulation). $y_i(n)$ is the corresponding output for the i^{th} block. The input-output relation can be written in matrix form as

$$\mathbf{y}_i = \mathbf{H}_i\mathbf{x}_i + \mathbf{w}_i = \mathbf{X}_i\mathbf{h}_i + \mathbf{w}_i \tag{1}$$

where $\mathbf{H}_i$ and $\mathbf{X}_i$ are the $(L+M)\times M$ and $(L+M)\times(L+1)$ Toeplitz channel and data matrices,

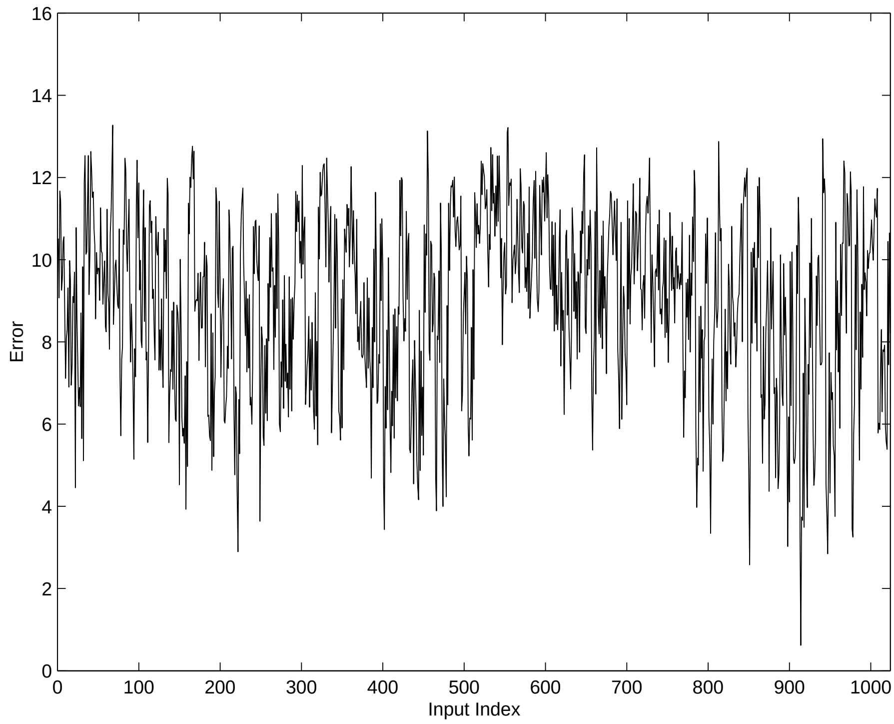

Figure 1. Error for different input sequences at SNR=10dB for BPSK modulation when the block length $M = 11$.

respectively. We assume that the channel order is L and $\mathbf{w}_i$ is the additive noise. Let $x_k(n)$ be one of the sequences obtained from the 2^{M-1} possible ones. The relation between $x_k(n)$ and the output, $y_i(n)$, can be built only with an error, i.e.

$$\mathbf{X}_k\mathbf{h}_k \approx \mathbf{y}_i \tag{2}$$

Here $\mathbf{h}_k$ is the channel corresponding to the sequence $\mathbf{x}_k$. Nevertheless we can find the channel $\mathbf{h}_k$ in a least squares sense with a minimum norm by

$$\mathbf{h}_k = \mathbf{X}_k^{\dagger}\mathbf{y}_i \tag{3}$$

where $\mathbf{X}_k^{\dagger}$ is the pseudoinverse of the matrix $\mathbf{X}_k$. We can define the error as

$$\mathbf{e}_k = \mathbf{X}_k\mathbf{h}_k - \mathbf{y}_i \tag{4}$$

Then the least squares error (LSE) can be defined as

$$E_k = \mathbf{e}_k^H\mathbf{e}_k \tag{5}$$

It is possible to prove that there is a unique solution up to a scale factor for $\mathbf{x}_i$ and $\mathbf{h}_i$ given y_i and the least squares error expression.

Lemma 1: Let A be the finite alphabetical set of all possible inputs $\{\mathbf{x}_k\}$. The number of distinct symbols in A is a. Given the length of the input sequence, M, there are $a^M/2$ possibilities for the input sequence in order to identify it within a scale factor given the output vector. If $\mathbf{y} = \mathbf{H}\mathbf{x}$ is the vector of convolution output of a LTI system corresponding to an input $\mathbf{x}$, then there is only a single input candidate (within a scale factor) from the input set which has the LSE $E_k = (\mathbf{y} - \mathbf{H}\mathbf{x_k}/\alpha)^H(\mathbf{y} - \mathbf{H}\mathbf{x_k}/\alpha) = 0$ where $\mathbf{x}_k = \alpha\mathbf{x}$ and $\alpha = \{1, -1\}$ is the scale factor.

Proof of the above lemma depends on the uniqueness of the zeros of the output sequence, which is a collection of the zeros of the input and the channel. LSE is always different from zero when the zeros of $x_k(n)$ are not the same as the zeros of $y(n)$. The error is zero only when all the zeros of $x_k(n)$ are the same as the zeros of $y(n)$ corresponding to the true input sequence.

An important issue is the scale factor or sign ambiguity. In the following section, we will use a LSE expression which is sign blind. In this respect, it is a very suitable error expression since we search half the search space or the size of the search space is $a^M/2$ instead of a^M.

The least squares error, E_k, can be found without explicitly implementing the convolution. Instead, E_k can be found from the singular value decomposition of $\mathbf{X}_k$ [11],

$$\mathbf{X}_k = \mathbf{U}_k \begin{bmatrix} \Sigma_k & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{V}_k^H \tag{6}$$

In the above equation, Σ_k is a diagonal matrix composed of the singular values of $\mathbf{X}_k$. Then the LSE expression is given as

$$E_k = \mathbf{y}_i^H(\mathbf{y}_i - \mathbf{X}_k\mathbf{h}_k) = \mathbf{y}_i^H\mathbf{U}_k(\mathbf{U}_k^H\mathbf{y}_i - \mathbf{U}_k^H\mathbf{X}_k\mathbf{V}_k\mathbf{V}_k^H\mathbf{h}_k) \tag{7}$$

The above error expression is obtained from the counterpart of the orthogonality principle and is based on the orthogonality of the left and right singular matrices $\mathbf{U}_k$ and $\mathbf{V}_k$ respectively. This expression is especially useful for the overdetermined matrices like $\mathbf{X}_k$.

Let us define

$$\mathbf{V}_k^H\mathbf{h}_k = \mathbf{b} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix}, \text{ and, } \mathbf{U}_k^H\mathbf{y}_i = \mathbf{c} = \begin{bmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \end{bmatrix} \tag{8}$$

where $\mathbf{b}_1$ and $\mathbf{c}_1$ are $(L+1) \times 1$ vectors as in [11]. The LSE can be written as

$$E_k = \mathbf{y}_i^H\mathbf{U}_k \begin{bmatrix} \mathbf{c}_1 - \Sigma_k\mathbf{b}_1 \\ \mathbf{c}_2 \end{bmatrix} \tag{9}$$

E_k is minimized when

$$\mathbf{c}_1 = \Sigma_k\mathbf{b}_1 \tag{10}$$

Therefore, LSE for the overdetermined set of equations is given as

$$E_k = \mathbf{y}_i^H\mathbf{U}_k \begin{bmatrix} \mathbf{0} \\ \mathbf{c}_2 \end{bmatrix} = \mathbf{y}_i^H\bar{\mathbf{U}}_k\bar{\mathbf{U}}_k^H\mathbf{y}_i \tag{11}$$

where $\bar{\mathbf{U}}_k$ is composed of the left singular vectors of $\mathbf{X}_k$,

$$\bar{\mathbf{U}}_k = \begin{bmatrix} \mathbf{u}_{L+1} & \mathbf{u}_{L+2} & \ldots & \mathbf{u}_{M+L-1} \end{bmatrix} \tag{12}$$

The LSE expression in (11) is a sign blind expression as opposed to those in (5) and (7) since the result does not change for $\mathbf{y}_i$ and $-\mathbf{y}_i$. This is mainly due to the choice in equation (10). Therefore it is sufficient to consider half of the search space and the true input sequence will be found within a sign ambiguity. Note that we do not need to compute $\bar{\mathbf{U}}_k$ each time. Instead, we can find the $\mathbf{B}_k = \bar{\mathbf{U}}_k \bar{\mathbf{U}}_k^H$ once offline and save it in advance. This can reduce the computational complexity dramatically at the expense of memory that is required to store the $\mathbf{B}_k$ matrices. For example, if we take $M = 16, L = 4$ and 16 bit representation is used, we need about 26 Mbytes of memory space for this purpose.

The true input sequence can be found as the one which minimizes the LSE, i.e.

$$\mathbf{x}_i = \arg\min_{\mathbf{x}_k} E_k \tag{13}$$

One of the important advantages of the proposed approach is that input is found with zero error when the SNR is sufficiently high. Therefore the channel can be identified by the pseudoinverse method effectively as in (3). The proposed approach has only some mild assumptions, which can be outlined as follows:

a1: Input is from a finite alphabet with known symbols

a2: Output sequence is completely available

a3: Input block length, M, is known.

Note that we do not need to know the channel order L. Instead a rough estimate will do the job since the proposed approach can be modified to predict the channel order as well. This point will be investigated in the following section. The blind identification method presented in this section has certain advantages. First, joint channel and symbol identification can be done. This is possible using a single output frame. The dimension of the search space does not depend on the channel order. Computational complexity can be reduced at the expense of memory and the algorithm is suitable for parallel computing and VLSI implementations. In case of VLSI implementations, real time processing is possible. The proposed approach is robust to noise since the evaluations are done with noiseless input sequences. In addition, the presented method works for any constellation as well as for nonstationary signals. The main disadvantage is the computational complexity. The search space increases exponentially with the block length.

We have performed several simulations in order to show the performance of the proposed method. The following examples summarize the cases of finite alphabet signals for BPSK and QPSK modulations.

Example 1: We simulated the performance of the proposed approach for BPSK modulation where the finite alphabet is composed of equiprobable -1 and 1 symbols or $A = \{-1, 1\}$. A Rayleigh fading channel is assumed with an order of L=4. The channel is normalized to have unit norm. Data block length is M=16. Average of the 100 trials is reported in Figure 2, where each trial a different channel and noise sequence are taken. The channel least squares error is small and the algorithm performs well at low SNR. It should be noted that since the computational complexity increases exponentially, it is hard to obtain bit-error rates for the proposed method. Nevertheless, we decreased the block length to M=9 and repeated the experiments for 5000 trials in order to get significant scores for the BER. Figure 3 shows the BER for this case.

Example 2: A QPSK modulated input signal is used in this case. The alphabet is composed of four symbols with the same symbol energy as in Example 1, namely $A = \{1 + j, 1 - j, -1 + j, -1 - j\}/\sqrt{2}$. The channel is chosen as a unit norm Rayleigh fading channel with an order of L=4 in each trial. Data block length is M=8. Average LSE of the 100 trials for the channel is given in Figure 4. The performance of the proposed algorithm is good for the QPSK modulation as well.

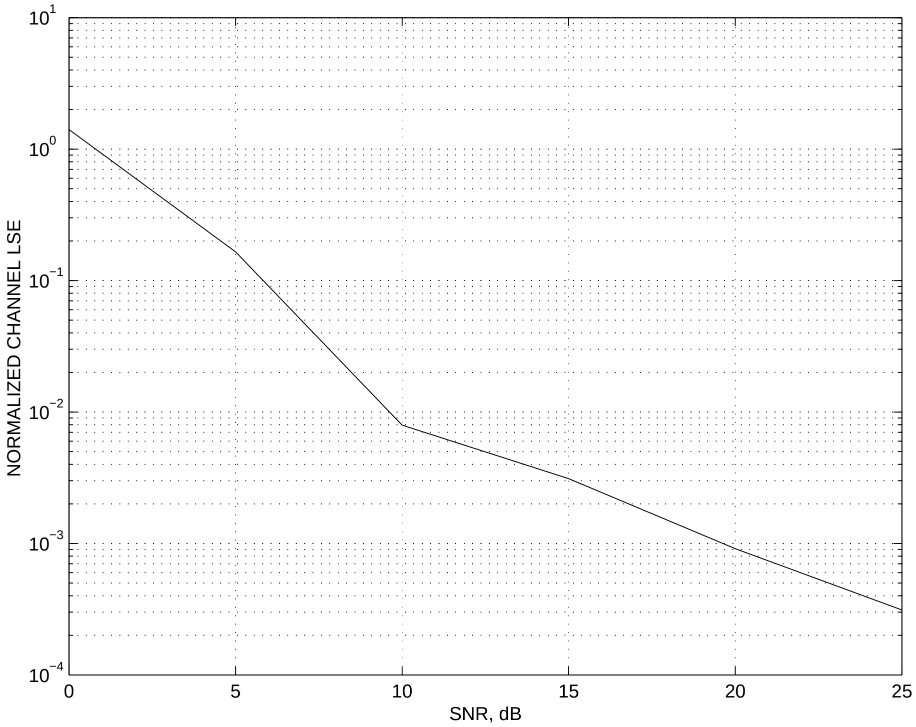

Figure 2. LSE performance for channel identification for BPSK modulation.

2.1. Channel Order Identification

The blind identification method of the previous section can be modified to determine the channel order as well. This can be achieved simply by considering different channel orders and their corresponding errors as in (11). The true channel order is identified as the one which returns the minimum LSE. In order to decrease the computational complexity, it is desired to have a rough estimate such as L_r for the channel order. Then we can search the neighborhood of this by comparing the LSE for each choice. Let $E_i^{L_p}$ be the minimum LSE for the i^{th} data block by using L_p as the channel order. Then the right channel order can be found as

$$L = \arg\min_{L_p} E_i^{L_p} = \arg\min_{L_p}(\arg\min_{x_k} E_k^{L_p}) \tag{14}$$

The computational complexity of the above expression is high. However, the performance of the proposed approach is significantly good, especially when the SNR is sufficiently high. In addition, this approach does not require any threshold or any additional parameter that should be known beforehand. In this respect, it is a straightforward method for the identification of channel order blindly. We have tested the performance of the proposed blind channel order identification approach. The parameters for the experiment are chosen as M=13, L=4 and the number of trials is 100. The channel is Rayleigh fading and the input is from the BPSK alphabet. Different channel orders are considered and $L_p \in [1, 7]$. Equation (14) is evaluated for each value of L_p. Figure 5 shows the percentage of the true identifications. It turns out that channel order is identified accurately after SNR=15dB.

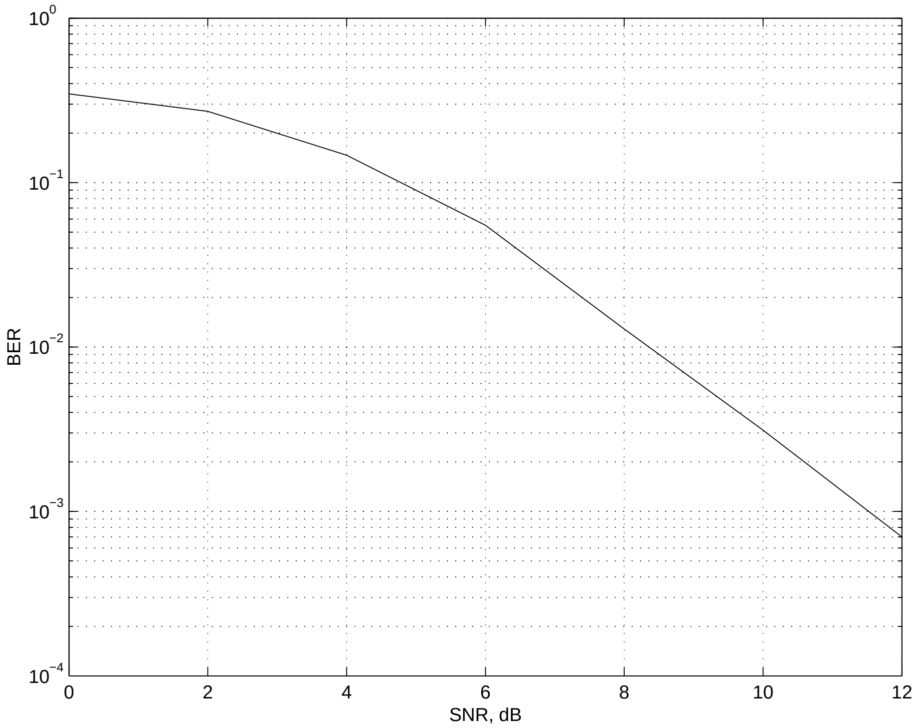

Figure 3. Bit error rate performance for the proposed method for BPSK modulation.

3. Multi-block Blind Identification: MRCCR Method

In a baud rate SISO system, the blind channel estimation problem is as follows. Given the $(M+L)\times 1$ channel output vector $\mathbf{y}$, find the $(L+1)\times 1$ channel vector $\mathbf{h}$ without knowing the $M\times 1$ input vector $\mathbf{x}$. The input-output relation is $\mathbf{y}=\mathbf{Hx}+\mathbf{u}$ where $\mathbf{H}$ is the Toeplitz channel matrix and $\mathbf{u}$ is the noise vector. For TZ transmission [1], the same relation is obtained at the receiver if we take $\mathbf{x}=\mathbf{Fs}$ where $\mathbf{F}$ is a $M\times M$ precoder matrix and $\mathbf{s}$ is a $M\times 1$ input symbol vector. A single equation like above cannot be used to find $\mathbf{h}$ unless the input is from a finite alphabet. In the multi-block case, we assume arbitrary input sequences and therefore a more general problem is considered. The SSM in [12] requires at least $M+L$ equations in the same form as above. However, if we employ the CR method [6], the minimum number of data blocks is only two to solve the unknowns $\mathbf{x}$ and $\mathbf{h}$. Let us assume that $\mathbf{N}$ output vectors, $\mathbf{y}_i$, are given corresponding to the input vectors $\mathbf{x}_i$, $(i=0,1,\ldots,N-1)$. Figure 6 shows the input and output blocks for the TZ transmission scheme where there is no inter block interference (IBI). The received signal $\mathbf{y}_i$ for the i^{th} block can be written as in the following equations:

$$y_i(n)=\sum_{m=0}^{M-1} x_i(n)h(n-m)+u_i(n) \tag{15}$$

Here n is the time index. The same equation in vector form is

$$\mathbf{y}_i=\mathbf{Hx}_i+\mathbf{u}_i \tag{16}$$

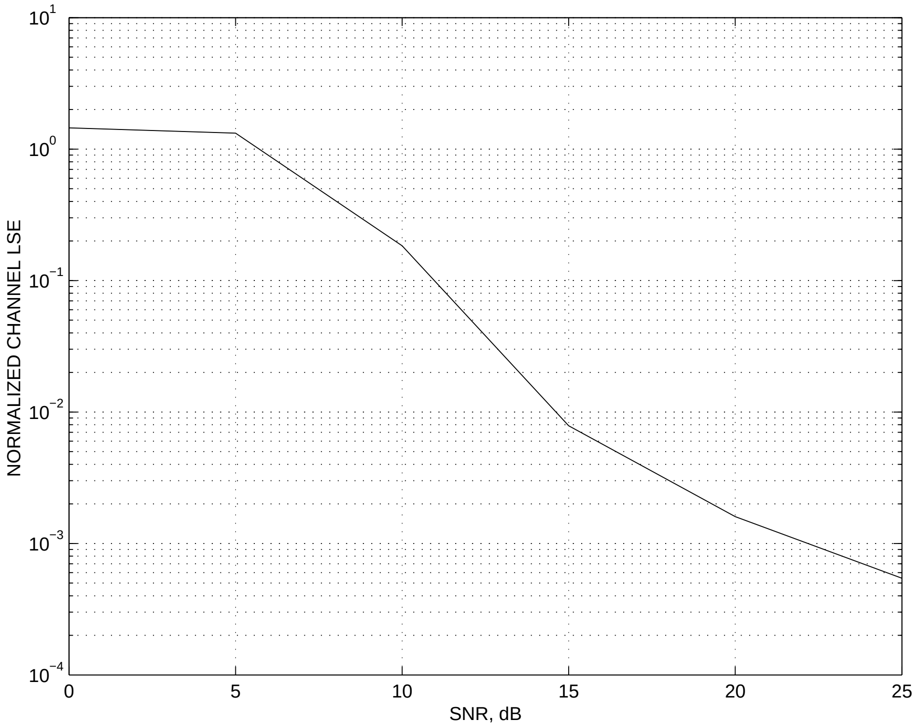

Figure 4. LSE performance for channel identification for QPSK modulation.

It is possible have $N(N-1)/2$ couples of $(\mathbf{y}_i, \mathbf{y}_j)$ with $i \neq j$. We can use the CR method for each of these couples in order to find $(\hat{\mathbf{x}}_i^j, \hat{\mathbf{x}}_j^i)$ couples. Here

$$\hat{\mathbf{x}}_i^j = c_i^j \mathbf{x}_i + \mathbf{v}_i^j \qquad i = 0, 1, \ldots, N-1 \quad j = 0, 1, \ldots, N-1 \quad j \neq i \tag{17}$$

and $\hat{\mathbf{x}}_i^j$ represents the estimate of $\mathbf{x}_i$ obtained from the $(\mathbf{y}_i, \mathbf{y}_j)$ couple together with a scale factor c_i^j and the noise term $\mathbf{v}_i^j$. In the CR approach, a linear relation between the observed vectors is obtained using the output couples. Let us assume that two observations are given, namely $\mathbf{y_i} = \mathbf{H}\mathbf{x_i} + \mathbf{v_i}$ and $\mathbf{y_j} = \mathbf{H}\mathbf{x_j} + \mathbf{v_j}$. Let $\mathbf{Y}_i$ and $\mathbf{Y}_j$ be the $(2M + L - 1) \times M$ Toeplitz matrices corresponding to $\mathbf{y}_i$ and $\mathbf{y}_j$ respectively. The following equation can be written neglecting the noise terms, i.e.

$$[\mathbf{Y_i} \quad -\mathbf{Y_j}] \begin{bmatrix} \mathbf{x_j} \\ \mathbf{x_i} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \tag{18}$$

Above is a homogeneous equation of the form

$$\mathbf{Y}_{ij} \mathbf{x}_{ij} = \mathbf{0} \tag{19}$$

Note that the equation in (18) is different than the one in [6] since $\mathbf{Y}_i$ and $\mathbf{Y}_j$ are full Toeplitz matrices as opposed to the ones in [6].

Remark: In both MRCCR and SSM methods, we need to solve a homogeneous equation like $\mathbf{A}\mathbf{x} = \mathbf{0}$. This equation can be solved up to a scale factor by two alternative approaches when the dimensionality of the null space of $\mathbf{A}$ is one. These two solutions are

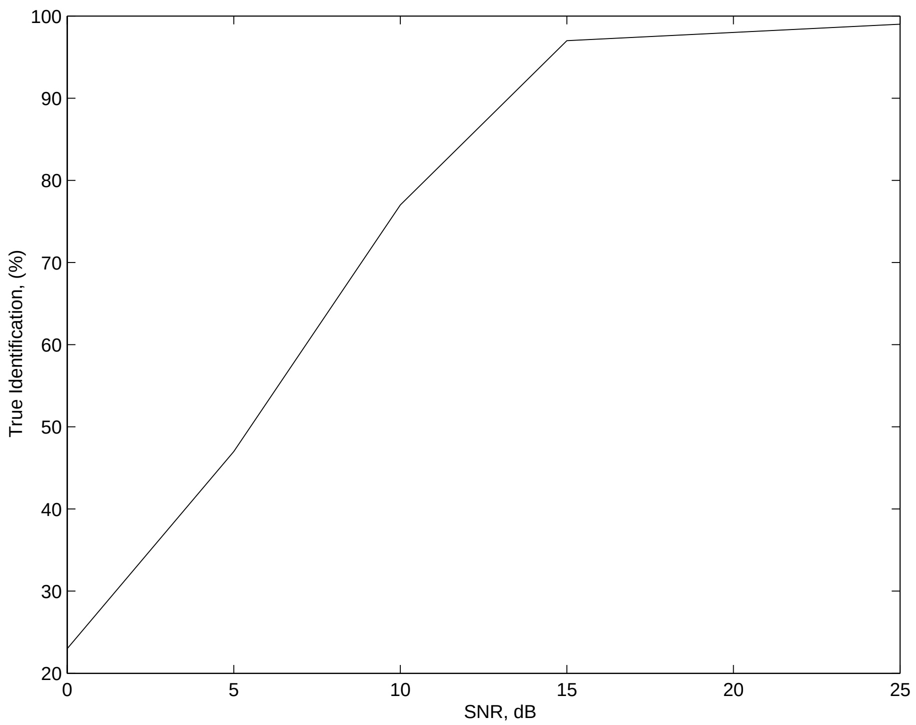

Figure 5. Percentage of the true channel identification for the proposed method.

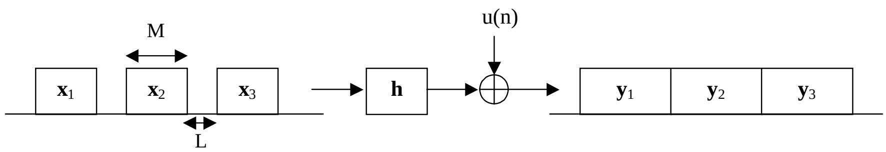

Figure 6. TZ transmission scheme and data blocks.

S1: The solution for $\mathbf{x}$ (within a scale factor) is the eigenvector corresponding to the null space of $\mathbf{A}$.
S2: Partitioning $\mathbf{A}$ and assuming that the first element of $\mathbf{x}$ is known,

$$[\mathbf{a_1} \quad \mathbf{A_1}] \begin{bmatrix} 1 \\ \bar{\mathbf{x}} \end{bmatrix} = \mathbf{0} \tag{20}$$

and

$$\mathbf{A}_1 \bar{\mathbf{x}} = -\mathbf{a}_1 \tag{21}$$

Then the least-squares optimum solution is

$$\bar{\mathbf{x}} = (\mathbf{A}_1^H \mathbf{A}_1)^{-1} \tag{22}$$

The solutions in **S1** and **S2** are both least-squares optimum. However, they are solutions to two different problems. It turns out that **S2** is not as robust to noise as **S1**. This is mainly due to the fact that $\mathbf{a}_1$ is a noisy observation as well as $\mathbf{A}_1$ and there is some noise enhancement after the Moore-Penrose pseudoinverse operation. The performance of the pseudoinverse approach for noisy observations is also reported in [15].

In the following part, we will prove the existence and uniqueness of the solution up to a scale factor. Before the proof, we will present a simple example for an extreme case, namely for a unit length input, to show that the proposed method can be used even for very short input sequences.

Example 3: Let us take $\mathbf{h} = [1 \;\; 2 \;\; 3 \;\; 4]$ and the two inputs $\mathbf{x}_1 = [1]$ and $\mathbf{x}_2 = [2]$. Since we assume that zero padding is done between each data block, we have the full convolution sequences, namely $\mathbf{y}_1 = [1 \;\; 2 \;\; 3 \;\; 4]$ and $\mathbf{y}_2 = [2 \;\; 4 \;\; 6 \;\; 8]$. We can construct the matrix $\mathbf{Y}_{12}$ as

$$\mathbf{Y}_{12} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} \tag{23}$$

The null eigenvector of $\mathbf{Y}_{12}$ is $\mathbf{x}_{12} = [0.8944 \;\; -0.4472]$. Therefore $\mathbf{x}_1$ and $\mathbf{x}_2$ are found as -0.4472 and 0.8944 respectively. These are the true values except for a scale factor.

Theorem 1 *(Existence and uniqueness) Let $\mathbf{y}_1 = \mathbf{H}\mathbf{x}_1$ and $\mathbf{y}_2 = \mathbf{H}\mathbf{x}_2$ be two outputs for a LTI system. Let $\mathbf{Y}_1$ and $\mathbf{Y}_2$ be the Toeplitz matrices corresponding to the $\mathbf{y}_1$ and $\mathbf{y}_2$ vectors respectively. $\mathbf{x}_1$ and $\mathbf{x}_2$ have no common zeros. Then $\mathbf{Y}_{12} = [\mathbf{Y}_1 \;\; -\mathbf{Y}_2]$ is column-rank deficient by one and $\mathbf{Y}_{12}\mathbf{x}_{12} = \mathbf{0}$ always has a unique solution within a scale factor where the solution is the null eigenvector of $\mathbf{Y}_{12}$.*

Proof: The existence of the solution can be easily seen when we consider the zeros of the output sequences. Consider the following convolution relations:

$$y_1(n) = h(n) * x_1(n), \qquad y_2(n) = h(n) * x_2(n) \tag{24}$$

Given the above relations, one can always find $x_1(n)$ and $x_2(n)$ sequences such that

$$y_1(n) * x_2(n) = y_2(n) * x_1(n) \tag{25}$$

The uniqueness of the solution can be proved by contradiction. Assume that there are $\bar{x}_1(n)$ and $\bar{x}_2(n)$ sequences which also satisfy equation (25) and $\bar{x}_1(n) \neq x_1(n)$ and $\bar{x}_2(n) \neq x_2(n)$. Then we can write

$$y_1(n) * \bar{x}_2(n) = y_2(n) * \bar{x}_1(n) \tag{26}$$

However, if two sequences are equal, then the zeros of those sequences should be the same. The zeros of $y_1(n)$ are composed of the zeros of $x_1(n)$ and $h(n)$. The zeros of $y_2(n)$ are composed of the zeros of $x_2(n)$ and $h(n)$. Then the equation in (26) is satisfied only if $\bar{x}_1(n) = \alpha_1 x_1(n)$ and $\bar{x}_2(n) = \alpha_2 x_2(n)$. This concludes the proof.

Note that if $\mathbf{Y}_{12}$ is obtained from arbitrary Toeplitz matrices $\mathbf{Y}_1$ and $\mathbf{Y}_2$, it is not column rank deficient by one. Rank deficiency occurs only when $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are obtained from sequences which satisfy the equation in (24). In this case, columns of $\mathbf{Y}_{12}$ have common zeros due to $\mathbf{h}$.

We employed the approach in **S1** for both MRCCR and SSM methods and the solution for the equation (19) is the eigenvector corresponding to the null space of $\mathbf{Y}_{ij}$ and $\mathbf{x}_{ij}$ is identified up to a scale factor. If we apply the CR method as above for each couple $(\mathbf{y}_i, \mathbf{y}_j)$, we obtain $(N-1)$ $\mathbf{x}_i$ estimates, $\hat{\mathbf{x}}_i^l$,

$(l = 0, 1, \ldots, N-2)$ for each i $(i = 0, 1, \ldots, N-1)$. It is possible to improve the instantaneous SNR using the maximal ratio combining idea for the estimates of each data block. The estimated symbol vectors for the i^{th} data block can be written in matrix form as

$$\hat{\mathbf{X}}_i = \left[\hat{\mathbf{x}}_i^0 \;\; \hat{\mathbf{x}}_i^1 \ldots \hat{\mathbf{x}}_i^{N-2}\right] = \mathbf{x}_i \mathbf{c}_i^H + \mathbf{V}_i = [c_0\mathbf{x}_i \;\; c_1\mathbf{x}_i \ldots c_{N-2}\mathbf{x}_i] + [\mathbf{v}_0 \;\; \mathbf{v}_1 \ldots \mathbf{v}_{N-2}] \tag{27}$$

For simplicity, we reorganized and dropped the upper indexes for the scale factors $\mathbf{c}_i^H = [c_0 \quad c_1 \ldots c_{N-2}]$ and noise terms in $\mathbf{V}_i$.

We will find the optimum weighting vector $\mathbf{w}_i$ for $\hat{\mathbf{X}}_i$ in order to combine $(N-1)$ estimates of $\mathbf{x}_i$ in an optimum manner and obtain the symbol vector $\tilde{\mathbf{x}}_i$,

$$\tilde{\mathbf{x}}_i = \hat{\mathbf{X}}_i \mathbf{w}_i = \mathbf{x}_i \mathbf{c}_i^H \mathbf{w}_i + \mathbf{V}_i \mathbf{w}_i \tag{28}$$

Figure 7 shows the general structure of the MRC system. MRC output for the i^{th} block at time instant k is $\tilde{x}_i(k)$,

$$\tilde{x}_i(k) = [0 \quad 0 \ldots 1 \ldots 0]\,\tilde{\mathbf{x}}_i = x_i(k)\mathbf{c}_i^H \mathbf{w}_i + \tilde{\mathbf{v}}_k \mathbf{w}_i \tag{29}$$

where $\tilde{\mathbf{v}}_k$ is the k^{th} row vector of $\mathbf{V}_i$.

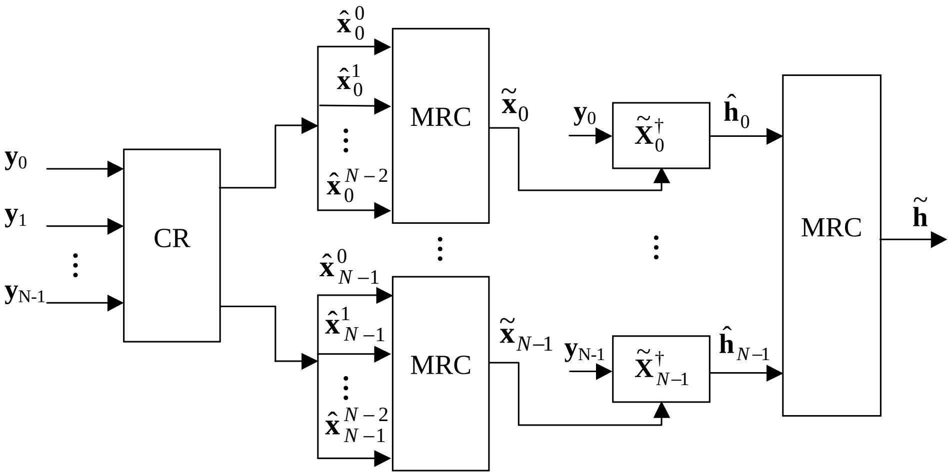

Figure 7. MRCCR method system structure.

Signal power at time instant k is

$$S = P_{x_i} \mathbf{w}_i^H \mathbf{c}_i \mathbf{c}_i^H \mathbf{w}_i \tag{30}$$

where P_{x_i} is the average signal power. If we assume that the column vectors of $\mathbf{V}_i$ are i.i.d. white Gaussian vectors with variances $\sigma_{\tilde{v}}^2 = \sigma_v^2$, the noise power, N, can be found as

$$N = E\{|\tilde{\mathbf{v}}_k \mathbf{w}_i|^2\} = \mathbf{w}_i^H E\{\tilde{\mathbf{v}}_k \tilde{\mathbf{v}}_k^H\} \mathbf{w}_i = \frac{1}{\sigma_v^2} \mathbf{w}_i^H \mathbf{w}_i \tag{31}$$

Then the instantaneous SNR can be written similar to the one in [14] as

$$SNR_{inst} = \frac{S}{N} = \frac{P_{x_i}}{\sigma_v^2} \frac{\mathbf{w}_i^H \mathbf{c}_i \mathbf{c}_i^H \mathbf{w}_i}{\mathbf{w}_i^H \mathbf{w}_i} \tag{32}$$

We can use the Cauchy-Schwarz inequality to find the optimum weight vector, i.e.

$$\left|\mathbf{w}_i^H \mathbf{c}_i\right|^2 \leq \left(\mathbf{w}_i^H \mathbf{w}_i\right)\left(\mathbf{c}_i^H \mathbf{c}_i\right) \tag{33}$$

For the above expression, equality holds if

$$\mathbf{w}_i = \alpha \mathbf{c}_i \tag{34}$$

Under this condition, SNR_{inst} in equation (32) becomes

$$SNR_{inst} = \frac{P_{x_i}}{\sigma_v^2} \mathbf{c}_i^H \mathbf{c}_i \tag{35}$$

At this point, we can find the maximum likelihood (ML) estimate of the scale factor vector $\mathbf{c}_i$, by considering the white Gaussian i.i.d. noise terms for each of the estimates in the columns of $\hat{\mathbf{X}}_i$ as given in equation (27). The k^{th} column vector in the noise matrix $\mathbf{V}_i$ is

$$v_k(m) = \hat{x}_i^k(m) - c_k x_i(m) \qquad m = 0, 1, \ldots, M-1 \tag{36}$$

Then the likelihood function can be written in terms of the conditional and marginal densities as

$$f_{\hat{x}_i^k, x_i; c_k}(\hat{x}_i^k, x_i; c_k) = f_{\hat{x}_i^k | x_i}(\hat{x}_i^k | x_i) f_{x_i}(x_i) \tag{37}$$

where

$$f_{\hat{x}_i^k | x_i}(\hat{x}_i^k | x_i) = \prod_{m=0}^{M-1} \frac{1}{\pi \sigma_v^2} e^{\frac{-\left|\hat{x}_i^k(m) - c_k x_i(m)\right|^2}{\sigma_v^2}} \tag{38}$$

We can take the log likelihood function and its derivative with respect to c_k^* as zero. Since $f_{x_i}(x_i)$ does not depend on c_k, we are left with only the terms due to $f_{\hat{x}_i^k|x_i}(\hat{x}_i^k|x_i)$. Then c_k factors are obtained as

$$c_k = \frac{\sum_{m=0}^{M-1} \hat{x}_i^k(m) x_i^*(m)}{\sum_{m=0}^{M-1} x_i(m) x_i^*(m)} \tag{39}$$

If we consider the definition of $\mathbf{c}$ as in equation (27), ML estimate $\mathbf{c}_{ML}$ is given as

$$\mathbf{c}_{ML} = \frac{\hat{\mathbf{X}}_i^H \mathbf{x}_i}{\mathbf{x}_i^H \mathbf{x}_i} \tag{40}$$

When this ML estimate is placed in equation (35),

$$SNR_{inst} = \frac{P_{x_i}}{\sigma_v^2} \frac{\mathbf{x}_i^H \hat{\mathbf{X}}_i \hat{\mathbf{X}}_i^H \mathbf{x}_i}{\left|\mathbf{x}_i^H \mathbf{x}_i\right|^2} \tag{41}$$

When a unit norm estimate for the symbol vector $\mathbf{x}_i$ is searched, we can use the Rayleigh principle. SNR_{inst} is maximized if $\mathbf{x}_i$ is the eigenvector corresponding to the largest eigenvalue of $\tilde{\mathbf{R}}_x = \hat{\mathbf{X}}_i \hat{\mathbf{X}}_i^H$. Therefore the optimum symbol estimate for the i^{th} data block, $\tilde{\mathbf{x}}_i$, is found from $\tilde{\mathbf{R}}_x$ according to the MRC approach.

The above analysis and the result can be applied for finding the optimum estimates for other blocks as well, i.e. $\tilde{\mathbf{x}}_i$, $i = 0, 1, \ldots, N-1$. Once the optimum symbol estimates are found, the next step is to find the corresponding channel estimates $\hat{\mathbf{h}}_i$, given the $(\tilde{\mathbf{x}}_i, \mathbf{y}_i)$ couples as shown in Figure 7. This can be done using the Moore-Penrose pseudoinverse for $i = 0, 1, \ldots, N-1$,

$$\hat{\mathbf{h}}_i = \left(\tilde{\mathbf{X}}_i^H \tilde{\mathbf{X}}_i\right)^{-1} \tilde{\mathbf{X}}_i^H \mathbf{y}_i = \tilde{\mathbf{X}}_i^{\dagger} \mathbf{y}_i \tag{42}$$

where $\tilde{\mathbf{X}}_i$ is the Toeplitz symbol matrix and $\hat{\mathbf{h}}_i$ is the least-squares optimum estimate of the channel for the i^{th} data block. Then the MRC approach can be used similarly for $\hat{\mathbf{h}}_i$. We form the channel matrix, $\hat{\mathbf{H}}$, which has $\hat{\mathbf{h}}_i$ as the column vectors, and obtain an expression similar to equation (41):

$$SNR_{inst} = \frac{P_h}{\sigma_{\hat{v}}^2} \frac{\mathbf{h}^H \hat{\mathbf{H}} \hat{\mathbf{H}}^H \mathbf{h}}{\left|\mathbf{h}^H \mathbf{h}\right|^2} \tag{43}$$

Therefore optimum channel estimate $\mathbf{h}_{opt}$ is obtained as the eigenvector corresponding to the largest eigenvalue of $\tilde{\mathbf{R}}_h = \hat{\mathbf{H}} \hat{\mathbf{H}}^H$.

4. Multi-block Blind Identification: Equal Gain Combining

The MRCCR method is optimum and has very good performance in channel identification. However, in certain cases, a suboptimum but computationally efficient alternative, such as EGCCR, is desired. Equal gain combining (EGC) [9],[17], is a very simple and computationally efficient way of combining the symbol and channel copies. The main idea is to equalize the gains of each symbol and channel copies and add them coherently. This approach increases the SNR by the number of copies. Below we will explain the approach for combining the input and channel estimates by assuming that these estimates are obtained by CR as discussed in the previous section. Let us rewrite the equation in (17),

$$\hat{x}_i^l(n) = c_i^l x_i(n) + v_i^l(n) \qquad i = 0, 1, \ldots, N-1 \quad l = 0, 1, \ldots, N-2, \tag{44}$$

and $n = 0, 1, \ldots, M-1$. Above c_i^l is a scale factor with gain and phase terms, i.e.

$$c_i^l = a_{i,l} e^{j\theta_{i,l}} \tag{45}$$

We can normalize each symbol copy with its first term, i.e.

$$\bar{x}_i^l(n) = \frac{\hat{x}_i^l(n)}{\hat{x}_i^l(0)} = \beta_l x_i(n) + \bar{v}_l(n) \tag{46}$$

Therefore the first term of each symbol sequence becomes identical after this normalization. If we ignore the effects of noise and take $\beta_l = \beta$, we obtain in phase symbol sequences, i.e.

$$\bar{x}_i^l(n) = \beta x_i(n) + \bar{v}_l(n) \tag{47}$$

Furthermore, noise components $\bar{v}_l(n)$ can be assumed as i.i.d. white Gaussian terms with variance $\sigma_{\bar{v}}^2$. Then the output of the equal gain combiner is

$$\tilde{x}_i(n) = \frac{1}{N-1} \sum_{l=0}^{N-2} \beta x_i(n) + \bar{v}_l(n) \tag{48}$$

The SNR at the output of equal gain combiner becomes

$$SNR_{eq} = \frac{(N-1)|\beta|^2 \sigma_x^2}{\sigma_{\bar{v}}^2} \tag{49}$$

where we assumed that the signal energy is σ_x^2. When we compare the SNR at the output of EGC, SNR_{eq}, with the SNR for a single symbol sequence, the improvement is by (N-1). We can also combine the copies of the channel vector in a similar manner and the channel estimate for equal gain combining becomes

$$\tilde{h}_i(n) = \frac{1}{N} \sum_{i=0}^{N-1} \gamma h_i(n) + \check{v}_i(n) \tag{50}$$

In this case, SNR improvement for the channel estimate is $10log_{10}(N)$ dB.

5. Performance Evaluation for MRCCR and EGCCR Methods

We compared the performances of MRCCR and EGCCR methods with the SSM method in [12]. We also implemented the approximate theoretical MSE in [13] in order to see the ideal case. In our comparisons, we used the LSE for simplicity. Note that this performance criterion favors the SSM method, since it is optimum with respect to this criterion. The channel estimate obtained by any of the three methods has a scale factor. The scale factor ambiguity is solved by normalization with a complex factor ρ,

$$\rho = \frac{h(k_{max})}{h_{est}(k_{max})}, \qquad |h(k_{max})| \geq |h(k)| \qquad \forall k \neq k_{max} \tag{51}$$

where $h_{est}(k)$ is the channel estimate obtained by any of the three methods. Then the normalized LSE for the channel is found as

$$NLSE = \frac{\|\mathbf{h}_{est} - \mathbf{h}\|^2}{\|\mathbf{h}\|^2} \tag{52}$$

In all of our simulations, input is chosen from a QPSK symbol set, and noise is a complex white Gaussian signal. The channel is also complex with Gaussian distributed taps leading to a Rayleigh fading channel. Channel variance is taken as $1/(L+1)$. SSM methods employ an IFFT precoder matrix $\mathbf{F}$ which has unit norm. In MRCCR and EGCCR methods, there is no precoding. In each of the experiments, except for the first one, we had 100 trials with different channel, input and noise signals. The results are reported as the average of these experiments.

Example 4: In this case, we chose the number of data blocks as $N = 2$ and the number of trials as 1000. Since the SSM method fails to give any estimate for this case, we will report only the performances of the MRCCR and EGCCR methods. The channel order is taken as $L = 4$, and the data block length is $M = 12$. Figure 8 shows the LSE performance for this case. The MRCCR and EGCCR methods perform similarly for high SNR when the number of data blocks is small. The symbol error rate is shown in Figure 9 for this case.

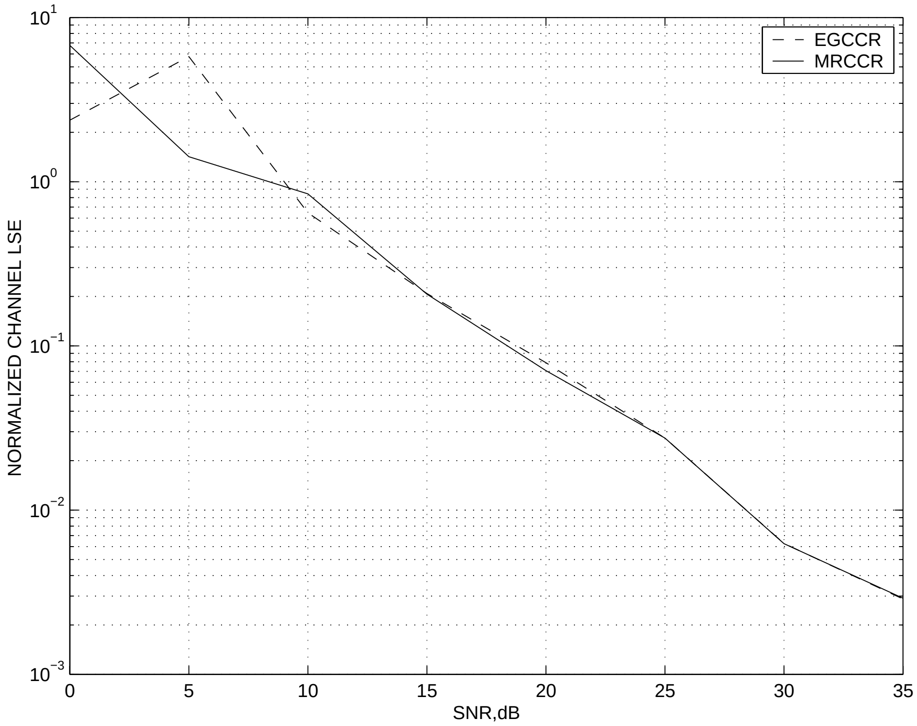

Figure 8. LSE performance of MRCCR and EGCCR methods for only two blocks (N=2).

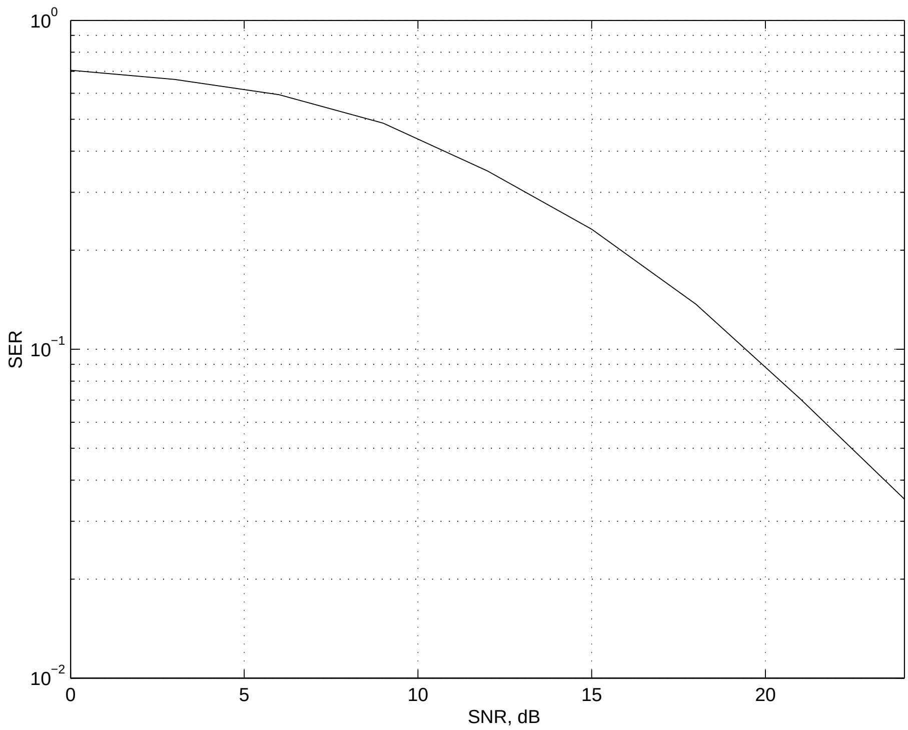

Figure 9. Symbol error rate for the MRCCR method in case of QPSK modulation (N=2).

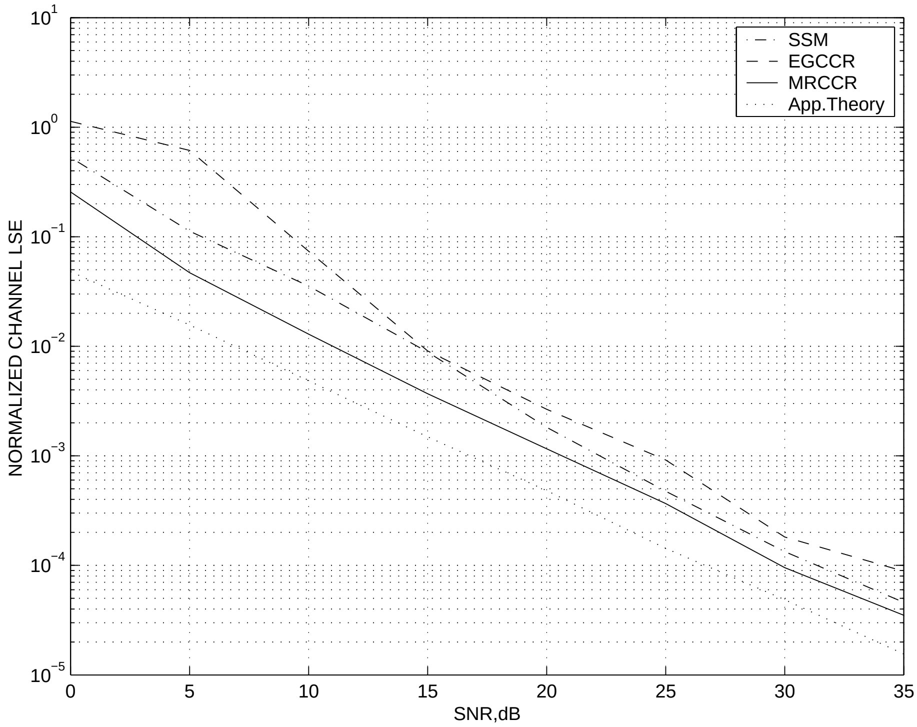

Figure 10. LSE performance of MRCCR, EGCCR and SSM methods for N=M+L=16.

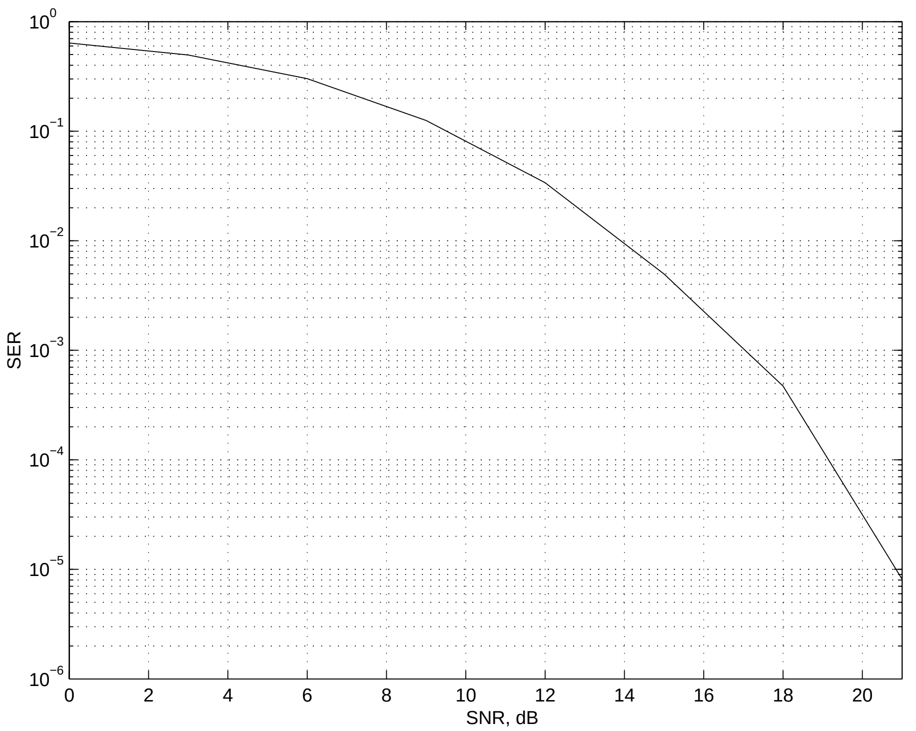

Figure 11. Symbol error rate for the MRCCR method in case of QPSK modulation and N=M+L=16.

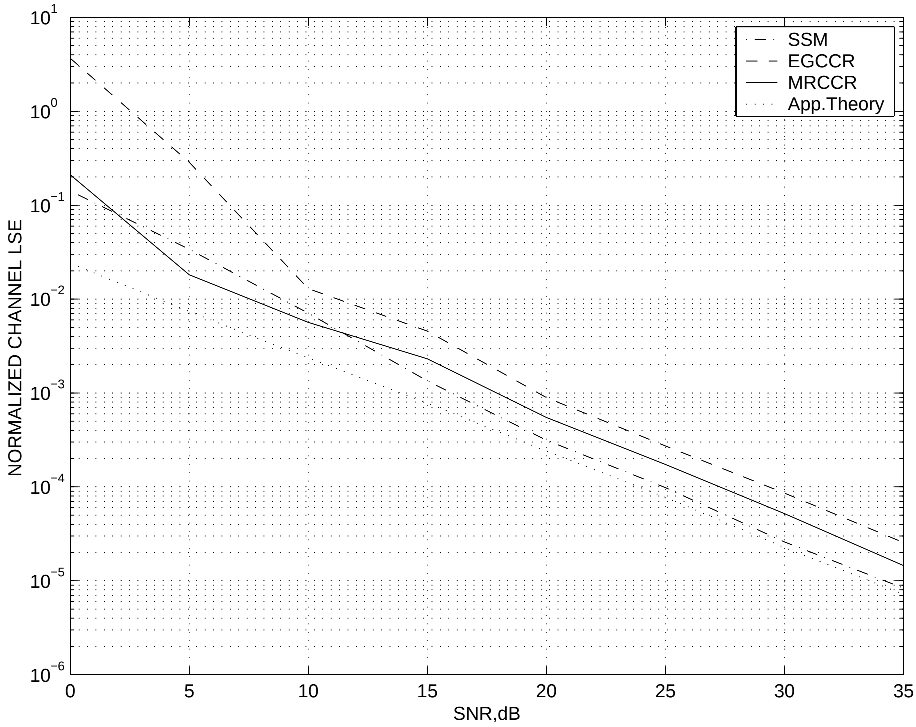

Figure 12. LSE performance of MRCCR, EGCCR and SSM methods for N=2(M+L).

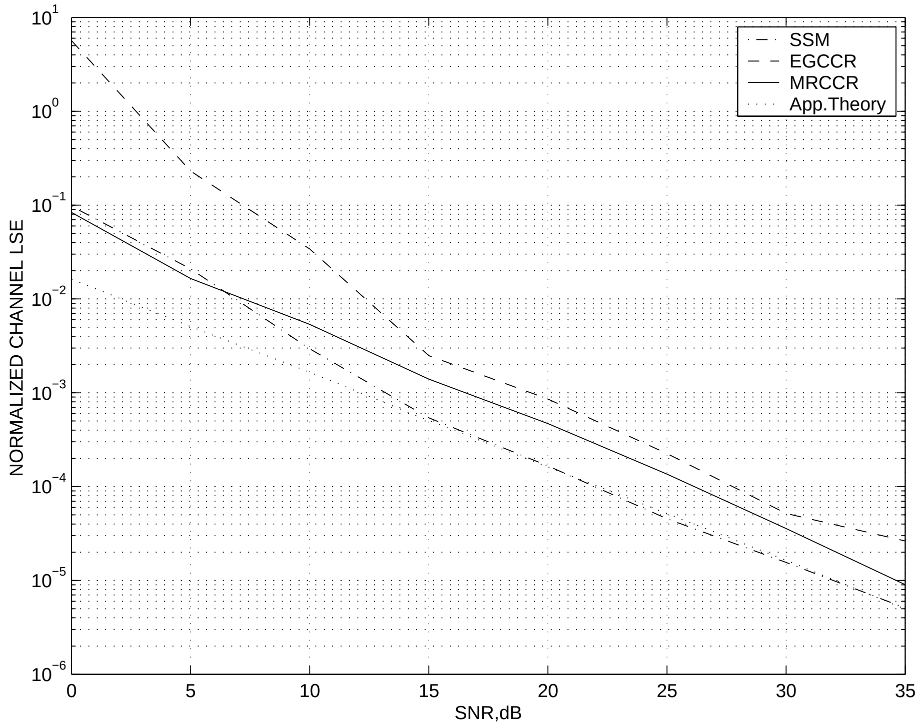

Figure 13. LSE performance of MRCCR, EGCCR and SSM methods for N=3(M+L).

Example 5: We took the number of data blocks $N \geq M+L$ and all three methods as well as the approximate MSE theoretical limit were considered. The channel order is $L = 4$, and the data block length $M = 12$. Figure 10 shows the LSE performance of the three methods when $N = M+L$. MRCCR has the best response followed by SSM and EGCCR. The LSE performance of the MRCCR method has approximately the same distance from the approximate theoretical limit for all SNR values. The symbol error rate is obtained by having 700 trials, as shown in Figure 11. When the number of blocks is increased, the performance of the SSM method gets closer to the theoretical limit. Figure 12 shows the LSE performance when $N = 2(M+L)$ is taken. As the number of blocks is increased, the MRCCR method has either an equal or better performance for low SNR. In Figure 13, $N = 3(M + L)$ is chosen and the performance of the SSM method is better except for low SNR.

Example 6: We also investigated the effects of increasing the data block and channel length on the proposed methods. In this case, channel order is selected as $L = 8$ and the block length is $M = 24$. The number of blocks is taken as $N = M+L$. We had 100 trials for this case as well. Figure 14 shows the LSE performances of all three methods. It turns out that SSM's relative performance improves for low and high SNR levels but MRCCR still has a better performance for low to medium SNR levels. This indicates that MRCCR and EGCCR methods are more effective especially for short data blocks.

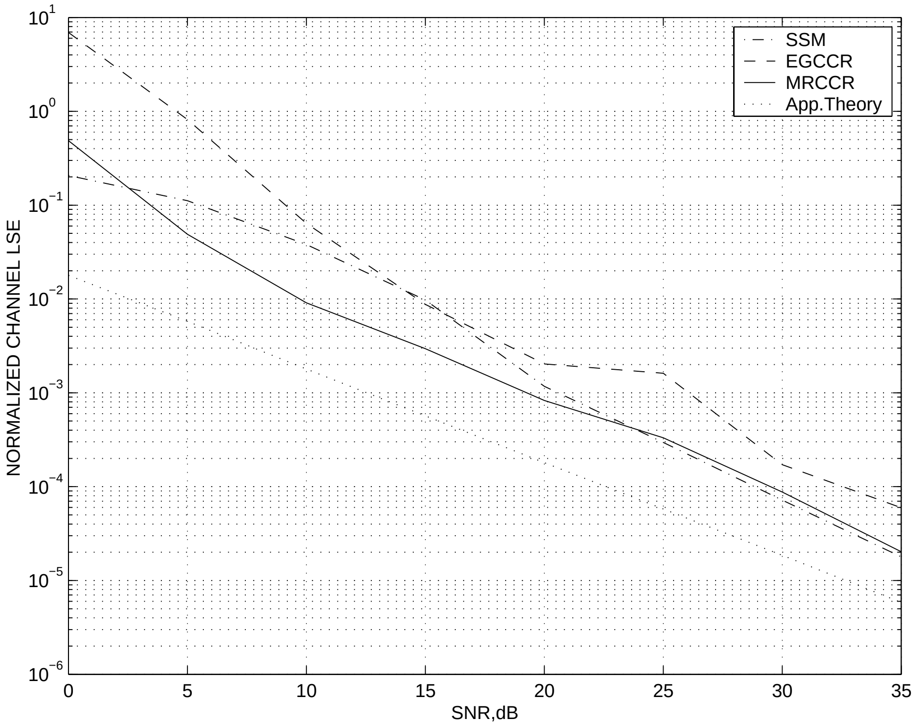

Figure 14. LSE performance of MRCCR, EGCCR and SSM methods for L=8, M=24 and N=(M+L).

6. Conclusion

We investigated the blind channel and symbol identification problem for SISO channels. A deterministic method is proposed which can operate with a single output data block. This method has a good performance even when the block length is short. It can be used to identify the channel order as well. In this respect, it is a good candidate in blind identification for wireless communications. Its main disadvantage is the exponential increase in the size of the search space. When there is more than one output data block for the same channel, there exist alternative approaches for blind identification. We have presented an optimum method, MRCCR, together with its suboptimum counterpart, EGCCR. MRCCR is optimum in terms of instantaneous SNR. Both of these methods are used to find both the channel and symbol estimates for TZ transmission or burst communication systems. MRCCR is an effective method for blind channel identification, especially for short data blocks and low SNR. In this respect, it is a good candidate for channel identification in wireless systems where there is a need to determine the channel response as fast as possible. MRCCR requires only two data blocks for identification and its performance gets better with additional data blocks. It is also possible to use EGCCR when low computational complexity is the prime factor. The performance of the EGCCR approach is not as good as MRCCR or SSM. However, when the number of data blocks is small, the difference is not very significant.

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