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YAPRAK YALÇIN

LEYLA GÖREN SÜMER

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Direct discrete-time control of port controlled Hamiltonian systems

Yaprak YALÇIN, Leyla GÖREN SÜMER
Department of Control Engineering, İstanbul Technical University
Maslak-34469, İstanbul-TURKEY
e-mail: yalciny@itu.edu.tr, gorenl@.itu.edu.tr

Abstract

The direct discrete time control of Port Controlled Hamiltonian Systems (PCHS) in the sense of energy shaping and damping injection is considered. In order to give a direct discrete time design method for PCHS, firstly an appropriate discrete gradient is proposed, which enables the derivation of a discrete time equation corresponding to the discrete time counterpart of Hamiltonian Systems. Using this proposed discrete-time model, the discrete-time counterpart of Passivity Based Control (PBC) technique is developed for n-degrees-of-freedom mechanical systems. The discrete-time control rules which correspond to the energy shaping and damping assignment are obtained directly using the discrete time model of the desired system and the discrete time model of the open loop systems. To illustrate the effectiveness of the proposed method, two non-separable and under actuated examples are investigated and the simulation results are given.

Key Words: *Hamiltonian systems, under-actuated systems, passivity based control*

1. Introduction

The port-controlled Hamiltonian (PCH) approach has been versatile not only for modeling of physical systems but also for control of a wide class of nonlinear systems [1, 2]. The PCH approach has been considered mostly for nonlinear systems especially when the systems have electrical and mechanical sub-systems which have to be considered together. Furthermore, the passivity-based control (PBC) is a powerful design technique for stabilizing nonlinear systems and especially for set point regulation problem both in Euler-Lagrange systems and PCH systems [3, 4].

In continuous-time context, the PBC design is completed in two-steps; first the energy shaping control rule $u_{es}(t)$ is designed to assign the desired energy function as the total energy of the system, second, the damping injection control rule $u_{di}(t)$ is designed to achieve asymptotic stability at desired equilibrium point, which corresponds to an isolated and strict minimum of the desired energy function. One can find the details of the design methodology in [5] and references therein.

On the other hand, technological advancements in digital processors, the widespread use of computer controlled systems in engineering practice are crucially in need of a theory to analysis and design sampled-data systems and techniques to obtain discrete-time model of non-linear systems. A framework on the issue of the stabilization of sampled-data non linear systems using their approximate discrete time models can be found in [6].

In control literature, to the best of our knowledge, number of works utilizing the discrete-time models of Hamiltonian systems for control applications are limited [7–9]. In these works, in order to obtain the discrete-time model, Euler method is used and as mentioned by the authors, "the Euler model is not Hamiltonian-conserving, but better preserves the Hamiltonian structure of the plant."

In this study, a gradient based method for deriving a discrete-time counterpart of continuous Hamiltonian systems is proposed and a stability analysis for both separable and non-separable case is done considering the energy interactions. Later the discrete-time complement of PBC technique is developed for an n-degrees-of-freedom mechanical system using this discrete-time counterpart of continuous Hamiltonian systems. The discrete-time control rules $u_{es}(k)$ and $u_{di}(k)$, which correspond to the energy shaping and damping injection, respectively, are derived directly using the discrete time model of the desired system and the discrete time model of the open loop system.

To illustrate the effectiveness of the proposed method, two non-separable examples are investigated and the simulation results are given. Throughout the this paper (for only the under-actuated case) it is assumed that the desired continuous time closed loop system is known. One can find the method of the direct discrete time controller design for fully actuated Hamiltonian systems in [10]. It should be mentioned that there are various control laws obtained using different approaches based on Lyapunov theory, feedback linearization, sliding mode control etc., aside from the passivity-based technique considered here, for the stabilization of the underactuated systems in continuous time context. Since the many underactuated systems are not feedback linearizable [11], the feedback controller design methods proposed in related literature are more complicated than passivity based technique.

2. Prelimineries

Consider the continuous-time Hamiltonian systems given in standard coordinates,

$$\begin{aligned} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} &= J \begin{bmatrix} \nabla_q H(q,p) \\ \nabla_p H(q,p) \end{bmatrix} + \begin{bmatrix} 0 \\ G(q) \end{bmatrix} u \\ y(t) &= G^T(q) \nabla_p H(q,p) \end{aligned} \tag{1}$$

where $(q,p) \in X \subset \Re^{2n}$ is an 2n-dimensional manifold, $y \in \Re^m$ is system output, $u \in \Re^m$ is the control input, $G(q) \in \Re^{nxm}$ is input force matrix and J is the standard skew-symmetric matrix, namely

$$J = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}$$

The notation $\nabla_{(\bullet)} H$ is used to denote the gradient vector of a scalar function $H(q,p)$ with respect to $(.)$. Furthermore, $H : \Re^{2n} \rightarrow \Re$ is the Hamiltonian function of the system or the energy function of the system in the form

$$H(q,p) = K(q,p) + V(q) = \frac{1}{2}p^T M^{-1}(q)p + V(q) \tag{2}$$

where $V(q)$ and $K(q,p)$ are potential and kinetic energy terms, respectively, and $M(q) = M(q)^T > 0$ is the generalized inertia matrix. If $M(q) = M \in \Re^{nxn}$, i.e. a constant matrix, the system is called a *separable Hamiltonian system*, and if $\boldsymbol{m{=}n}$ the system is said to be *fully actuated*.

In order to solve the stabilization problem for Hamiltonian systems the IDA-PBC design method was developed by Ortega et al., [5]. The main idea in this method was to design a stabilizing controller which assigns a desired energy function

$$H_d(q,p) = K_d(q,p) + V_d(q) = \frac{1}{2}p^T M_d^{-1}(q)p + V_d(q) \tag{3}$$

which has an isolated equilibrium point $(q^*, 0)$ of the closed system. It is easily observed that this problem can be solved by assigning only desired potential energy function for fully actuated Hamiltonian systems. The stabilization problem for Hamiltonian systems for the under actuated case is known as a challenging problem since it needs the appropriate choice of the desired energy function and also assignment of a new interconnection matrix. In literature, the desired system is considered as

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = (J_d - R_d) \begin{bmatrix} \nabla_q H_d \\ \nabla_p H_d \end{bmatrix}, \quad H_d(q,p) = \frac{1}{2}p^T M_d(q)^{-1}p + V_d(q) \tag{4}$$

$$J_d = \begin{bmatrix} 0 & M^{-1}M_d \\ -M_d M^{-1} & J_2 \end{bmatrix}, \quad R_d = \begin{bmatrix} 0 & 0 \\ 0 & GK_vG^T \end{bmatrix}$$

where $J_2(q,p) = -J_2^T(q,p)$, $R_d(q,p) = R_d^T(q,p), K_v = K_v^T \geq 0$. The controller $u_{es}(t)$ is obtained as a solution to the equation

$$\begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \begin{bmatrix} \nabla_q H \\ \nabla_p H \end{bmatrix} + \begin{bmatrix} 0 \\ G(q) \end{bmatrix} u_{es} = \begin{bmatrix} 0 & M^{-1}M_d \\ -M_d M^{-1} & J_2 \end{bmatrix} \begin{bmatrix} \nabla_q H_d \\ \nabla_p H_d \end{bmatrix} \tag{5}$$

Note that for separable Hamiltonian systems, the parameter J_2 can be set to zero [12]. In case the matrix $G(q)$ is full column rank, it should be constructed such that the $M_d(q)$ and $V_d(q)$ holds the constraint

$$G^{\perp}\left\{\nabla_q H - M_d(q)M^{-1}(q)\nabla_q H_d + J_2(q,p)M_d^{-1}(q)p\right\} = 0 \tag{6}$$

or the equivalent constraints

$$G^{\perp}(q)\left\{\nabla_q(p^T M^{-1}(q)p) - M_d(q)M^{-1}(q)\nabla_q(p^T M_d^{-1}p) + 2J_2(q,p)M_d^{-1}(q)p\right\} = 0 \tag{7}$$

$$G^{\perp}(q)\left\{\nabla V - M_d(q)M^{-1}(q)\nabla V_d\right\} = 0, \tag{8}$$

with $G^{\perp}$ a full rank left annihilator of G. If the PDE s' (7) and (8) are solvable, the energy shaping controller is derived as

$$u_{es}(t) = (G^T G)^{-1} G^T \left\{\nabla_q H - M_d(q)\, M^{-1}(q)\, \nabla_q H_d + J_2(q,p)\, \nabla_p H_d\right\} \tag{9}$$

Since the resulting closed loop system under this control rule is also a Hamiltonian system, the damping injection control rule, which yields an asymptotically stable system is obtained as

$$u_{di}(t) = -K_v G^T \nabla_p H_d \quad , \quad K_v > 0 \tag{10}$$

3. Main results

In this section, first a discrete-time counterpart of continuous Hamiltonian systems with input is derived using the concept of the discrete gradient. To obtain this result, the discrete gradient definition given in [13, 14] and restated below will be considered.

Definition 1: Let $H(x)$, be a differentiable scalar function in $x \in \Re^n$; then $\bar{\nabla} H(x_{k+1}, x_k)$ is *a discrete gradient* of $\boldsymbol{H}$ if it is continuous in $\boldsymbol{x}$ and,

$$\begin{array}{l} \bar{\nabla}^T H(x_{k+1}, x_k)\,[x_{k+1} - x_k] = H(x_{k+1}) - H(x_k) \\ \bar{\nabla} H(x_k, x_k) = \nabla H(x_k) \end{array} \tag{11}$$

where $x_k = k\,\Delta\,t,\ x_{k+1} = (k+1)\,\Delta\,t\ = x_k + \Delta\,t$. □

It should be noted that the main results of this study will be derived under the assumption that there exists a discrete gradient, i.e. $\bar{\nabla} H(x_{k+1}, x_k)$, $x = \{q, p\} \in \Re^{2n}$ which satisfies the conditions in Definition 1, exactly. A detailed stability discussion will be given later, considering the energy relation when the conditions in Definition 1 are not precisely satisfied.

Approximating to derivatives of state variables (1) by Forward Euler with sampling period T_s

$$\dot{q} = \frac{q_{k+1} - q_k}{T_s}, \qquad \dot{p} = \frac{p_{k+1} - p_k}{T_s} \tag{12}$$

and replacing the gradient term in (1) with the discrete gradient $\bar{\nabla} H(x_{k+1}, x_k)$, $x = \{q, p\} \in \Re^{2n}$ the gradient-based discrete-time description of the system (1) can be obtained as follows

$$\left[\begin{array}{c} q_{k+1} - q_k \\ p_{k+1} - p_k \end{array}\right] = T_s\, J \left[\begin{array}{c} \bar{\nabla}_q H \\ \bar{\nabla}_p H \end{array}\right] + T_s \left[\begin{array}{c} 0 \\ G(q_k) \end{array}\right] u, \quad H(q,p) = \frac{1}{2} p^T M(q)^{-1} p + V(q). \tag{13}$$

Furthermore, the similar expressions can also be obtained for the discrete-time description of the desired system as

$$\left[\begin{array}{c} q_{k+1} - q_k \\ p_{k+1} - p_k \end{array}\right] = T_s (J_d - R_d) \left[\begin{array}{c} \bar{\nabla}_q H_d \\ \bar{\nabla}_p H_d \end{array}\right], \quad H_d(q,p) = \frac{1}{2} p^T M_d(q)^{-1} p + V_d(q) \tag{14}$$

$$J_d = \left[\begin{array}{cc} 0 & M^{-1} M_d \\ -M_d M^{-1} & J_2 \end{array}\right], \quad R_d = \left[\begin{array}{cc} 0 & 0 \\ 0 & G K_v G^T \end{array}\right]$$

If the right hand side of (13) is equated to the right hand side of (14), the discrete time control rule responsible for energy shaping is obtained as follows in terms of discrete gradients,

$$u_{es}\,(k) = (G^T G)^{-1} G^T \left\{ \bar{\nabla}_q H - M_d(q_k) M^{-1}(q_k) \bar{\nabla}_q H_d + J_2(q_k, p_k)\, \bar{\nabla}_p H_d \right\} \tag{15}$$

and for the resulting closed-loop Hamiltonian system the damping injection control rule can be written as

$$u_{di}\,(k) = -K_v G^T \bar{\nabla}_p H_d \tag{16}$$

It should be noted that these discrete time control rules (15) and (16) have been derived under the assumption that the desired continuous time closed loop system is given.

In the sequel, the following discrete gradient definition that is inspired from the mean value theorem and the first condition given in Definition 1 will be used.

Definition 2: Consider a differentiable function in $\boldsymbol{x}$ given as $H(x) = {}^1/_2 x^T Z(x)x$ and its gradient given in the form of $\nabla H(x) = Q(x)x$, then the discrete gradient of a $H(x)$ is defined as

$$\bar{\nabla} H(x) = \hat{Q}(x_{k+1}, x_k) \left[\frac{x_{k+1} + x_k}{2}\right] \tag{17}$$

where,

$$\hat{Q}(x_{k+1}, x_k) = [Q(x_{k+1}) + Q(x_k)]/2 \tag{18}$$

The discrete gradient definition given here which is based on midpoint is slightly different than the one introduced by Gonzalez [13]. If $Z(x)$ is constant matrix, then it can be easily shown that, the discrete gradient given in Definition 2 exactly satisfies both of the conditions given in Definition 1. However, in general it does not satisfy the first condition of Definition 1, precisely.

To give the stabilizability discussion for this approximate case, consider the continuous-time Hamiltonian systems with dissipation and input

$$\dot{x}(t) = [J(x) - R(x)] \nabla H(x) + G(x)u(t) \tag{19}$$

where $x \in \Re^n$ denotes the states, $u \in \Re^m$ is the control input of the system and $J(x) = -J^T(x)$, $R(x) = R^T(x) \geq 0$, when $R(x) > 0$, $u(t) = 0$, and $\boldsymbol{H(x)}$ has a local (global) strict minimum at $x = x^*$; then this system has a local (global) asymptotically stable equilibrium at point $\boldsymbol{x}^*$, and the following inequality holds [1]:

$$\dot{H}(t) = \nabla^T H(x) [J(x) - R(x)] \nabla H(x) < 0 \tag{20}$$

On the other hand, the analogy between continuous and discrete cases would give rise to a similar energy relation as the one in (20) for the discrete case,

$$\bar{\nabla}^T H \;\; [x_{k+1} - x_k] = T_s \bar{\nabla}^T H(x) \;\; [J(x_k) - R(x_k)] \;\; \bar{\nabla} H(x) \tag{21}$$

After some algebraic manipulations, the following energy relation is obtained for the discrete time description of the Hamiltonian systems

$$\frac{H(x_{k+1}) - H(x_k)}{T_s} = \bar{\nabla}^T H \;\; [J(x_k) - R(x_k)] \;\; \bar{\nabla} H + \varepsilon(x_{k+1}, x_k) \tag{22}$$

This relation implies that the discrete time system creates an extra energy or extra dissipation according to the sign of $\varepsilon(x_{k+1}, x_k) \in \Re$. Obviously, for $T_s \to 0$ this extra term tends to zero, i.e. $\varepsilon(x_{k+1}, x_k) \to 0$. As a consequence of the above analysis the following Remark can be given on the stabilizability property.

Remark : When the discrete gradient given in Definition 2 is used to derive the control rules given in (15) and (16), to stabilize the sampled-data Hamiltonian system, the extra term does not effect stabilizability condition if $\varepsilon(x_{k+1}, x_k) < 0$. On the other hand, if $\varepsilon(x_{k+1}, x_k) > 0$ the control rules should be designed considering this fact, especially when slow sampling is used. As it can be obviously followed from (22), the stabilizability condition of continuous Hamiltonian system under the discrete-time control rules remains same, i.e. stability can be achieved by adding an extra dissipation. Moreover, since $u(t) = u_{es}(t) + u_{di}(t)$ given with (9) and (10) is an Asymptotically Stabilizing (AS) controller in continuous time setting, the discrete time control rules have the properties

$$\lim_{T_s \to 0} u_{es}(kT_s) \to u_{es}(t) \quad , \quad \lim_{T_s \to 0} u_{di}(kT_s) \to u_{di}(t)$$

Practically, AS property of the control rules given in (15) and (16) are guaranteed [6].

In order to obtain the discrete time control rules (15) and (16), we have to construct discrete gradient of $H(q,p)$. Recall that

$$H(q,p) = K(q,p) + V(q) = \frac{1}{2} p^T M^{-1}(q) p + V(q) \tag{23}$$

and assume

$$\nabla H(q,p) = \begin{bmatrix} V_{gr}(q) & S(q,p) \\ 0 & M^{-1}(q) \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix} = Q(q,p) \begin{bmatrix} q \\ p \end{bmatrix}$$
$$S(q,p) = \begin{bmatrix} \left[p^T \frac{\partial M^{-1}(q)}{\partial q_1} \right] \\ \left[p^T \frac{\partial M^{-1}(q)}{\partial q_2} \right] \\ \\ \left[p^T \frac{\partial M^{-1}(q)}{\partial q_n} \right] \end{bmatrix} \tag{24}$$

in which the matrices $V_{gr}(q)$ are described by the relation

$$\nabla V(q) = V_{gr}(q)\, q \tag{25}$$

then the discrete gradient expression is obtained as

$$\bar{\nabla} H(q,p) = \Phi(k+1,k) \begin{bmatrix} q_{k+1} + q_k \\ p_{k+1} + p_k \end{bmatrix} \tag{26}$$

where $\Phi(k+1,k) = \frac{1}{4}\left(Q(q_k,p_k) + Q(q_{k+1},p_{k+1})\right)$.

Since the desired energy function also has same structure with (23), the corresponding expressions for discrete-gradient of the desired energy function can be similarly obtained as

$$\bar{\nabla} H_d(q,p) = \Phi_d(k+1,k) \begin{bmatrix} q_{k+1} + q_k \\ p_{k+1} + p_k \end{bmatrix} \tag{27}$$

where $\Phi_d(k+1,k) = \frac{1}{4}\left(Q_d(q_k,p_k) + Q_d(q_{k+1},p_{k+1})\right)$ and $\nabla H_d(q,p) = Q_d(q,p) \begin{bmatrix} q \\ p \end{bmatrix}$

Therefore, the discrete-time control rules $u_{es}(k)$ and $u_{di}(k)$ given in (15) and (16), respectively, which correspond to the energy shaping and damping injection respectively have been designed directly using the discrete time model of the desired system and the discrete time model of the open loop systems.

4. Examples

In the literature, there are various studies on IDA-PBC control method for under-actuated systems [12, 15–8]. In these studies, some classes of admissible desired systems for the considered Hamiltonian systems are given in continuous time setting. In this paper, two well-known underactuated Hamiltonian systems —*Cart Pendulum* and *Ball and Beam systems*—are considered in discrete time. In the examples to avoid non-causality, we employ the approximations

$$\begin{aligned} \bar{\nabla} H(q_{k+1}, p_{k+1}, q_k, p_k) &\approx \bar{\nabla} H(q_k, p_k, q_{k-1}, p_{k-1}) \\ \bar{\nabla} H_d(q_{k+1}, p_{k+1}, q_k, p_k) &\approx \bar{\nabla} H_d(q_k, p_k, q_{k-1}, p_{k-1}) \end{aligned} \tag{28}$$

for the discrete gradient. One has to consider that the computational complexity of the control rule using this discrete gradient expression is nearly the same as the computational complexity of the emulation controller; this property might provide an important advantage especially in industrial applications. Furthermore, it should be emphasized that the emulation controller makes the closed loop system unstable in both examples for sampling period that are used in direct discrete control rules proposed here.

Example 1: Pendulum on a Cart

The dynamic equation of the pendulum on a cart that is shown in Figure 1 are given as

$$M = \begin{bmatrix} 1 & b\cos(q_1) \\ b\cos(q_1) & c \end{bmatrix}, \quad V(q) = a\cos(q_1) \tag{29}$$

$$G = e_2, \ a = \frac{g}{l}, \ b = \frac{1}{l}, \ c = \frac{M+m}{ml^2}$$

in [15], with $n = 2$, $m = 1$, where q_1 is the pendulum angle with the upright vertical, q_2 is the cart position, m is the mass of the pendulum, l is the length of the pendulum, M is the mass of the cart and g is acceleration due to gravity. The equilibrium to be stabilized is the upward position of the pendulum with the cart placed in any desired position, which corresponds to $q_{1*} = 0$ and any arbitrary q_{2*}

As presented in main result section, the discrete-time energy shaping control rule and the damping injection control rule have been obtained as

$$u_{es}(k) = (G^T G)^{-1} G^T \left\{ \bar{\nabla}_q H - M_d(q_k) M^{-1}(q_k) \bar{\nabla}_q H_d + J_2(q_k, p_k) \bar{\nabla}_p H_d \right\}$$

$$u_{di}(k) = -K_v G^T \bar{\nabla}_p H_d$$

for the discrete gradient terms in these control rules, the expressions given in (26) and (27) are utilized. On the other hand, the $H_d(q,p)$ and $J_2(q_k, p_k)$ terms in these expressions are solutions of the continuous-time design method described in [16], which will be explained briefly in the sequel. The construction of M_d, V_d and J_d are completed after a coordinate transformation and the relationships between the new coordinate system and the original coordinate system are as in [15]:

$$\begin{aligned} &T = M, \quad S = \nabla_q (T\tilde{p}), \quad p = T(q)\,\tilde{p} \\ &M_d = T\tilde{M}_d T^T, \quad V_d = \tilde{V}_d \\ &J_2(q,p) = T\tilde{J}_2\left(q, T^{-1}p\right) T^T + S\left(q, T^{-1}p\right) M^{-1} T\tilde{M}_d T^T - T\tilde{M}_d T^T M^{-1} S^T\left(q, T^{-1}p\right) \end{aligned} \tag{30}$$

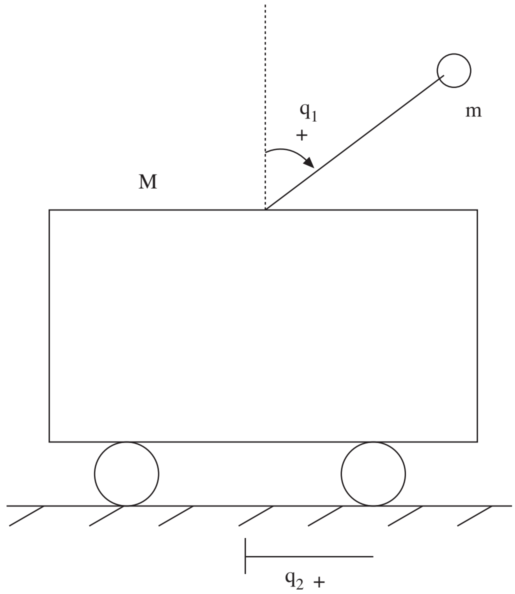

Figure 1. The cart and pendulum system modeled.

where T is the coordinate transformation matrix which is taken as $T = M$ for this system and $\tilde{M}_d$, $\tilde{V}_d$ and $\tilde{J}_d$ are the new design parameters in the new coordinates. When $\tilde{M}_d$ and $\tilde{J}_d$ are take the form

$$\tilde{M}_d = \begin{bmatrix} \frac{kb^2}{3}\cos^3(q_1) & -\frac{kb}{2}\cos^2(q_1) \\ -\frac{kb}{2}\cos^2(q_1) & k\cos q_1 + m_{22}^0 \end{bmatrix}, \quad \tilde{J}_2 = \left(\tilde{p}^T \tilde{M}_d^{-1} \begin{bmatrix} \alpha_1(q) \\ \alpha_2(q) \end{bmatrix}\right) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{31}$$

with the free parameter $m_{22}^0 \geq 0$, then M_d, J_d can be generated using (30). Here, the free functions $\alpha_1(q)$ and $\alpha_2(q)$ are taken as in [16]:

$$\alpha_1 = -k^2 b^3 \cos^4(q_1)\sin(q_1)/12, \quad \alpha_2 = k^2 b^2 \cos^3(q_1)\sin(q_1)/12 \tag{32}$$

with the free parameter $k > 0$. Thus, the positivity of $\tilde{M}_d$ for $q_1 \in (-\pi/2, \pi/2)$ is guaranteed [15, 16]. Moreover, the desired potential energy function $\tilde{V}_d$ is taken as in [15],

$$\tilde{V}_d = \frac{3a}{kb^2\cos^2 q_1} + \frac{K_p}{2}\left[q_2 - q_{2*} + \frac{3}{b}\ln(\sec q_1 + \tan q_1) + \frac{6m_{22}^0}{kb}\tan q_1\right]^2 \tag{33}$$

with $K_p > 0$ arbitrary and q_{2*} the cart position to be stabilized.

In simulations, the system parameters are taken as $g = 9.81ms^{-2}$, $l = 1\,m$, $M = 5\,kg$, $m = 1\,kg$, and the discrete time controller design parameters are chosen as $K_v = 0.5, K_p = 1.0$, $k = 0.1$, $m_{22}^0 = 0.002$. The simulations are carried out for the sampling period $T_s = 0.01s$, the initial conditions $(q(0), p(0)) = (\pi/4,\ -0.1,\ 0.1,\ 0.5)$ and the desired cart position $q_{2^*} = 20$. The simulation results are presented in Figure 2 and 3 which illustrate time domain responses under the direct discrete-time control and the continuous-time control together for comparison. The Figure 4 and the Figure 5 exhibit the control input versus time and phase portrait of the system, respectively.

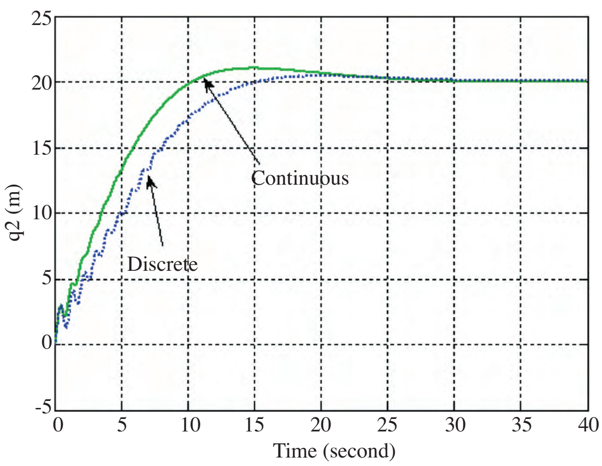

Figure 2. A graph of cart position as a function of time.

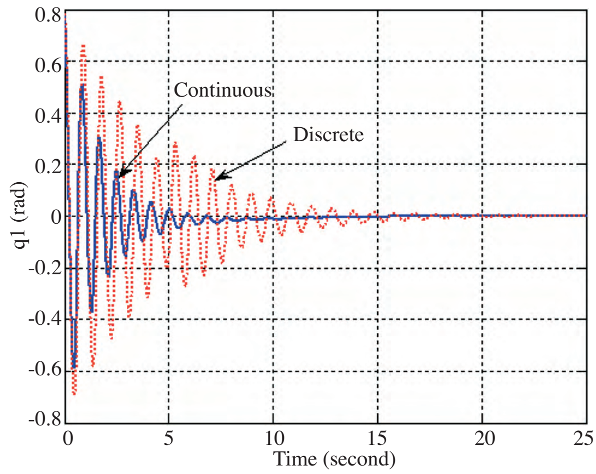

Figure 3. The pendulum angle as a function of time.

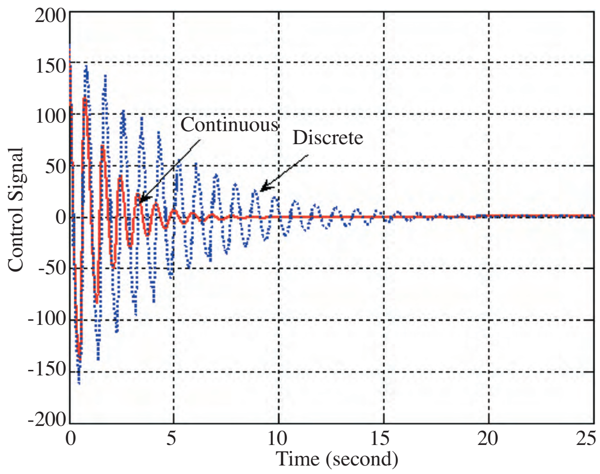

Figure 4. Evolution of the amplitude of the discrete and continuous control signals over time.

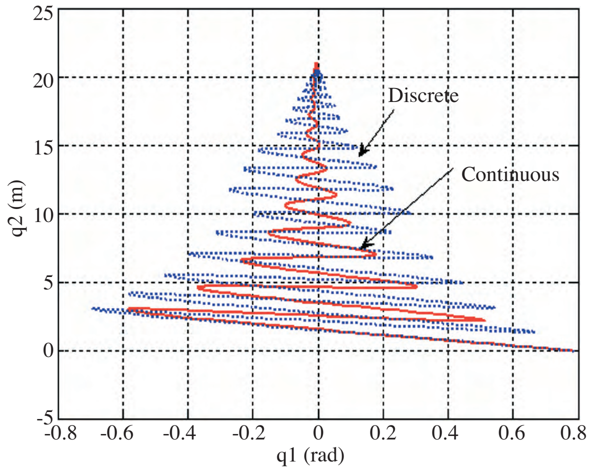

Figure 5. The phase diagram showing the evolution of q_1 and q_2.

Example 2: Ball and Beam

Let's consider the ball and beam system that is shown in Figure 6 whose Hamiltonian model is defined in [12] with

$$M = \begin{bmatrix} 1 & 0 \\ 0 & L^2 + q_1^2 \end{bmatrix}, \quad V(q) = g q_1 \sin(q_2), \quad G = \begin{bmatrix} 0 & 1 \end{bmatrix}^T \tag{34}$$

where q_1 is the ball position, q_2 the angle of the beam and L is the beam length.

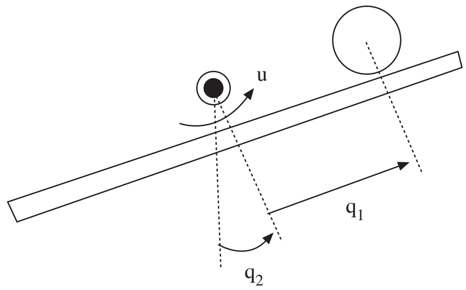

Figure 6. Diagram of the modeled ball and beam system.

The control objective is to stabilize the ball and beam in its rest position $q_{1*} = q_{2*} = 0$. To design the discrete time controller, the desired continuous time system proposed in [12] will be used. The matrix M_d and J_d and the desired potential energy function V_d completely describe the closed loop dynamics, as follows:

$$M_d = \left(L^2 + q_1^2\right) \begin{bmatrix} \sqrt{\frac{2}{L^2+q_1^2}} & 1 \\ 1 & \sqrt{2\left(L^2+q_1^2\right)} \end{bmatrix}, \quad J_2(q,p) = \begin{bmatrix} 0 & j(p,q) \\ -j(p,q) & 0 \end{bmatrix}$$
$$V_d(q) = g\left(1 - \cos\left(q_2\right)\right) + \Phi\left(z\left(q\right)\right) \tag{35}$$

where $j = q_1 \left(p_1 - \sqrt{\frac{2}{L^2+q_1^2}} p_2\right)$, $\Phi(z) = \frac{K_p}{2} z^2$ and $z(q) \triangleq q_2 - \arcsin h\left(\frac{q_1}{L}\right)/\sqrt{2}$.

In simulations, the discrete time controller design parameters are chosen as $K_v = 50$, $K_p = 1.0$, $k = 0.1$ for the system parameters $g = 9.81 ms^{-2}$ and $L = 10\,m$. Figures 7–10 illustrate the results of the simulation for the sampling period $T_s = 0.01s$, the initial conditions $(q(0), p(0)) = (10.0, 0.0, 0.0, 0.0)$ and the desired ball position $q_{1^*} = 0$. The Figure 7 and 8 demonstrate the time domain responses under the discrete-time control and the continuous-time control together for comparison of results. The control input versus time and the phase portrait of the system are shown in the Figure 9 and the Figure 10, respectively.

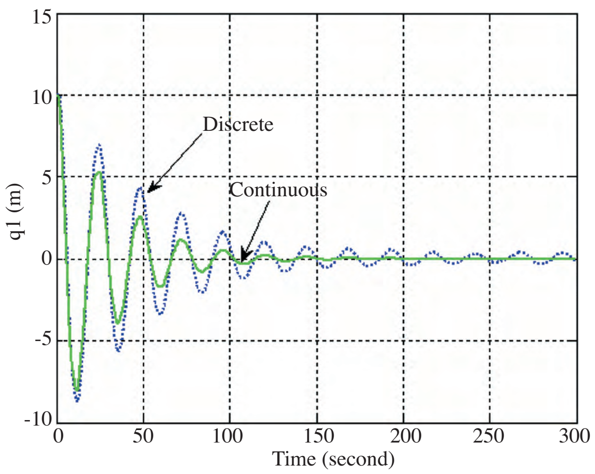

Figure 7. Evolution of the ball position as a function of time.

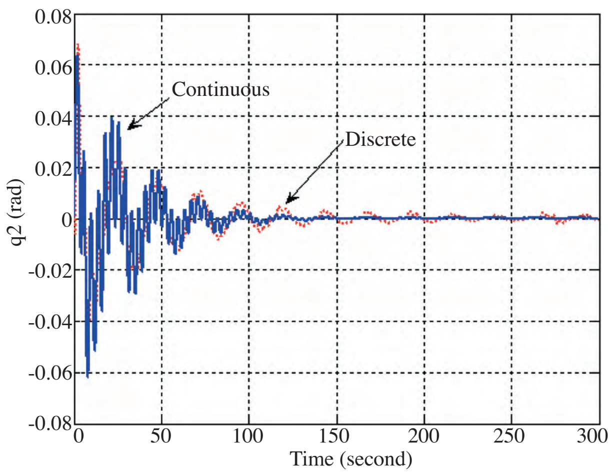

Figure 8. Evolution of the beam angle as a function of time.

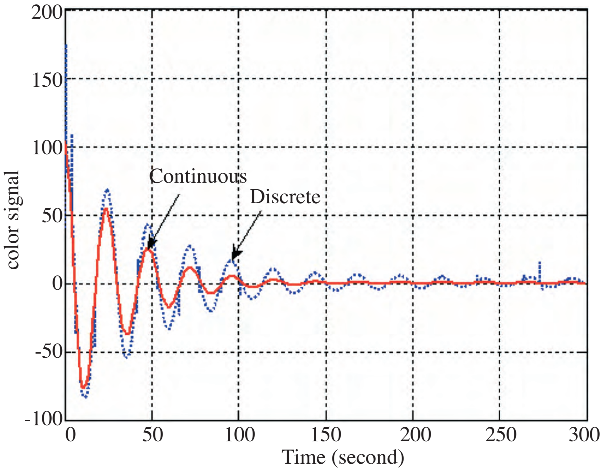

Figure 9. Evolution of the discrete and continuous control signals over time.

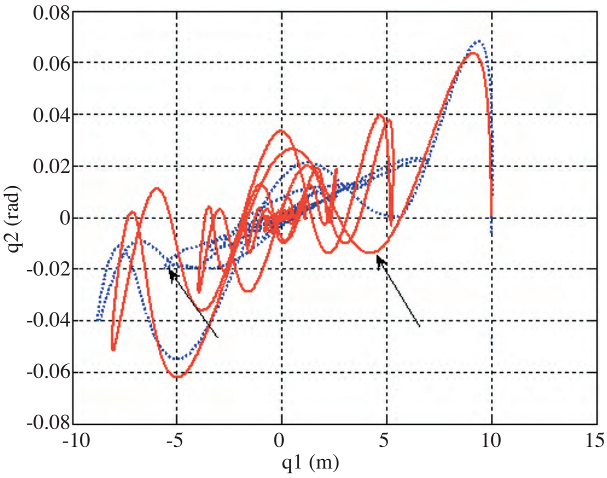

Figure 10. Phase diagram showing the evolution of q_1 and q_2.

The simulation results given in this paper show that the discrete control rule generated via the method proposed here can be used instead of the continuous control rule, since the closed loop systems under the discrete control rule and the continuous control rule have almost same behavior as seen from Figures 2–4 of cart and pendulum system and Figures 7–9 of ball and beam system. It should be pointed out that the emulations of the continuous controllers have destabilized the system in both examples.

5. Conclusions

In order to design the direct discrete time control for the continuous Hamiltonian systems, a gradient based method has been presented. Moreover, stability analysis of the proposed method is completed considering the energy interactions. The discrete-time complement of PBC technique has been derived for n-DOF mechanical system using this method. The discrete-time control rules which correspond to the energy shaping and damping injection have been obtained directly using the discrete time model of the desired system and the discrete time model of the open loop systems. It should be noted that the desired continuous time closed loop system is known. One can find the method of the direct discrete time controller design for fully actuated Hamiltonian systems in [10].

To analyze the effectiveness of the proposed method, two non-separable and underactuated examples are considered and the simulations have been done. These results reveal that the direct discrete time controller design method proposed in this study yields a good performance for sampled data Hamiltonian systems since the emulation of the continuous controllers have destabilized the systems.

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