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A short note on a new approach to Rayleigh-Bénard-Chandrasekhar convection in weakly electrically conducting viscoelastic liquids

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Abstract: The onset of magnetoconvection (known as Rayleigh-Bénard-Chandrasekhar convection) in two relaxation time viscoelastic liquids is studied here without seeking explicit recourse to a normal stress formulation as is usually done in these studies. Magnetoconvection refers to the flow of fluid in the presence of both thermal gradients (Rayleigh-Bénard convection) and a magnetic field. When these two effects are combined, they can lead to interesting and complex patterns of fluid motion. Understanding magnetoconvection in viscoelastic liquids is crucial for various industrial and scientific applications. The hyperbolic-type of linear momentum equation is decomposed into two first-order equations in time by cleverly separating the viscoelastic effect from the other effects in a clever manner as reported in a recent paper. The results of Maxwell, Rivlin-Ericksen, Walters' liquid B, and Newtonian liquids are obtained as limiting cases of the present study. This research contributes to the understanding of magnetoconvection in viscoelastic liquids by using a novel approach that decouples the viscoelastic effect from other influences. The results obtained shed light on the behaviour of various types of viscoelastic materials and provide valuable insights for practical applications in fields such as materials science, engineering, and geophysics.

Key words: Rayleigh-Benard Convection, viscoelastic liquid, electrically conduction

1. Introduction

Rayleigh-Bénard convection problems in electrically conducting Newtonian liquids have received widespread attention because of their implications in heat transfer, in the control of convection, and in other engineering applications (see Chandrasekhar [4], Weiss [18, 19], Platten and Legros [9], Siddheshwar [14] and references therein). It is now a well-known fact that viscoelastic liquids are practically realisable and a good discussion on these is available in Bird et al. [3] and Joseph [7]. There are numerous papers dealing with the onset of convection in viscoelastic liquids (see Sekhar and Jayalatha [11, 12], Siddheshwar et al. [15, 16] and references therein). In the presence of suitable additives that render liquids electrically conductive are present then viscoelastic liquids respond to electromagnetic fields. This can be exploited to control convection using an external magnetic field. Such a problem is called magnetoconvection. Few authors have considered magnetoconvection in viscoelastic liquids. Bhatia and Steiner [1] have investigated the problem of overstability in a layer of a Maxwellian liquid heated from below when a uniform magnetic field is present, acting in a direction parallel to that of gravity, is present and when the liquid is confined between two free boundaries. The magnetic field was shown to have a

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stabilising effect on the oscillatory mode of convection. Bhatia and Steiner [2] have studied overstability in a layer of Maxwell's liquid confined between two rigid boundaries. Sharma [13] has investigated the stability of a layer of Oldroyd-B liquid heated from below and subjected to a magnetic field. The magnetic field is found to have a stabilising effect. The analytical condition for the absence of overstability is given. Eltayeb [6] has studied thermal instability in a Maxwell liquid layer in hydromagnetics using an asymptotic technique which is effective only for large values of the Chandrasekhar number. Ciancio [5] has also carried out research which, by using a procedure of classical irreversible thermodynamics with internal variables, some possible interactions between heat conduction and heat conduction and viscous-elastic flows for rheological media. From the literature review reported above, the following facts emerge, (i) There is a reported work on linear stability analysis of magnetoconvection in Oldroyd-B liquid, a viscoelastic liquid with two relaxation times, but a detailed discussion on direct and Hopf bifurcations for this problem is not available. (ii) There is very little literature available on magnetoconvection in viscoelastic liquids with a single relaxation time of the Maxwell type and no reported work on magnetoconvection in Rivlin-Ericksen liquid. A detailed analysis is also lacking in the reported works. All the works on linear stability of viscoelastic fluid convection, both single and double relaxation time ones, make use of the hyperbolic type of linear momentum equation to obtain the critical Rayleigh number. Recently, Siddheshwar et al. [15] proposed a novel approach to study convection in viscoelastic liquids. In this paper we incorporate the idea of Siddheshwar et al. [15] and study the onset of magnetoconvection in viscoelastic liquids with two relaxation times. In the second section part of this article, the normal mode solution was used, and in section 3, the results were plotted based on the critical Rayleigh number (R_c) and the critical wave number(k_c) for (Chandrasekhar number) Q=0 and Q=10.

2. Mathematical formulation and solution

The physical configuration considered in this paper is shown in Figure 1. It consists of a weakly electrically conducting Oldroyd-B liquid layer of infinite horizontal extent heated from below and cooled from above, to which a uniform magnetic field H_0 is applied in the vertical direction. The layer has thickness d and is bounded by two free isothermal planes. The upper and lower planes are at constant temperatures T_0 and $T_0 + \Delta T$ respectively. We consider two-dimensional longitudinal rolls and therefore assume that all quantities are independent of y. For a weakly electrically conducting Boussinesq-Oldroyd-B liquid, the governing equations are:

Continuity equation:

$$q_{i,j} = 0. \tag{2.1}$$

Conservation of linear momentum for a weakly electrically conducting fluid:

$$\rho_0[\frac{\partial q_i}{\partial t} + \ q_j\frac{\partial q_i}{\partial x_j}] = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}^{'}}{\partial x_i} + \rho.g_i - \mu_m^2\sigma H_0^2 q_i \tag{2.2}$$

Constitutive relationship:

$$(1+\lambda_1\frac{\partial}{\partial t})\tau_{ij}^{'} = (1+\lambda_2\frac{\partial}{\partial t})[\mu(\frac{\partial q_i}{\partial x_j} + \frac{\partial q_i}{\partial x_i})]. \tag{2.3}$$

Conservation of energy:

$$\frac{\partial T}{\partial t} + q_j \frac{\partial T}{\partial x_j} = \kappa \frac{\partial^2 T}{\partial xj \partial xj} \tag{2.4}$$

Density equation of state:

$$\rho = \rho_0[1 - \alpha(T - T_0)] \tag{2.5}$$

where $q_i = (u, v, w)$ are the components of the velocity of the liquid, $\rho(T)$ is the density at temperature T, $\rho_0 = \rho(T_0)$, H_i are the components of the applied magnetic field, p is the pressure, μ_m the magnetic permeability, $g_i = (0, 0, -g)$ are the gravitational acceleration components, σ is the electrical conductivity, μ is the viscosity, λ_1 is the stress relaxation coefficient, λ_2 is the strain retardation coefficient, κ is the thermal diffusivity and α is the thermal expansion coefficient.

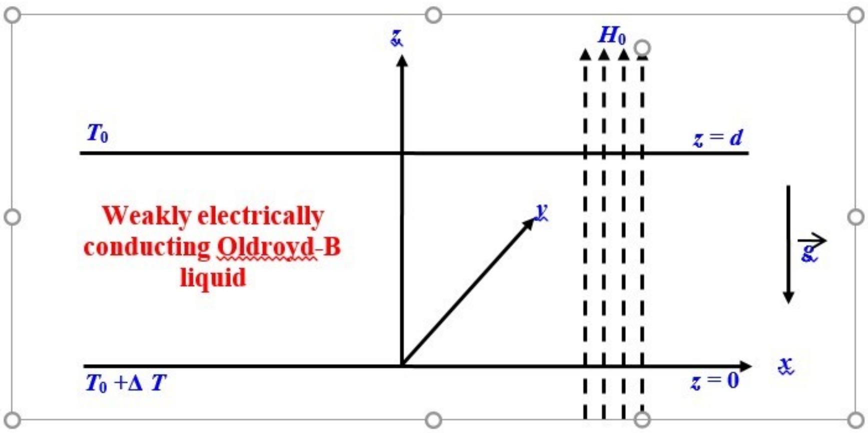

Figure 1: Schematic of the problem.

The basic state is quiescent. When we perturb the basic state and introduce the stream function $\psi(x, z)$ as follows:

$$u = -\frac{\partial \psi}{\partial z}, w = \frac{\partial \psi}{\partial x} \tag{2.6}$$

and then make the equations (2.1) to (2.5) dimensionless, we get the equations governing finite amplitude perturbations in the form:

$$\mathrm{Pr}^{-1}\left(1 + \Lambda_1 \frac{\partial}{\partial t}\right) \frac{\partial}{\partial t}\left(\nabla^2 \psi\right) = \left(1 + \Lambda_1 \frac{\partial}{\partial t}\right)\left[R \frac{\partial T}{\partial x} - Q \nabla^2 \psi\right] + \left(1 + \Lambda_2 \frac{\partial}{\partial t}\right) \nabla^4 \psi \tag{2.7}$$

$$\frac{\partial T}{\partial t} = \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial z}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial z} + \nabla^2 T \tag{2.8}$$

To arrive at equation (2.7), we have eliminated p and ρ between these four equations, one of which is a vector equation, by following the classical procedure used in all convection problems. The dimensionless parameters appearing in equations (2.7) and (2.8) are the following:

$\mathrm{Pr} = \frac{\mu_0}{\rho_0 \kappa}$ (Prandtl number),

$R = \frac{\alpha \rho_0 g d^3 \Delta T}{\mu_0 \kappa}$ (Rayleigh number),

$Q = \frac{\mu_m^2 H_0^2 d^2 \sigma_m}{\mu_0}$ (Chandrasekhar number),

$\Lambda_1 = \lambda_1 \frac{\kappa}{d^2}$ (Scaled stress-relaxation parameter or Deborah number),

and

$\Lambda_2 = \lambda_2 \frac{\kappa}{d^2}$ (Scaled strain-retardation parameter).

We now follow the Siddheshwar-decomposition method [16] and rearrange the equation (2.7) as two first-order equations in time as follows:

$$\Lambda_1 \frac{\partial M}{\partial t} = -M + (1-\Lambda)\nabla^4 \psi \tag{2.9}$$

where M is such that

$$\mathrm{Pr}^{-1} \frac{\partial}{\partial t}\left(\nabla^2 \psi\right) = R \frac{\partial T}{\partial x} + \Lambda \nabla^4 \psi + M - Q\nabla^2 \psi \tag{2.10}$$

subject to the boundary condition:

$$\psi = \nabla^2 \psi = T = M = 0 \quad at \quad z = 0, 1 \tag{2.11}$$

We represent the stream function, the temperature distribution, and M in the form:

$$\psi = A\, e^{i\omega\tau} \sin\left(k_c x\right) \sin\left(\pi z\right), \tag{2.12}$$

$$T = B e^{i\omega\tau} \cos\left(k_c x\right) \sin\left(\pi z\right) \tag{2.13}$$

$$M = C e^{i\omega\tau} \sin\left(k_c x\right) \cos\left(\pi z\right) \tag{2.14}$$

where k_c is the critical wave number. Substituting the normal mode solution (2.12)- (2.14) into the equations (2.8)- (2.10) we get the following algebraic system:

$$\begin{bmatrix} i\omega + \mathrm{Pr}\left(\Lambda + Q'\right) & -\mathrm{Pr}\left(1 + Q'\right) & \mathrm{Pr}\left(1-\Lambda\right) \\ R' & -\left(i\omega + 1\right) & 0 \\ 1 & 0 & -\left(i\omega\Gamma + 1\right) \end{bmatrix} \begin{bmatrix} X \\ Y \\ N \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.15}$$

where the scaled amplitudes and other scaled quantities are given by:

$X = \frac{A k_c \pi}{\sqrt{2}\delta^2}, \quad Y = \frac{B\pi R'}{\sqrt{2}}, \quad N = \frac{C k_c \pi}{\sqrt{2}(1-\Lambda)\delta^6}, \quad R' = \frac{k_c^2 R}{\delta^4(\delta^2 + Q)}.$

$\delta^2 = \pi^2 + k_c^2, \quad \Gamma = \Lambda_1 \delta^2, \quad \Lambda = \frac{\Lambda_2}{\Lambda_1}$ and $Q' = \frac{Q}{\delta^2}$

For a nontrivial solution of the above homogeneous system, the following condition needs to be satisfied.

$$\begin{vmatrix} i\omega + \Pr(\Lambda + Q') & -\Pr(1+Q') & \Pr(1-\Lambda) \\ R' & -(i\omega+1) & 0 \\ 1 & 0 & -(i\omega\Gamma+1) \end{vmatrix} = 0. \tag{2.16}$$

Rearranging equation (2.16) for and separating its expression into real and imaginary parts, we get

$$\Pr(\Lambda + Q')\left(1-\omega^2\Gamma\right) - \omega^2(\Gamma+1) - \Pr(1+Q')R' + \Pr(1-\Lambda) = 0 \tag{2.17}$$

$$\left(1-\omega^2\Gamma\right) + (\Gamma+1)\Pr(\Lambda+Q') - R'\Pr(1+Q')\Gamma + \Pr(1-\Lambda) = 0 \tag{2.18}$$

From (2.17) we get

$$R' = \frac{\Pr(\Lambda+Q')\left(1-\omega^2\Gamma\right) - \omega^2(\Gamma+1) + \Pr(1-\Lambda)}{\Pr(1+Q')} \tag{2.19}$$

and

$$R' = \frac{\left(1-\omega^2\Gamma\right) + (\Gamma+1)\Pr(\Lambda+Q') + \Pr(1-\Lambda)}{\Pr(1+Q')\Gamma} \tag{2.20}$$

If we use (2.19) and (2.20) we obtain

$$\omega^2 = \frac{\Pr(1-\Lambda)(\Gamma-1) - \Pr(\Lambda+Q')}{\Gamma^2(\Pr(\Lambda+Q')+1)} \tag{2.21}$$

If we use $Q' = 0$ in (2.21), then

$$\omega^2 = \frac{\Pr(1-\Lambda)(\Gamma-1) - \Pr\Lambda}{\Gamma^2(\Pr\Lambda+1)} \tag{2.22}$$

From (2.21),

$$\omega^2\Gamma = \frac{\Pr(1-\Lambda)(\Gamma-1) - \Pr(\Lambda+Q') + \Gamma(\Pr\Lambda+1) - \Gamma(\Pr\Lambda+1)}{\Gamma(\Pr(\Lambda+Q')+1)} \tag{2.23}$$

$$1-\omega^2\Gamma = \frac{\Pr(\Lambda+Q') + \Gamma(\Pr\Lambda+1) - \Pr(1-\Lambda)(\Gamma-1)}{\Gamma(\Pr\Lambda+1)} \tag{2.24}$$

Substituting in equation (2.20), we get

$$\frac{(\Pr^2\Gamma^2\Lambda^2 + ((2\Gamma+\Gamma^2)\Pr + (2\Gamma^2 Q + \Gamma Q + \Gamma)\Pr^2)\Lambda + (\Gamma Q^2 + \Gamma^2 Q^2 + \Gamma Q)\Pr^2 + \Gamma + (Q + 2\Gamma.Q + 1 + \Gamma^2 Q)\Pr)}{(\Gamma^2(\Pr\Lambda + \Pr Q + 1)\Pr(1+Q)^2)}. \tag{2.25}$$

Alternately differentiate (2.20) wrt “Λ to get

$$\frac{\partial R'}{\partial \Lambda} = \left\{ \frac{\partial}{\partial \Lambda} \left(1 - \omega^2 \Gamma\right) + \Gamma \Pr \right\} \frac{1}{\Pr (1 + Q') \Gamma} \tag{2.26}$$

Now consider

$$\frac{\partial}{\partial \Lambda} \left(1 - \omega^2 \Gamma\right) = \frac{\Pr \Gamma}{\Gamma^2 (\Pr \Lambda + 1)^2} \left\{ \Gamma (\Pr + 1) - \Pr (Q' + 1) \right\}$$

$\frac{\partial}{\partial \Lambda} \left(1 - \omega^2 \Gamma\right) > 0$ provided $\Gamma (\Pr + 1) - \Pr (Q' + 1) > 0$
i.e. $Q' < \frac{\Gamma(\Pr+1) - \Pr}{\Pr}$
i.e. $Q' + 1 < \frac{\Gamma(\Pr+1)}{\Pr}$
ie.. $Q' < \frac{\Gamma(\Pr+1)}{\Pr} - 1$
Since $Q' > 0$ we have the condition
$0 < Q' < \frac{\Gamma(\Pr+1)}{\Pr} - 1$ and $\frac{\Gamma(\Pr+1)}{\Pr} > 1$
i.e. $\Gamma > \frac{\Pr}{1+\Pr}$
We may thus conclude that $\frac{\partial R'}{\partial \Lambda} > 0$ provided $\Gamma > \frac{\Pr}{1+\Pr}$ and $Q' < \frac{\Gamma(\Pr+1)}{\Pr} - 1$
i.e. $Q < \delta^2 \left[\frac{\delta^2 \Lambda_1 (\Pr+1)}{\Pr} - 1 \right]$ and $\Lambda_1 > \frac{\Pr}{\delta^2 (\Pr+1)}$

3. Results and discussion

Single relaxation time and double relaxation time viscoelastic liquids are physically realisable and are practically important in many nonisothermal applications (see Bird et al., [3], and Joseph [7]). It is the elasticity in these liquids that renders them attractive for investigation. An applied magnetic field is known to induce in an elastic effect in an electrically conducting liquid. It is therefore interesting to consider the interplay between the natural elasticity and the elasticity imposed by a magnetic field. As mentioned in the introduction to this paper, the onset of convection in two-relaxation-time viscoelastic liquids is studied here and is likely to yield interesting results.

Classical linear stability analysis reveals that the following results are true:

$R_c^{Rivlin-Ericksen} = R_c^{Newtonian} > R_c^{Oldroyd-B} > R_c^{Maxwell}$ (for Q = 0.10)
$a_c^{Rivlin-Ericksen} = a_c^{Newtonian} < a_c^{Oldroyd-B} < a_c^{Maxwell}$ (for Q = 0.10)
$\omega_c^{Oldroyd-B} < \omega_c^{Maxwell}$ (for Q=0,10)
$R_{oc}|_{Q>0} > R_{oc}|_{Q=0}$ (true for all the 4 liquids considered),
$a_c|_{Q>0} > a_c|_{Q=0}$ (true for Newtonian, Rivlin-Ericksen, and Oldroyd-B liquids),
$a_c|_{Q>0} < a_c|_{Q=0}$ (true for Maxwell liquid),
$\omega_c|_{Q>0} < \omega_c|_{Q=0}$ (true for Oldroyd-B and Maxwell liquids).

Figure 2 illustrates the stationary convection curves in the (R_c, k_c^2) plane for different values of the Chandrasekhar number (Q), while keeping other parameters constant. The results indicate that as the Chan-

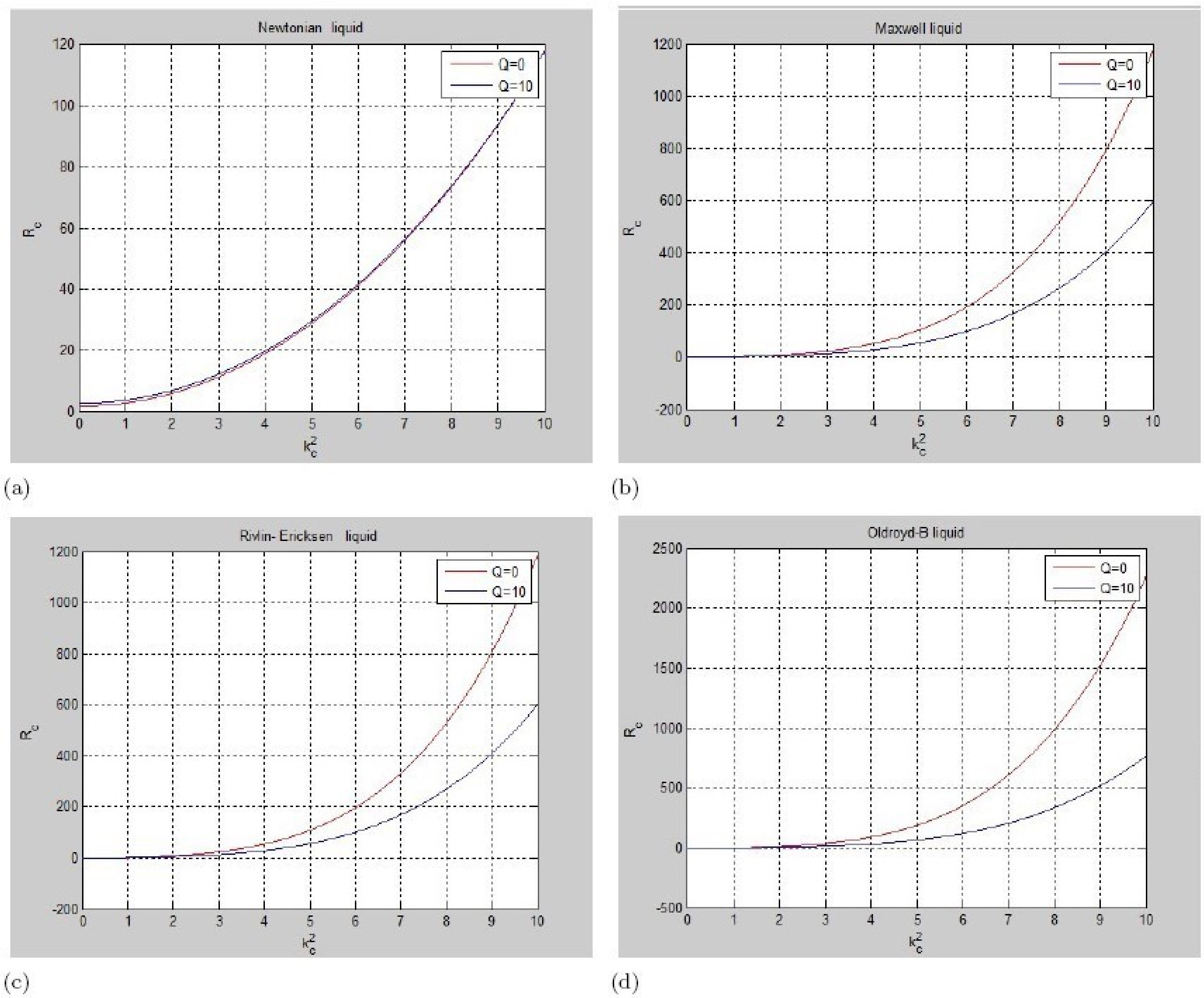

Figure 2: R_c *versus* k_c^2 with $\mathrm{Q} = 0$ and $\mathrm{Q} = 10$ for, (a) Newtonien liquid , (b) Maxwell liquid (c) Rivlin-Rricsen liquid (d) Oldroyd-B liquid

drasekhar number (Q) increases, so does the Rayleigh number (R_c). This suggests that the magnetic field has a stabilising effect on the liquid layer.

Also in Figure 2 (a) it can be observed that the graphical values for $Q = 0$ and $Q = 10$ are quite similar for the Newtonian fluid, which gives results for smaller values of R_c. For the same values, in Figure 2 (b) and in Figure 2 (c), Maxwell and Rivlin-Ericksen fluids give similar results. On the other hand, in Figure 2 (d), Oldroyd-B fluid gives different results from the above flows for $Q = 0$ and $Q = 10$. It can also be seen that R_c reaches higher values.

The table 1 lists the fluid models considered in this study. The results obtained from Maxwell, Rivlin-Ericksen, Walters' Liquid B, and Newtonian fluids form limiting cases within this general study. These cases represent different fluid behaviours, including fluids with predominantly viscous behaviour with Maxwell, fluids with both viscous and elastic behaviour with Rivlin-Ericksen, fluids with time-dependent and stress-dependent behaviour with Walters' Liquid B, and fluids with constant viscosity with Newtonian fluids. The following table provides information on Q, Λ, and Λ_1 values compared to literature data.

Table 1: Various models covered by the present study.

Q	$\Lambda_1 \in [0,1)$	$\Lambda \in [0,1]$	Model
0	$\neq 0$	$\neq 0$	Oldroyd-B liquid (Siddheshwar et al., [15])
0	=0	$\neq 0$	Rivlin-Ericksen liquid (Siddheshwar and Srikrishna [17], Siddheshwar et al., [15])
0	$\neq 0$	=0	Maxwell liquid (Siddheshwar et al., [15])
0	=0	=0	Newtonian liquid (Lorenz [8], Saltzman [10])
> 0	$\neq 0$	$\neq 0$	magnetoconvection in Oldroyd-B liquid
> 0	=0	$\neq 0$	magnetoconvection in Rivlin-Ericksen liquid
> 0	$\neq 0$	=0	magnetoconvection in Maxwell liquid
> 0	=0	=0	magnetoconvection in Newtonian liquid

4. Conclusion:

a) The effect of the magnetic field on the Rayleigh-Bénard system is to increase the critical Rayleigh number.

b) The magnetic field has a rheostatic effect on convection in viscoelastic liquids and can therefore be used as an external means of controlling convection.

c) The results of Maxwell, Rivlin-Ericksen, Walters' Liquid B, and Newtonian liquids are limiting cases of the present general study.

It can be concluded that the stability transition principle is similar for Maxwell fluid and Rivlin-Ericksen liquid for Q = 0 and Q = 10. In section 3, the obtained results are utilized to examine the behavior of viscoelastic liquids concerning the critical Rayleigh number as it varies with the critical wave number. Finally, we note that our analysis was carried out on liquids, which is very important for the comparison of the available studies in the literature.

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