Turkish Journal of Electrical Engineering and Computer Sciences

Volume 21
Number 7 *Sup. 1*

Article 6

1-1-2013

Improving power systems operation through multiobjective optimal location of optimal unified power flow controller

AFSHIN LASHKAR ARA

AHAD KAZEMI

MOHAMMAD BEHSHAD

Follow this and additional works at: https://journals.tubitak.gov.tr/elektrik

Part of the Computer Engineering Commons, Computer Sciences Commons, and the Electrical and Computer Engineering Commons

Recommended Citation

ARA, AFSHIN LASHKAR; KAZEMI, AHAD; and BEHSHAD, MOHAMMAD (2013) "Improving power systems operation through multiobjective optimal location of optimal unified power flow controller," *Turkish Journal of Electrical Engineering and Computer Sciences*: Vol. 21: No. 7, Article 6. https://doi.org/10.3906/elk-1203-40

Available at: https://journals.tubitak.gov.tr/elektrik/vol21/iss7/6

This Article is brought to you for free and open access by TÜBİTAK Academic Journals. It has been accepted for inclusion in Turkish Journal of Electrical Engineering and Computer Sciences by an authorized editor of TÜBİTAK Academic Journals. For more information, please contact academic.publications@tubitak.gov.tr.

Turkish Journal of Electrical Engineering & Computer Sciences
http://journals.tubitak.gov.tr/elektrik/
Research Article
Turk J Elec Eng & Comp Sci
(2013) 21: 1893 – 1908
© TÜBİTAK
doi:10.3906/elk-1203-40

Improving power systems operation through multiobjective optimal location of optimal unified power flow controller

Afshin LASHKAR ARA,[1,*] **Ahad KAZEMI,**[2] **Mohammad BEHSHAD**[1]
[1]Department of Electrical Engineering, Islamic Azad University, Dezful Branch, Dezful, Iran
[2]Department of Electrical Engineering, Iran University of Science and Technology, Narmak, Tehran, Iran

Received: 09.03.2012 • **Accepted:** 08.06.2012 • **Published Online:** 24.10.2013 • **Printed:** 18.11.2013

Abstract: This paper presents a multiobjective optimization methodology to find the optimal location for an optimal unified power flow controller (OUPFC) device. Moreover, the installation cost function of the OUPFC device is developed. The objective functions are the total fuel cost, power losses, and system loadability with and without the minimum cost of the OUPFC installation. The ε-constraint approach is implemented for the multiobjective mathematical programming formulation. The proposed algorithm is implemented in the IEEE 30- and 118-bus test systems. The solution procedure uses nonlinear programming to optimally locate the OUPFC incorporated in an optimal power flow problem considering these objective functions and improves the operation of the power system. The optimization problem is modeled in the general algebraic modeling system software using a MINOS solver. Furthermore, the results obtained by the OUPFC are compared to those of the phase shifting transformer (PST) and unified power flow controller (UPFC) devices. The performance and applicability of the OUPFC is highlighted compared to the PST and UPFC from an analytical and technical point of view.

Key words: Phase shifting transformer, optimal unified power flow controller, unified power flow controller, flexible AC transmission systems, optimal location, multiobjective, optimal power flow

1. Introduction

It is well documented that the planning and the optimum operation of an electrical system involve developing evolution scenarios of the electrical energy requirement such as installation and implementation of flexible AC transmission systems (FACTS) controllers [1]. In addition, during more than 99% of the operating time, the focus of the control system is on loss minimization and loop flow control relative to neighboring networks, which can be done through the installation of controllable devices in the transmission system such as FACTS devices [2]. Many investigations have been carried out on these devices and various types of equipment have been presented. The unified power flow controller (UPFC), its concept proposed by Gyugyi in 1991 [3], is one of the most brilliant of these devices. The UPFC can simultaneously control and regulate all 3 of the variables of the transmission line power flow, i.e. the line impedance, voltage, and phase angle [3–7]. Another device used to control and transfer the power through certain paths is the phase shifting transformer (PST). The ability of the PST to control the transmission line power flow has long been recognized in a power system [8–10], and its models and operational characteristics are well established [4,5,11–13]. The other most significant device is the optimal unified power flow controller (OUPFC). A steady-state model of the OUPFC and its operational

*Correspondence: lashkarara@iust.ac.ir

characteristics were introduced in [14]. The OUPFC combines a conventional PST and a UPFC as a more cost-effective device in comparison with a standalone UPFC due to the expensiveness of the high-power UPFC [14].

The optimal choice and the allocation of FACTS devices are very important since the installation of FACTS devices in any power system is an investment issue. This needs an offline simulation of the power system with different candidate FACTS device locations to assess the system operation improvement. Among the different assessment tools used for this purpose, optimal power flow (OPF) seems to be the most suitable [15]. In the OPF problem, a specific objective function to address the operator concerns is optimized by adjusting the control variables of the power system, while a set of operational and physical constraints is satisfied [16,17]. The optimal location and the OPF of FACTS devices can be formulated as a multiobjective optimization problem. It can offer additional opportunities for operational improvement through the integration of economic and technical objectives.

Recently, several multiobjective methods, as listed below, have been investigated for solving the multiobjective optimal location problem of FACTS devices. The multiobjective evolutionary algorithm (EA) has been applied to the optimal location and parameters of the UPFC [18] and thyristor-controlled phase-shifting transformer [19] in order to minimize real power loss and to improve the voltage profile. In [20], the particle swarm optimization (PSO) technique was used to find the optimal location of the thyristor-controlled series capacitor (TCSC), static VAR compensator (SVC), and UPFC with a minimum cost of installation and to improve the system loadability. The fuzzy multiobjective optimal location of the TCSC, SVC, and static synchronous compensator (STATCOM) based on fuzzy decision making and the genetic algorithm (GA) was explained in [21]. The multiobjective optimal placement of the TCSC and SVC has been investigated using PSO [22–24], GA [24,25], simulated annealing [24,26], EA [19], and evolutionary programming [27]. In [28], the reactive power planning with SVC and STATCOM devices was studied using bacterial foraging (BF) oriented by the PSO algorithm (BF-PSO). It was found that much attention has been paid to the heuristic algorithm to solve the multiobjective optimal location of FACTS devices.

The multiobjective mathematical programming (MMP) methods, such as generation methods, despite having significant advantages, are less popular due to their computational complexity and the lack of widely available software [29]. Considering our present knowledge, no research work in this area has considered the MMP methods for optimal location of OUPFC devices. The main contribution of this paper is to propose the ε-constraint method to solve the multiobjective optimal location of OUPFC devices using the general algebraic modeling system (GAMS), as well as to introduce the installation cost function of the OUPFC device. In addition, the results obtained by the OUPFC device are compared to those of the PST and UPFC. For this application, MATLAB feeds the parameters to the GAMS routine, and the optimization model is solved using GAMS as nonlinear programming (NLP). The NLP optimization problem is modeled in the GAMS software using the MINOS solver [30]. In order to select the 'best' compromised solution among the Pareto-optimal solutions of the multiobjective optimization problem, a fuzzy decision-making tool is adopted. Furthermore, the power injection models of the FACTS devices are investigated in the IEEE 30- and 118-bus test systems and the best location for the FACTS is selected to optimize the total fuel cost, power losses, system loadability, and cost of FACTS installation as objective functions while satisfying the power system constraints.

The paper is organized as follows. Section 2 describes the modeling of the FACTS devices. In Section 3, the problem formulation including the OPF with the FACTS and the optimal location is explained. Section 4 presents the MMP. Section 5 contains the simulation results followed by the conclusions.

2. Modeling of FACTS devices

2.1. Modeling of the PST

The schematic diagram of a PST is shown in Figure 1, which is used to control and transfer the power through certain paths in an interconnected power system. The PST proves to be a valuable means of control, although the technology is relatively old.

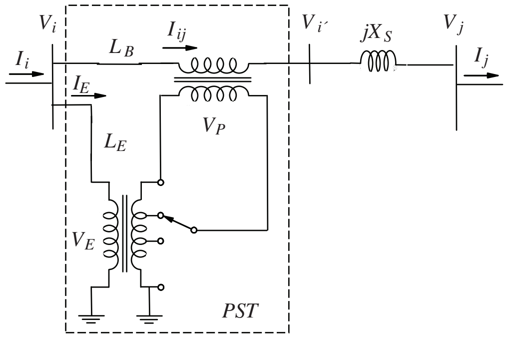

Figure 1. Per-phase schematic diagram of the PST.

The power injection model of the PST is shown in Figure 2, where:

$$P_{ss} = -b_s k V_i V_j \sin(\theta_i - \theta_j + \sigma), \tag{1}$$

$$Q_{ss} = -b_s k V_i^2 (k + 2\cos(\sigma)) + b_s k V_i V_j \cos(\theta_i - \theta_j + \sigma), \tag{2}$$

$$P_{sr} = -P_{ss}, \tag{3}$$

$$Q_{sr} = b_s k V_i V_j \cos(\theta_i - \theta_j + \sigma). \tag{4}$$

Here, k is the transfer ratio of the PST; and σ is the PST phase angle; b_s is $1/(X_S + X_B)$, where X_S is the transmission line reactance and X_B is the series transformer leakage reactance. In this study, the transfer ratio of the PST, k, is equal to $\tan(\sigma)$ as a quadrature booster.

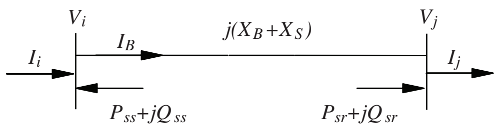

Figure 2. The power injection model of the FACTS devices.

2.2. Modeling of the UPFC

The basic schematic of the UPFC is shown in Figure 3. The power injection model of the UPFC is the same as the PST in Figure 2, where:

$$P_{ss} = -b_s r V_i V_j \sin(\theta_i - \theta_j + \gamma), \tag{5}$$

$$Q_{ss} = -b_s r V_i^2 (r + 2\cos(\gamma)) + b_s r V_i V_j \cos(\theta_i - \theta_j + \gamma), \tag{6}$$

$$P_{sr} = -P_{ss}, \tag{7}$$

$$Q_{sr} = +b_s r V_i V_j \cos(\theta_i - \theta_j + \gamma). \tag{8}$$

Here, r is the radius of the UPFC operating region and γ is the UPFC phase angle [7].

2.3. Modeling of the OUPFC [14]

The OUPFC comprises a PST and a UPFC, as shown in Figure 4.

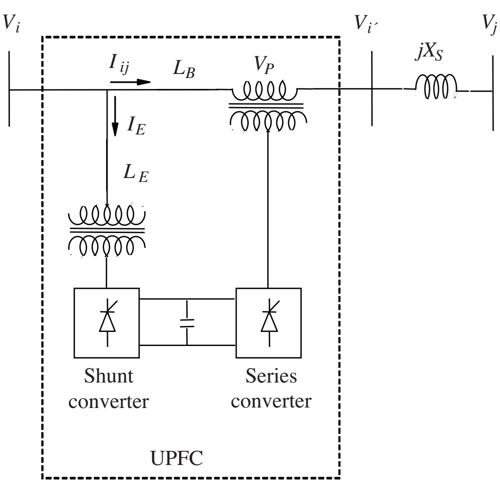

Figure 3. Basic schematic diagram of the UPFC.

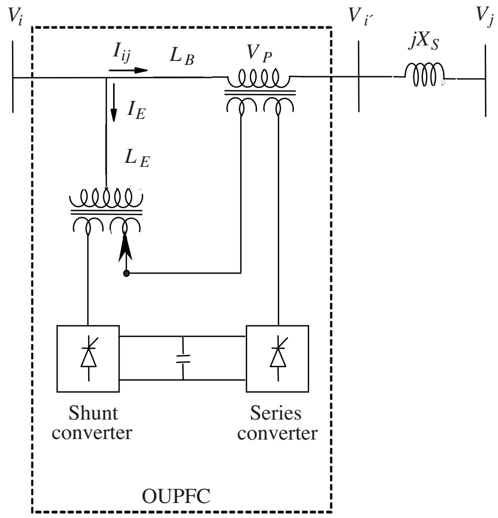

Figure 4. Per-phase schematic diagram of the OUPFC.

The power injection model in Figure 2 can also be used for the OUPFC model, where:

$$P_{ss} = -b_s k V_i V_j \sin(\theta_i - \theta_j + \sigma) - b_s r V_i V_j \sin(\theta_i - \theta_j + \rho), \tag{9}$$

$$\begin{aligned} Q_{ss} = & -b_s V_i^2 (k^2 + r^2) - 2b_s k r V_i^2 \cos(\sigma - \rho) - 2b_s k V_i^2 \cos(\sigma) - 2b_s r V_i^2 \cos(\rho) \\ & + b_s k V_i V_j \cos(\theta_i - \theta_j + \sigma) + b_s r V_i V_j \cos(\theta_i - \theta_j + \rho) \end{aligned}, \tag{10}$$

$$P_{sr} = -P_{ss}, \tag{11}$$

$$Q_{sr} = b_s k V_i V_j \cos(\theta_i - \theta_j + \sigma) + b_s r V_i V_j \cos(\theta_i - \theta_j + \rho). \tag{12}$$

Here, k is the transfer ratio of the PST, σ is the PST phase angle, r is the radius of the UPFC operating region, and ρ is the UPFC phase angle.

3. Problem formulation

The operator of the power system seeks to optimize the steady state operating conditions of the power system in terms of one or more objective functions while satisfying several equality and inequality constraints. Generally, the problem can be formulated as follows [1,15].

3.1. Objective functions

In this paper, 4 objective functions are considered, which are the total fuel cost, the power loss, the system loadability, and the cost of FACTS installation. These objective functions are formulated as follows.

3.1.1. The total fuel cost

The first objective function is to minimize the total fuel cost, which can be expressed as:

$$F_1 = \sum_{i=1}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^2 (\$/h), \tag{13}$$

where P_{Gi} is the active power output of the ith generator; NG is the total number of generators; and a_i, b_i, and c_i are the fuel cost coefficients of the ith generator [1].

3.1.2. The real power losses

The second objective function is to minimize the real power losses in the transmission lines, which can be defined as:

$$F_2 = \sum_{l=1}^{N_l} P_l = \sum_{i=1}^{n}\sum_{j=1}^{n} |Y_{ij}||V_i||V_j| \cos(\angle Y_{ij} + \angle V_j - \angle V_i), \tag{14}$$

where P_l is the real power losses at line l and N_l is the number of transmission lines [1].

3.1.3. The system loadability

The third objective function is to maximize the system loadability, which can be described as [1,20]:

$$F_3 = \rho(x, u), \tag{15}$$

and ρ can be obtained by assuming the constant power factor at each load in both the real and reactive power balance equations, as follows:

$$P_G - \rho P_D = f_p(x, u), \tag{16}$$

$$Q_G - \rho Q_D = f_q(x, u), \tag{17}$$

where P_G and Q_G are the vectors of the generator's real and reactive power, respectively; P_D and Q_D are the vectors of loads' real and reactive power, respectively; and f_p and f_q are the vectors of the real and reactive power flow functions, respectively.

3.1.4. The cost of FACTS installation

The fourth objective function is to minimize the cost of FACTS installation, which can be mathematically formulated and is given by the following equation:

$$F_4 = \frac{C_{FACTS}}{8760 \times 5} (\$/h), \tag{18}$$

where C_{FACTS} is the cost of the FACTS installation in US\$. Based on the ABB and Siemens database, the cost functions for the PST, UPFC, and OUPFC are developed as [20,31,32]:

$$C_{PST} = 12 \times S_{PST} \times 1000,$$

$$C_{UPFC} = (0.0003S_{UPFC}^2 - 0.2691S_{UPFC} + 188.22) \times S_{UPFC} \times 1000, \tag{19}$$

$$C_{OUPFC} = [(12 \times S_{PST}) + ((0.0003S_{UPFC}^2 - 0.2691S_{UPFC} + 188.22) \times S_{UPFC})] \times 1000,$$

where S_{FACTS} is the operating range of the FACTS devices in MVA. In this paper, a 5-year period is applied to evaluate the cost function since the FACTS devices will be in service for many years.

3.2. Constraints

The OPF problem has 2 sets of constraints, including equality and inequality constraints. These constraints can be expressed in the following compact form [1,15]:

$$g(x, u) = 0, \tag{20}$$

$$h(x, u) \leq 0 \tag{21}$$

where u and x are the set of control and dependent variables, respectively. The control variables include the active power and voltage magnitude of the generator buses, voltage angle and magnitude of the swing bus, and FACTS control parameters, while the dependent variables include the active and reactive power of the swing bus, voltage angle and reactive power of generator buses, and voltage angle and magnitude of load buses; $g(x, u)$ and $h(x, u)$ are the set of equality and inequality constraints, respectively, which is explained in detail in the next 2 subsections.

3.2.1. Equality constraints

The equality constraints of the OPF problem, which comprise the real and reactive power balance equations, can be written as:

$$P_{Gi} - P_{Di} - f_{Pi}(x, u) = 0, \tag{22}$$

$$Q_{Gi} - Q_{Di} - f_{Qi}(x, u) = 0, \tag{23}$$

where f_{Pi} and f_{Qi} are the real and reactive power flow functions at bus i, respectively, where the FACTS device parameters are considered; P_{Gi} and Q_{Gi} are the generator real and reactive power at bus i, respectively; and P_{Di} and Q_{Di} are the load real and reactive power at bus i, respectively.

3.2.2. Inequality constraints

The inequality operation constraints in the OPF problem include:

Generation constraints: These constraints are composed of the generator voltages, real power outputs, and reactive power outputs, and are restricted by their lower and upper limits as follows:

$$V_{Gi}^{\min} \leq V_{Gi} \leq V_{Gi}^{\max}, \ i = 1, ..., NG, \tag{24}$$

$$P_{Gi}^{\min} \leq P_{Gi} \leq P_{Gi}^{\max}, \ i = 1, ..., NG, \tag{25}$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max}, \ i = 1, ..., NG. \tag{26}$$

Security constraints: These are the voltages at the load buses and transmission line loadings, as follows:

$$V_{Li}^{\min} \leq V_{Li} \leq V_{Li}^{\max}, \ i = 1, ..., Nd, \tag{27}$$

$$S_{Li} \leq S_{Li}^{\max} \qquad , \ i = 1, ..., Nl, \tag{28}$$

where *Nd* and *Nl* are the number of load buses and transmission lines, respectively.

FACTS constraints: The FACTS settings are bounded as follows:

$$\sigma^{\min} \leq \sigma \leq \sigma^{\max} \text{ for PST}, \tag{29}$$

$$\left.\begin{array}{l} r^{\min} \le r \le r^{\max} \\ \rho^{\min} \le \rho \le \rho^{\max} \\ \sigma^{\min} \le \sigma \le \sigma^{\max} \end{array}\right\} \text{ for OUPFC,} \tag{30}$$

$$\left.\begin{array}{l} r^{\min} \le r \le r^{\max} \\ \gamma^{\min} \le \gamma \le \gamma^{\max} \end{array}\right\} \text{ for UPFC.} \tag{31}$$

3.3. Problem statement

In general, when aggregating the objectives and constraints, the problem can be mathematically formulated as a nonlinear constrained multiobjective optimization problem as follows:

$$\begin{gathered} \text{minimize } F_1, \\ \text{minimize } F_2, \\ \text{maximize } F_3, (32) \\ \text{minimize } F_4, \\ \text{subject to: } g(x,u) = 0, \\ h(x,u) \le 0. \end{gathered} \tag{32}$$

The multiobjective optimization problem can be converted into a single objective optimization and then solved using the ε-constraint method given in the next section.

4. Multiobjective mathematical programming

For solving MMP problems that have more than one objective function, a well-organized method based on the ε-constraint strategy is proposed. In this section, the concept of optimality is used instead of the Pareto optimality (or efficient, nondominated, noninferior) solution. For optimal solution, the ‘most preferred’ solution is then considered. The mentioned solution is not a single optimal solution and cannot improve an objective function without deteriorating its performance. Furthermore, there is no optimal solution for the optimization of several objective functions at the same time. For the ‘most preferred’ solution, both the fuzzy decision-making tool and the ε-constraint methods are deduced as follows.

4.1. The ε-constraint method

Since the various strategies are used as constraints, the ε-constraint method has been considered for the optimization of one objective function [33]:

$$\min F_{1,k}(x), \tag{33}$$

$$\text{subject to } F_{2,k}(x) \le e_{2,k} F_{3,k}(x) \le e_{3,k} \ldots F_{p,k}(x) \le e_{p,k},$$

where the number of transmission lines including the FACTS device and the number of objective functions are indicated by k and p, respectively. For the range of every objective function that is calculated from the payoff table, at least the$p$-1 objective function is considered. The payoff table, which is constructed by the calculation of the individual optima of the objective functions, is applied in order to manage this method [33]. The optimum value of the ith objective function, which is placed by the FACTS device on the kth line, is indicated by $F^*_{i,k}$ and the values of the other objective functions are denoted by $F_1^{i,k}$, ..., $F_{i-1}^{i,k}$, $F_{i+1}^{i,k}$, ..., $F_p^{i,k}$. By computing

$F^i_{1,k}$, ..., $F^i_{i-1,k}$, $F^*_{i,k}$, $F^i_{i+1,k}$, ..., $F^i_{p,k}$, the ith row of the payoff table is carried out. Hence, all of the rows of the payoff table are calculated. For obtaining the total (q_j+ 1) grid points for the jth objective function, its values from the jth column of the payoff table are applied. The range of the jth objective function is obtained among its minimum and maximum values. Moreover, the deviation of the jth objective function to q_j equal intervals using (q_j − 1) intermediate equidistant grid points gives (q_2+ 1) × (q_3+ 1) × ... × (q_p+ 1), which are the total number of optimizations for placing the FACTS device on each line. As the number of grid points to the denser representation is high, the values of q_i can help to control the density of the Pareto optimal set representation. On the other hand, the cost of the higher computation time should be noted. In this paper, 4 intervals for the objective functions is the best selection. Moreover, a tradeoff between the computation time and the Pareto optimal set cannot be ignored [34].

Four objective functions F_1, F_2, F_3, and F_4, described in Eqs. (13), (14), (15), and (18), respectively, are considered for their relation with the power system operation and its MMP problem. Therefore, the optimization subproblems become the following form:

$$\min F_{1,k}(x), \tag{34}$$

$$\text{subject to } F_{2,k}(x) \leq e_{2,i,k} F_{3,k}(x) \geq e_{3,j,k} F_{4,k}(x) \leq e_{4,l,k}$$

$$e_{2,i,k} = \max(F_{2,k}) - \left(\frac{\max(F_{2,k}) - \min(F_{2,k})}{q_2}\right) \times i, \quad i = 0, 1, ..., q_2, \tag{35}$$

$$e_{3,j,k} = \min(F_{3,k}) + \left(\frac{\max(F_{3,k}) - \min(F_{3,k})}{q_3}\right) \times j, \quad j = 0, 1, ..., q_3, \tag{36}$$

$$e_{4,l,k} = \max(F_{4,k}) - \left(\frac{\max(F_{4,k}) - \min(F_{4,k})}{q_4}\right) \times l, \quad l = 0, 1, ..., q_4, \tag{37}$$

where max(.) and min(.) represent the maximum and minimum values of the individual objective function while the FACTS device is installed on the kth line, respectively. NLP is the commonly used solution for optimization subproblems, which is applied by the mentioned constraints of the MMP problem in order to obtain Pareto optimal solutions.

4.2. The fuzzy decision-making tool

After placing a FACTS device on each transmission line of the power system, the Pareto optimal solutions are obtained by solving the optimization subproblems. Thereafter, the decision-maker needs to choose the optimal location for the FACTS device according to the best compromise among the Pareto optimal solutions. In this paper, a fuzzy decision-making approach is proposed for the optimal location process, wherein a linear membership function (μ_i) is defined for each objective function, as follows:

$$\underset{i=1,2,4}{\mu^n_{i,k}} = \begin{cases} 1 & F^n_{i,k} \leq \min(F_i) \\ \frac{\max(F_i) - F^n_{i,k}}{\max(F_i) - \min(F_i)} & \min(F_i) \leq F^n_{i,k} \leq \max(F_i) \\ 0 & F^n_{i,k} \geq \max(F_i) \end{cases} \tag{38}$$

for the minimized objective functions and

$$\underset{i=3}{\mu^n_{i,k}} = \begin{cases} 0 & F^n_{i,k} \leq \min(F_i) \\ \frac{F^n_{i,k} - \min(F_i)}{\max(F_i) - \min(F_i)} & \min(F_i) \leq F^n_{i,k} \leq \max(F_i) \\ 1 & F^n_{i,k} \geq \max(F_i) \end{cases} \tag{39}$$

for the maximized objective functions, where $F_{i,k}^{n}$ and $\mu_{i,k}^{n}$ are the values of the ith objective function in the nth Pareto optimal solution of the kth transmission line, which includes the FACTS device and its membership function, respectively. The membership functions are used to evaluate the optimality degree of the Pareto optimal solutions. The most preferred degree of the Pareto optimal solutions can be expressed as follows:

$$\mu_{opt} = \max_{k=1}^{Nl} \left\{ \sup_{n \in k} \frac{\sum_{i=1}^{p} w_i \cdot \mu_{i,k}^{n}}{\sum_{n=1}^{M} \sum_{i=1}^{p} w_i \cdot \mu_{i,k}^{n}} \right\}, \tag{40}$$

where

$$w_i \geq 0, \; \sum_{i=1}^{p} w_i = 1. \tag{41}$$

Here w_i is the weight value assigned to the ith objective function and M is the number of Pareto optimal solutions in each transmission line, which includes the FACTS device. By giving a relatively large value to w_i, it is possible to favor F_i over other objective functions. The weight values w_i can be selected by the power system dispatcher based on the importance of the economic and technical aspects. Therefore, the optimal location and settings of a FACTS device based on the adopted weight factors are obtained by the proposed algorithm as the best Pareto optimal solution.

5. Case studies

The effectiveness of the proposed approach is investigated on the IEEE 30- and 118-bus test systems. The optimal location of the PST, OUPFC, and UPFC, and their settings to optimize total fuel cost, power losses, system loadability, and cost of the FACTS installation as objective functions, using NLP as the solution procedure, are obtained and discussed below. The simulation studies are performed with MATLAB and GAMS software. The NLP optimization problem is modeled in the GAMS software using a MINOS solver. The GAMS/MINOS algorithm is designed to solve linear and nonlinear programming problems to find solutions that are locally optimal. If the nonlinear objective and constraint functions are convex within a certain region, any optimal solution obtained will be a global optimum [30]. The data on the test systems are taken from [35]. The characteristics of the FACTS devices are given in the Appendix.

5.1. IEEE 30-bus test system

In order to study the effect of the FACTS location and its settings on the indices of the power system operation, the performance of the PST, OUPFC, and UPFC are investigated on the IEEE 30-bus system. The results of single objective and multiobjective optimization are obtained and compared for the objective functions. Since the PST cost is low compared to that of the UPFC and OUPFC [20,31,32], it is not minimized but its value is computed.

The results of single objective optimization are presented in Table 1. The results obtained with the OUPFC and UPFC are the same in the minimization of power losses without minimization of the FACTS installation cost, although the investment cost of the UPFC is higher than that of the OUPFC. In the case of maximum system loadability, the OUPFC gives the best performance compared to the PST and UPFC. It is noticeable that by maximizing the system loadability without minimizing the cost of UPFC installation, the loadability index of the UPFC is equal to 1.437. After adding the cost of the FACTS device into the total fuel cost, the OUPFC shows a large improvement compared to the PST and UPFC in minimum power losses and minimum total fuel cost as single objective functions. It is observed that the UPFC losses are higher than those of the PST and OUPFC, which is neglected in this study.

Table 1. Results of the single objective optimization in the IEEE 30-bus system.

Objective function	Parameters	Without FACTS	PST (without F_4 minimized)	OUPFC (without F_4 minimized)	OUPFC (with F_4 minimized)	UPFC (with F_4 minimized)
F_1	Total fuel cost ($/h)	802.25	800.54	791.50	793.93	793.97
	$\sum P_{loss}$ (MW)	9.447	8.968	6.696	7.327	7.207
	$\sum Q_{loss}$ (MVAr)	37.789	36.019	27.587	30.161	29.498
	Loadability index	1	1.00	1.00	1.00	1.00
	Investment cost ($/h)	-	14.98	82.00	47.49	192.34
	FACTS size (MVA)	-	54.56	94.86	52.96	47.86
	FACTS location	-	Line 2-5	Line 1-3	Line 1-3	Line 2-5
	FACTS settings	-	$\sigma = 6.221$	$\sigma = 2.775$ $r = 0.15$ $\rho = 95.544$	$\sigma = -5.446$ $r = 0.410$ $\rho = 36.391$	$r = 0.097$ $\gamma = 86.302$
F_2	$\sum P_{loss}$ (MW)	3.291	3.040	2.031	2.179	2.332
	Total fuel cost ($/h)	968.12	967.52	965.12	965.47	965.83
	$\sum Q_{loss}$ (MVAr)	16.245	15.535	11.668	12.271	12.827
	Loadability index	1	1.00	1.00	1.00	1.00
	Investment cost ($/h)	-	14.54	52.48	36.28	132.27
	FACTS size (MVA)	-	53.10	69.38	41.63	32.38
	FACTS location	-	Line 2-5	Line 2-5	Line 2-5	Line 2-5
	FACTS settings	-	$\sigma = 6.032$	$\sigma = 4.352$ $r = 0.122$ $\rho = 99.616$	$\sigma = -7.656$ $r = 0.15$ $\rho = 25.512$	$r = 0.064$ $\gamma = 87.934$
F_3	Loadability index	1.402	1.446	1.454	1.442	1.436
	Total fuel cost ($/h)	1319.402	1367.72	1377.01	1363.44	1355.01
	$\sum P_{loss}$ (MW)	12.532	14.328	14.842	14.125	13.555
	$\sum Q_{loss}$ (MVAr)	51.846	59.029	61.066	58.168	56.507
	Investment cost ($/h)	-	2.62	4.16	2.67	13.27
	FACTS size (MVA)	-	9.57	13.25	7.95	3.10
	FACTS location	-	Line 24-25	Line 24-25	Line 24-25	Line 27-30
	FACTS settings	-	$\sigma = 2.562$	$\sigma = 4.131$ $r = 0.01$ $\rho = 180.00$	$\sigma = 1.362$ $r = 0.015$ $\rho = 40.267$	$r = 0.026$ $\gamma = 68.793$

In order to study the conflict among the objective functions, the multiobjective optimization is performed in 4 cases. In case 1, the total fuel cost and the power losses are minimized. The total fuel cost and the system loadability are optimized in case 2. Case 3 is planned to optimize the power losses and the system loadability. The total fuel cost, the power losses, and the system loadability are optimized in case 4. All of the cases are carried out with and without the minimum cost of FACTS installation as the objective function. In all cases, the same weight values are assigned to the objective functions. According to the adopted weight factors, the most preferred compromise solution is selected among the Pareto optimal solutions using the fuzzy decision-making process in each case. Therefore, the best Pareto optimal location and settings of the FACTS devices are obtained. The results of the multiobjective optimization are shown in Table 2. Some interesting observations that can be derived from Table 2 are as follows.

In case 1, the optimal location of the FACTS device is obtained on line 2-5. It is observed that the OUPFC achieves the best performance in comparison to that without FACTS devices. The total fuel cost and loadability index obtained with the OUPFC are lower than that of the UPFC in case 2; therefore, operating the

OUPFC is similar to operation of the UPFC because maximizing the system loadability increases the total fuel cost. Since maximizing the system loadability increases the power losses, the results obtained with the OUPFC and UPFC show nearly the same performance as in case 3. In case 4, the UPFC effectiveness is better than that of the OUPFC due to the lower power losses and higher loadability index of the UPFC with respect to the OUPFC. Furthermore, the optimal location of the UPFC is on line 2-5, while the optimal location of the OUPFC is on line 1-3 in case 4.

Table 2. Results of the multiobjective optimization in the IEEE 30-bus system.

Case	Parameters	Without FACTS	PST (without F_4 minimized)	OUPFC (without F_4 minimized)	OUPFC (with F_4 minimized)	UPFC (with F_4 minimized)
Case 1 (F_1 & F_2)	Total fuel cost ($/h)	839.03	868.35	818.71	819.68	845.63
	$\sum P_{loss}$ (MW)	5.448	4.223	3.602	3.602	2.897
	$\sum Q_{loss}$ (MVAr)	24.050	19.372	16.545	16.591	14.250
	Loadability index	1.00	1.00	1.00	1.00	1.00
	Investment cost ($/h)	-	14.50	67.67	61.69	294.15
	FACTS size (MVA)	-	52.94	78.08	72.81	76.02
	FACTS location	-	Line 2-5	Line 2-5	Line 2-5	Line 2-5
	FACTS settings	-	$\sigma = 5.993$	$\sigma = -2.626$ r = 0.15 $\rho = 84.725$	$\sigma = 0.995$ r = 0.144 $\rho = 95.640$	r = 0.150 $\gamma = 89.225$
Case 2 (F_1 & F_3)	Total fuel cost ($/h)	1042.53	901.81	954.44	954.44	966.47
	Loadability index	1.221	1.094	1.142	1.142	1.154
	$\sum P_{loss}$ (MW)	12.933	10.982	11.825	11.825	11.855
	$\sum Q_{loss}$ (MVAr)	52.914	44.003	47.419	47.419	47.681
	Investment cost ($/h)	-	33.99	10.01	10.01	83.28
	FACTS size (MVA)	-	8.00	2.33	2.33	19.94
	FACTS location	-	Line 6-8	Line 24-25	Line 24-25	Line 4-6
	FACTS settings	-	$\sigma = -0.232$	$\sigma = -1.902$ r = 0.04 $\rho = -16.597$	$\sigma = -1.902$ r = 0.04 $\rho = -16.597$	r = 0.010 $\gamma = 92.880$
Case 3 (F_2 & F_3)	$\sum P_{loss}$ (MW)	6.789	6.211	7.190	4.152	7.412
	Loadability index	1.200	1.170	1.229	1.166	1.240
	Total fuel cost ($/h)	1126.16	1100.26	1149.77	1091.12	1160.17
	$\sum Q_{loss}$ (MVAr)	30.039	27.374	31.075	19.700	32.661
	Investment cost ($/h)	-	0.09	27.71	35.82	34.04
	FACTS size (MVA)	-	0.32	37.63	51.32	8.97
	FACTS location	-	Line 24-25	Line 6-9	Line 2-5	Line 27-30
	FACTS settings	-	$\sigma = 0.081$	$\sigma = -6.442$ r = 0.117 $\rho = 11.316$	$\sigma = 2.304$ r = 0.106 $\rho = 109.887$	r = 0.069 $\gamma = 78.230$
Case 4 (F_1 & F_2 & F_3)	Total fuel cost ($/h)	996.12	1137.69	966.77	972.44	1083.77
	$\sum P_{loss}$ (MW)	8.822	7.403	8.356	8.354	6.162
	Loadability index	1.160	1.227	1.166	1.165	1.228
	$\sum Q_{loss}$ (MVAr)	36.867	31.902	34.382	34.705	27.058
	Investment cost ($/h)	-	11.62	83.49	54.74	235.25
	FACTS size (MVA)	-	42.43	96.61	63.01	59.47
	FACTS location	-	Line 2-5	Line 1-3	Line 1-3	Line 2-5
	FACTS settings	-	$\sigma = 4.827$	$\sigma = 3.447$ r = 0.15 $\rho = 96.882$	$\sigma = -5.364$ r = 0.15 $\rho = 38.376$	r = 0.119 $\gamma = 86.778$

5.2. IEEE 118-bus test system

Simulations are carried out for the optimal location of the PST, OUPFC, and UPFC devices in the IEEE 118-bus system to highlight the performance of the proposed algorithm. In order to compare the results obtained by the single objective and multiobjective optimization problems, the effects of the FACTS device settings and placement on the objective functions in terms of the operation planning are investigated in the current study.

The results of the single objective optimization are shown in Table 3. After adding the cost of the FACTS device into the total fuel cost, the performance of the OUPFC is highlighted compared to those of the PST and UPFC in minimization of the total fuel cost with minimization of the FACTS installation cost simultaneously. After placing FACTS devices on line 69-75, the results obtained with the OUPFC are similar to that of the UPFC for system loadability and cost of FACTS installation objective functions, although the investment cost of the OUPFC is lower than that of the UPFC. In the case of power losses minimization with minimization of the FACTS installation cost, the UPFC delivers the best efficiency in comparison with the PST and OUPFC. The power losses of the UPFC are equal to 7.801 in minimization of power losses without minimization of UPFC installation cost.

Table 3. Results of the single objective optimization in the IEEE 118-bus system.

Objective function	Parameters	Without FACTS	PST (without F_4 minimized)	OUPFC (without F_4 minimized)	OUPFC (with F_4 minimized)	UPFC (with F_4 minimized)
F_1	Total fuel cost ($/h)	129,660.997	129,467.56	129,378.15	129,195.52	129,194.73
	$\sum P_{loss}$ (MW)	77.407	73.222	71.075	67.395	67.367
	$\sum Q_{loss}$ (MVAr)	507.2500	479.4020	465.114	455.322	455.513
	Loadability index	1.00	1.00	1.00	1.00	1.00
	Investment cost ($/h)	-	54.79	170.13	86.38	375.12
	FACTS size (MVA)	-	200	200	100	100
	FACTS location	-	Line 25-27	Line 25-27	Line 25-27	Line 25-27
	FACTS settings	-	$\sigma = 17.542$	$\sigma = 17.739$ $r = 0.15$ $\rho = 123.921$	$\sigma = 3.345$ $r = 0.15$ $\rho = 96.678$	$r = 0.158$ $\gamma = 80.445$
F_2	$\sum P_{loss}$ (MW)	9.2476	8.891	7.938	8.040	7.860
	Total fuel cost ($/h)	166,390.383	165,950.28	166,482.15	166,456.01	166,506.41
	$\sum Q_{loss}$ (MVAr)	69.3829	68.066	64.580	64.978	64.327
	Loadability index	1.00	1.00	1.00	1.00	1.00
	Investment cost ($/h)	-	51.94	87.39	66.40	405.11
	FACTS size (MVA)	-	189.57	101.19	80.95	109.27
	FACTS location	-	Line 89-90	Line 80-96	Line 80-96	Line 80-96
	FACTS settings	-	$\sigma = -19.4036$	$\sigma = 6.287$ $r = 0.15$ $\rho = 102.499$	$\sigma = 4.736$ $r = 0.138$ $\rho = 109.338$	$r = 0.191$ $\gamma = 75.496$
F_3	Loadability index	2.0385	2.283	2.284	2.280	2.280
	Total fuel cost ($/h)	347,709.059	417,113.21	417,113.21	409,551.59	409,551.60
	$\sum P_{loss}$ (MW)	205.1381	278.702	275.767	278.719	278.719
	$\sum Q_{loss}$ (MVAr)	1165.5570	1553.053	1543.378	1543.90	1543.907
	Investment cost ($/h)	-	54.79	170.13	38.23	375.12
	FACTS size (MVA)	-	200	200	100	100
	FACTS location	-	Line 69-75	Line 69-75	Line 69-75	Line 69-75
	FACTS settings	-	$\sigma = 14.345$	$\sigma = 13.922$ $r = 0.15$ $\rho = 117.066$	$\sigma = 8.671$ $r = .0271$ $\rho = 147.58$	$r = 0.1335$ $\gamma = 16.338$

Similar to cases of the previous subsection, the multiobjective optimization problem is performed in 4 cases. In each case, the best Pareto optimal location and settings for the FACTS devices are selected among the Pareto optimal solutions based on the adopted weight factors in the fuzzy decision-making process. The results of the multiobjective optimization are represented in Table 4, where some remarkable observations can be made, as follows.

Table 4. Results of the multiobjective optimization in the IEEE 118-bus system.

Case	Parameters	Without FACTS	PST (without F_4 minimized)	OUPFC (without F_4 minimized)	OUPFC (with F_4 minimized)	UPFC (with F_4 minimized)
Case 1 (F_1 & F_2)	Total fuel cost ($/h)	134,197.355	135,478.78	135,845.21	135,845.96	131,916.37
	$\sum P_{loss}$ (MW)	29.2948	22.0424	21.519	21.519	31.478
	$\sum Q_{loss}$ (MVAr)	204.617	162.913	164.088	164.112	234.482
	Loadability index	1.00	1.00	1.00s	1.00	1.00
	Investment cost ($/h)	-	54.79	89.73	87.01	495.60
	FACTS size (MVA)	-	200	103.95	100.75	138.54
	FACTS location	-	Line 25-27	Line 80-96	Line 80-96	Line 89-90
	FACTS settings	-	σ = 17.628	σ = 6.906 r = 0.15 ρ = 103.713	σ = 6.246 r = 0.015 ρ = 102.419	r = 0.139 γ = 87.692
Case 2 (F_1 & F_3)	Total fuel cost ($/h)	276,425.255	259,036.43	198,927.93	301,499.87	198,885.91
	Loadability index	1.8308	1.7373	1.404	1.963	1.403
	$\sum P_{loss}$ (MW)	146.2734	126.4205	82.391	173.224	81.513
	$\sum Q_{loss}$ (MVAr)	854.5147	754.3217	542.715	1011.382	538.949
	Investment cost ($/h)	-	19.55	149.66	146.58	668.46
	FACTS size (MVA)	-	71.34	175.25	171.58	200.00
	FACTS location	-	Line 77-80	Line 69-75	Line 69-75	Line 69-75
	FACTS settings	-	σ =–4.4414	σ = 10.867 r = 0.15 ρ = 111.356	σ = 10.708 r = 0.15 ρ = 111.056	r = 0.253 γ = 75.732
Case 3 (F_2 & F_3)	$\sum P_{loss}$ (MW)	82.4453	69.3179	59.905	117.008	60.608
	Loadability index	1.796	1.7299	1.706	1.963	1.706
	Total fuel cost ($/h)	300,656.625	294,909.09	190,244.04	331,384.59	290,373.58
	$\sum Q_{loss}$ (MVAr)	457.1453	404.8569	363.381	679.59	365.321
	Investment cost ($/h)	-	7.63	170.13	77.61	471.42
	FACTS size (MVA)	-	27.85	200	105.25	130.52
	FACTS location	-	Line 88-89	Line 69-75	Line 69-75	Line 69-75
	FACTS settings	-	σ =–1.1588	σ = 10.287 r = 0.15 ρ = 110.013	σ = 5.066 r = 0.117 ρ = 101.860	r = 0.167 γ = 65.946
Case 4 (F_1 & F_2 & F_3)	Total fuel cost ($/h)	201,216.679	253,202.12	200,904.02	200,953.53	200,993.91
	$\sum P_{loss}$ (MW)	87.8151	118.5713	80.912	82.064	82.741
	Loadability index	1.415	1.7060	1.415	1.415	1.415
	$\sum Q_{loss}$ (MVAr)	566.3427	718.5768	521.715	528.86	531.802
	Investment cost ($/h)	-	44.23	170.13	119.11	550.32
	FACTS size (MVA)	-	161.45	200	138.72	171.21
	FACTS location	-	Line 69-75	Line 25-27	Line 25-27	Line 26-30
	FACTS settings	-	σ = 11.510	σ = 17.758 r = 0.15 ρ = 123.955	σ = 10.580 r = 0.15 ρ = 110.811	r = 0.138 γ = 85.917

In case 1, after placing the OUPFC on line 80-96, the related objective functions are minimized and the reactive power losses are reduced compared to those of the UPFC device. Although the total fuel cost of the UPFC on line 89-90 is lower than that of the OUPFC, its power losses are higher than those of the OUPFC. In cases 2–4, the results obtained with the OUPFC and UPFC achieve nearly the same performance. In addition, the system loadability objective function of the OUPFC is higher than that of the UPFC, whereby its total fuel cost is increased in cases 2 and 3.

6. Conclusions

FACTS devices can relieve congestion and improve the efficiency of an existing network. The exact type is determined by the dispatcher requirement from an operational planning point of view. It is important that the operation planning be provided by efficient optimization strategies. The operational planning optimization with the aims of the real power losses, the total fuel cost, the system loadability, and the cost of the FACTS installation, accompanied by the optimal location of the FACTS and optimal power flow problems, was dealt with in this paper. The single objective and multiobjective optimization problems were performed on the IEEE 30- and 118-bus test systems with the PST, OUPFC, and UPFC as 3 viable options for the efficiency improvement of the optimization. The MMP was developed to involve the objective functions using the ε-constraint method for generating the Pareto-optimal solutions. The fuzzy decision-making approach is proposed to obtain the best Pareto-optimal location and settings of the FACTS devices among the Pareto-optimal solutions. The results show the capability of the proposed algorithm to optimally locate the FACTS devices. In addition, the results illustrate that the OUPFC can be applied for the best satisfaction of the dispatcher requirement based on the technical and economic aspects.

Appendix

The data of the PST, OUPFC, and UPFC are tabulated in Table 5.

Table 5. Data of the PST, OUPFC, and UPFC.

Data	PST	OUPFC	UPFC
Range of r	-	$0 \le r \le 0.15$	$0 \le r \le 1$
Range of phase angles	$-20^\circ \le \sigma \le 20^\circ$	$-20^\circ \le \sigma \le 20^\circ$ $-\pi \le \rho \le \pi$	$-\pi \le \gamma \le \pi$
X_B (p.u.)	0.007	0.007	0.007
X_E (p.u.)	0.001	0.001	0.001
S_{base} (MVA)	100	100	100

Acknowledgments

This work was supported by the Research Fund of the Islamic Azad University, Dezful Branch, under Project Number 10149.

References

[1] A. Lashkar Ara, A. Kazemi, S.A. Nabavi Niaki, "Optimal location of hybrid flow controller considering modified steady-state model", Applied Energy, Vol. 88, pp. 1578–1585, 2011.

[2] R. Grunbaum, P. Lundberg, G. Stromberg, B. Berggren, "Congestion relief: FACTS – the key to congestion relief", ABB Review, Vol. 2, pp. 28–32, 2007.

[3] L. Gyugyi, "Unified power-flow control concept for flexible AC transmission systems", IEE Proceedings - Generation Transmission and Distribution, Vol. 139, pp. 323–331, 1992.

[4] Y.H. Song, A.T. Johns, Flexible AC Transmission Systems (FACTS), London, IEE Press, 1999.

[5] N.G. Hingorani, L. Gyugyi, Understanding FACTS: Concepts and Technology of Flexible AC Transmission Systems, New York, Wiley IEEE Press, 2000.

[6] S.A. Nabavi Niaki, M.R. Iravani, "Steady-state and dynamic modeling of unified power flow controller (UPFC) for power system studies", IEEE Transactions on Power Systems, Vol. 11, pp. 1937–1943, 1996.

[7] M. Noroozian, L. Angquist, M. Ghandhari, G. Andersson, "Use of UPFC for optimal power flow control", IEEE Transactions on Power Systems, Vol. 12, pp. 1629–1634, 1997.

[8] W.J. Lyman, "Controlling power flow with phase-shifting equipment", Transactions of the American Institute of Electrical Engineers, Vol. 49, pp. 825–831, 1930.

[9] Z. Han, "Phase-shifter and power flow control", IEEE Transactions on Power Systems, Vol. PAS 101, pp. 3790–3795, 1982.

[10] M. Noroozian, G. Anderson, "Power flow control by use of controllable series components", IEEE Transactions on Power Delivery, Vol. 8, pp. 1420–1429, 1993.

[11] M.R. Iravani, D. Maratukulam, "Review of semiconductor-controlled (static) phase shifters for power system applications", IEEE Transactions on Power Systems, Vol. 9, pp. 1833–1839, 1994.

[12] M.R. Iravani, P.L. Dandeno, D. Maratukulam, K.H. Nguyen, D. Zhu, "Applications of static phase shifters in power systems", IEEE Transactions on Power Delivery, Vol. 9, pp. 1600–1608, 1994.

[13] S.A. Nabavi Niaki, "A novel steady-state model and principles of operation of phase-shifting transformer comparable with FACTS new devices", Proceedings of the IEEE International Conference on Power System Technology, Vol. 3, pp. 1450–1457, 2002.

[14] A. Lashkar Ara, A. Kazemi, S.A. Nabavi Niaki, "Modelling of optimal unified power flow controller (OUPFC) for optimal steady-state performance of power systems", Energy Conversion and Management, Vol. 52, pp. 1325–1333, 2011.

[15] M.R. Alrashidi, M.E. El-Hawary, "Application of computational intelligence techniques for solving the revived optimal power flow problem", Electric Power Systems Research, Vol. 79, pp. 694–702, 2009.

[16] H.W. Dommel, W.F. Tinney, "Optimal power flow solutions", IEEE Transactions on Power Apparatus and Systems, Vol. PAS 87, pp. 1866–1876, 1968.

[17] J. Peschon, D.W. Bree Jr, L.P. Hajdu, "Optimal power-flow solutions for power system planning", Proceedings of the IEEE, Vol. 60, pp. 64–70, 1972.

[18] I. Marouani, T. Guesmi, H.H. Abdallah, A. Ouali, "Application of a multiobjective evolutionary algorithm for optimal location and parameters of FACTS devices considering the real power loss in transmission lines and voltage deviation buses", Proceedings of the International Multi-Conference on Systems, Signals and Devices, pp. 1–6, 2009.

[19] Q.H. Wu, Z. Lu, M.S. Li, T.Y. Ji, "Optimal placement of FACTS devices by a group search optimizer with multiple producer", Proceedings of the IEEE Congress on Evolutionary Computation, pp. 1033–1039, 2008.

[20] M. Saravanan, S.M.R. Slochanal, P. Venkatesh, J.P.S. Abraham, "Application of particle swarm optimization technique for optimal location of FACTS devices considering cost of installation and system loadability", Electric Power Systems Research, Vol. 77, pp. 276–283, 2007.

[21] A. Deihimi, H. Javaheri, "A fuzzy multi-objective multi-case genetic-based optimization for allocation of FACTS devices to improve system static security, power loss and transmission line voltage profiles", International Review of Electrical Engineering, Vol. 5, pp. 1616–1626, 2010.

[22] R. Benabid, M. Boudour, M.A. Abido, "Optimal placement of FACTS devices for multiobjective voltage stability problem", Proceedings of the IEEE PES Power Systems Conference & Exhibition, pp. 1–11, 2009.

[23] A. Laifa, M. Boudour, "FACTS allocation for power systems voltage stability enhancement using MOPSO", Proceedings of the International Multi-Conference on Systems, Signals and Devices, pp. 1–6, 2008.

[24] K.Y. Lee, M.M. Farsangi, H. Nezamabadi-pour, "Hybrid of analytical and heuristic techniques for FACTS devices in transmission systems", Proceedings of the IEEE Power Engineering Society General Meeting, pp. 1–8, 2007.

[25] D. Radu, Y. Besanger, "A multi-objective genetic algorithm approach to optimal allocation of multi-type FACTS devices for power systems security", Proceedings of the IEEE Power Engineering Society General Meeting, pp. 1–8, 2006.

[26] M. Gitizadeh, M. Kalantar, "A novel approach for optimum allocation of FACTS devices using multi-objective function", Energy Conversion and Management, Vol. 50, pp. 682–690, 2009.

[27] W. Ongsakul, P. Jirapong, "Optimal allocation of FACTS devices to enhance total transfer capability using evolutionary programming", Proceedings of the IEEE International Symposium on Circuits and Systems, Vol. 5, pp. 4175–4178, 2005.

[28] R.A. Hooshmand, M. Ezatabadi Pour, "Corrective action planning considering FACTS allocation and optimal load shedding using bacterial foraging oriented by particle swarm optimization algorithm", Turkish Journal of Electrical Engineering & Computer Sciences, Vol. 18, pp. 597–612, 2010.

[29] G. Mavrotas, Generation of efficient solutions in multiobjective mathematical programming problems using GAMS, effective implementation of the ε-constraint method (online). Available at: http://www.gams.com/modlib/adddocs/epscm.pdf.

[30] General Algebraic Modeling System (GAMS), 2010. Available at: http://www.gams.com.

[31] L.J. Cai, I. Erlich, "Optimal choice and allocation of FACTS devices using genetic algorithm", Proceedings on 12th Intelligent Systems Application to Power Systems Conference, pp. 1–6, 2003.

[32] D. Bonmann, Technical information on the ABB phase shifting transformer, 2001. Available at: dietrich.bonmann@de.abb.com.

[33] J.L. Cohon, Multiobjective Programming and Planning, New York, Academic Press, 1978.

[34] G. Mavrotas, "Effective implementation of the ε-constraint method in multi-objective mathematical programming problems", Applied Mathematics and Computation, Vol. 213, pp. 455–465, 2009.

[35] Power Systems Test Case Archive, 2008. Available at: http://www.ee.washington.edu/research/pstca.